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Optimization of Battery Charging and Purchasing at Electric Vehicle Battery Swap Stations

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An operator of a network of battery swap stations for electric vehicles must make a long term investment decision on the number of batteries and charging bays in the system and periodic short term decisions on when and how many batteries to recharge. Both decisions must be made concurrently, because there exists a trade-off between the long term investment in batteries and charging bays, and short term expenses for operating the system. Costs for electric energy as well as demand rates for batteries are stochastic and we consider an infinite time horizon for operation of the system. We derive a complex optimization problem, which cannot be solved optimally in a reasonable time for real world instances. In various small test cases we show fundamental effects of the different parameters using an optimal solution algorithm. We then develop a near-optimal solution heuristic based on Monte-Carlo sampling for the infinite horizon dynamic program. We show that operating battery swap stations in a network where lateral transshipments are allowed can substantially decrease expected operating costs.

Key words: Battery Swapping, Approximate Dynamic Programming, Lateral Transshipments, Stochastic Optimization

History:

1. Introduction

Increasing emissions of green house gases and oil supply risks on the rise have intensified the search for alternative power-train technologies (World Energy Council 2007). Because CO_2 emissions induced by the transportation sector grow fastest among all energy consuming sectors (U.S.

Environmental Protection Agency 2006), a key to a greener environment is seen in the development of alternative fuel vehicles. Especially electric vehicles (EVs) have lately attracted increasing attention to cut down green house gas emissions and to limit the automotive industry's impact on the environment.

Compared to vehicles powered by internal combustion engines, EVs do not have tailpipe (local) emissions and thus lower total well to wheel emissions, even with the current mix of energy sources in power generation systems of more developed countries (MIT Electric Vehicle Team 2008). Growing shares of renewable energy sources in energy generation would even further increase this benefit. The batteries of EVs could be used by the power grid as a storage helping to bridge phases of low availability of intermittent energy sources in power generation (Nationale Plattform Elektromobilität 2013).

However, adoption of EVs is still slow, because there are a number of challenges to overcome before EVs could reach significant shares in car sales. Three fundamental problems limiting the success of EVs are their low range on a single charge, long charging times, and high battery prices. Current battery technology admits EVs to travel distances of 150 to 250 kilometers between charges (Boston Consulting Group 2010, Hensley et al. 2012) and the fear to run out of energy before reaching the destination, also referred to as range anxiety, still discourages many potential customers from purchasing an EV. Additionally, recharging a battery with 30 kWh at a standard 110-volt outlet takes more than 18 hours. This time reduces considerably when charging at specialized fast charging systems, but high costs are incurred for installation of outlets with increased power. Another factor limiting the success of EVs is that the cost of batteries is still high (around USD 500 per kWh) and expected to remain above USD 250 per kWh of energy storage capacity (Boston Consulting Group 2010) in the near future.

Because major breakthroughs in battery technology are not expected any time soon (Boston Consulting Group 2010), other approaches to cope with the problems of limited range and long charging times were developed.

One concept to handle the problem is to establish a system of battery swap stations at which discharged batteries are replaced by charged ones. A car that requires battery swapping drives onto a fully automated station, where the depleted battery is removed from the EV and replaced by a fully charged battery. The removed battery is then recharged in specialized charging bays at the station. This system promises to swap a battery in less than three minutes and solves the problem of limited range; of course only if a swap station is en route (e.g., Mak et al. 2013, Avci et al. 2013). Typically, batteries are not owned by car owners, but by the service provider.

Two companies have started to pursue the concept of battery swapping. While Better Place has already installed swap networks in Israel and Denmark (Better Place 2013), EV manufacturer

Tesla has only recently announced plans to install swap stations exclusively for their customers (Tesla Motors 2013).

Battery swapping is an approach that relies on building a pervasive infrastructure and can only be profitable if the infrastructure is provided at low cost. There are a number of issues, which may be disadvantageous if not addressed properly. The most obvious is that battery swapping requires more batteries than cars in a swapping network, and thus - because the cost of batteries is high - it is crucial to determine the optimal number of batteries in circulation. Besides costs, the optimal number of batteries is influenced by many factors such as the total demand rate, charging time and capacity, and energy price volatility. Determining the optimal solution requires solving a complex stochastic optimization problem. Another important factor determining infrastructure and operating costs is the charging capacity, i.e., the number of batteries that can be recharged concurrently at a station. The energy required to charge an EV battery is high and battery charging puts a heavy strain on the power distribution network. As a result, possible reinforcements on the power distribution network must also be accounted for, which makes the charging capacity a major cost factor.

In this paper, we analyze the characteristics of battery swap stations and determine the optimal number of batteries and charging bays. We first focus on a model with a single swap station and assess how demand parameters, volatility of energy prices, and equipment costs affect the optimal battery and charging capacity configuration. There exists a trade-off between equipment costs for batteries and charging bays and the long term operating costs of a station. For example, a low number of batteries at a station reduces investment expenses, but a station operator cannot exploit varying energy prices when making a decision on when to recharge batteries without compromising the desired service level. Another aspect to consider is that more batteries in stock can replace additional charging bays and vice versa, because the risk of stocking out can be reduced either way. We model the problem as a cyclic dynamic program and solve the infinite horizon problem optimally.

We then extend our model to a network of swap stations in which the single station models are building blocks that are interlinked to perform lateral transshipments of batteries. We quantify the network effect on the optimal configuration of batteries and charging capacity in the system. Again, we formulate a dynamic program for the swap network model. Because the problem is very large, we solve it by an algorithm based on approximate dynamic programming (ADP), which combines Monte-Carlo simulation and mathematical programming.

Our contribution is threefold. First, we present a dynamic programming formulation of a battery swap station. Unlike previous research, we allow energy prices and demand for batteries to be stochastic and non-stationary and charging capacity at the station to be finite. Second, we show that the dynamic program is in general not coordinatewise convex, and we identify parts of the state space on which coordinatewise convexity is present. Third, based on our analyses we develop an algorithm that solves the single station model optimally and an approximation that produces empirically observed near-optimal solutions for the network model. Our algorithms concurrently determine the number of batteries and charging bays for each station along with the operating policy for the station. In the network model, the algorithm additionally makes decisions on transshipment of batteries between stations.

We also contribute three important managerial implications. Based on an extensive numerical study using public real-world data, we identify the main parameters influencing the optimal number of batteries and charging bays at a swap stations with and how these numbers are interrelated. We show that at the current cost of batteries and energy price fluctuations, it is not profitable to exploit arbitrage of energy prices. Our results also indicate that substantial savings in infrastructure costs are possible, if transshipments of batteries between stations is allowed. Finally, we show that it might be beneficial for station operators to use incentives to level demand for batteries over the day, because demand volatility is a key driver of infrastructure costs.

This paper is organized as follows. In Section 2, we review the literature that is related to our problem. In Section 3, we present our model for a single battery swap station and prove a number of properties and derive a lower bound. In Section 4, we extend the single station model to a network of swap stations and derive several properties. In Section 5, we present our solution approach for both models. In Section 6, we discuss our findings from a numerical study, which is based on actual data for energy prices, demand profiles and infrastructure costs. We conclude the paper in Section 7. Notation is summarized in the appendix.

2. Literature Review

In this section we review literature related to our work. Our model shares many characteristics with classical inventory management models, especially with research on closed-loop supply chains with repairable items. We first review literature on closed-loop supply chains and then literature on extensions of these models to account for transshipment in inventory networks. We conclude by reviewing the emerging body of literature specifically pertaining to battery swap stations.

The problem of managing a battery swap station is similar to a closed-loop inventory system in which failed items (depleted batteries) are returned and replaced by functioning ones (charged batteries). The returned items are then repaired (recharged) and absorbed as stock. A large body of literature on this topic exists (see, e.g., Muckstadt 2005 and Nahmias 1981 for an overview). Most work is based on the METRIC system of Sherbrooke (1968) and makes use of the implicit assumption of infinite repair capacity (Graves 1985). As a result, optimization only involves the level of spares and not the repair capacity and no trade-off between inventories and capacity must be made. There exist extensions of these models to capacitated systems that are modeled in continuous time as queues or in discrete time as periodic review models. Queuing models typically assume single-server queues (Gross and Harris 1971), exponential repair times (Gross et al. 1983), or only approximate the steady state probability distribution (Díaz and Fu 1997). In general, costs for the repair of items and demand rates are assumed to be stationary, while our paper specifically takes non-stationary energy costs and demands into account to quantify the effect of their volatility on the optimal solution. Periodic review models assume per-period capacity limits (e.g., Glasserman 1997, Glasserman and Tayur 1994, and Roundy and Muckstadt 2000). However in our problem charging capacity is not limited per period, but limits the total number of batteries that can be recharged concurrently.

A second stream of literature related to our work is concerned with transshipments in an inventory network. In our extended network model, we allow for lateral shipments of batteries between swap stations. The literature distinguishes between models for repairable items and those pertaining to consumable items. Both types of models are either queuing models assuming stationary independent demands and ample repair capacity (Axsäter 1990 and Lee 1987) or periodic review models that assume instantaneous transshipments (Muckstadt 2005). All models considering lateral shipments in the context of repairable items assume a multi-echelon structure with a central repair depot replenishing the decentral facilities at which demand for replacement occurs. In our model, each base runs its own charging bay to recharge batteries. Similar to models with repairable items, consumable item models with transshipments have been proposed as continuous time or periodic review models. Periodic review models have been solved optimally for zero lead and transshipment times for two stations (Archibald et al. 1997) and approximately in other settings (see, e.g., Jönsson and Silver 1987, Minner et al. 2003, and Tagaras 1999). For recent advances in inventory models with lateral transshipments the reader is referred to the overview by Paterson et al. (2011).

In summary, none of the models found in the literature incorporates all characteristics of the problem considered in this paper, especially capacitated decentral charging facilities, non-zero charging and transshipment times, and non-stationary cost and demand parameters in an infinite time horizon setting. Additionally, none of the models allows for concurrent optimization of the number of batteries and the charging capacity in the system.

Because it is still a very new concept, research on managing operations of electric vehicles is scarce. Kaschub et al. (2012) show feasibility of battery swapping for urban public transport in a case study, but no optimization is performed. To the best of our knowledge, Mak et al. (2013) and Avci et al. (2013) are the only papers that study a battery swap station model. The former focuses on developing a model that aids planning the locations of stations in a battery swap network. Avci et al. (2013) consider the battery management problem to assess the effectiveness of a swapping network in reducing carbon emissions and oil dependence. Both papers apply a repairable item inventory model such as the METRIC model in the context of a battery swap station and assume stationary demand rates and energy prices along with ample charging capacity at the swap station. In many applications, costs for electric energy and demand rates are highly volatile with strong periodicity. Additionally, assuming ample capacity at swapping stations is not always reasonable, because due to the strain induced on power grids by battery charging costly upgrades to transmission capacities in local power distribution networks might have to be performed. We relax both, the assumption of stationary parameters and infinite charging capacity, because they lead to an underestimation of the optimal battery inventory (see, e.g., Gross 1982 and Díaz and Fu 1997) and an inaccurate estimate of expected operating costs of a station in the long run.

Our paper addresses both issues by developing a dynamic programming model for battery swap stations. We are the first to address the problem of managing and concurrently determining the optimal number of batteries and charging capacities at a battery swap station and performing a numerical analysis based on public real-world data. We solve small instances of the dynamic program optimally and approximate larger instances with an algorithm based on the ideas of ADP. For an overview on ADP, the reader is referred to Powell (2007), Bertsekas and Tsitsiklis (1996), Sutton and Barto (1998), and Powell and Van Roy (2004); for recent applications of ADP to inventory management and resource allocation problems see, e.g., Topaloglu and Kunnumkal (2006), Choi et al. (2006), and Van Roy et al. (1997).

3. Single Station

We begin by introducing our model of a single battery swap station, which is a building block for the multi-station model with transshipments introduced subsequently. We next describe the model, a number of properties, a lower bound to the optimal solution, and the structure of the optimal solution.

3.1. Model

We consider a battery swap station providing charged batteries for arriving customers. Our goal is to determine the optimal number of charging bays, total number of batteries required in the system, and operating policy at a station.

We consider a fixed planning horizon, e.g., a week, that is divided into $T+1$ time periods, e.g., into $T+1=168$ hours. The planning horizon is cycled through infinitely many times as we want to assess infinite horizon operating costs. We allow probability distributions for random variables and cost parameters to be non-stationary over time to model the fact that electricity prices and demand for batteries depend on time of the day and day of the week.

In our model, batteries require L time periods to be fully recharged, i.e., all batteries require the same charging time. This is a mild assumption if the majority of returned batteries has a state of charge close to zero. The reason lies in the physical characteristics of recharging lithium-ion batteries, because states of charge show a sharp increase in initial charging periods, followed by a saturation phase. As a result, total charging time is approximately equal for returned batteries with different states of charge in terms of time periods, if the period length is sufficiently large compared to recharging times (e.g., Mak et al. 2013 and Chen 2007).

The sequence of events in each time period is as follows. At the beginning of period t, batteries that have been charged for L periods become available and backlogged demand for batteries is filled from on-hand inventory. We assume that demand that cannot be satisfied from on-hand inventory is backlogged, because drivers most likely will have to wait for the next available battery at the station and cannot keep driving. Next, after current prices for electric energy E_t are revealed, the station operator decides how many batteries x_t to start charging. The station has finite charging capacity of K and a number of uncharged batteries available, which constrain the charging decision. Then, demand D_t is realized and filled from on-hand inventory. Demand that cannot be satisfied is backlogged. At the end of the period, holding and penalty costs of batteries are charged.

We model demand as a discrete non-negative random variable that can be arbitrarily distributed. Demand in our model follows a cyclic distribution, i.e., to follow the same distribution each time period t is visited, but we assume demand to be independent across periods $t=0,\dots,T-1$.

Since we study the infinite time horizon setting, in what follows $t=0,1,\dots$. We denote the number of uncharged batteries at the station by U_t and the number of charged batteries that become available in n periods by $W_{t,n}$. The current number of batteries on hand $W_{t,0}$ may also take negative values in case of backlogged demand.

The number of uncharged batteries at the beginning of period t after on-hand inventory was assigned to satisfy backlogged demand depends on the on-hand inventory, the charging decision, the number of batteries that became available and the demand for batteries in period $t-1$. If demand exceeds on-hand inventory, then at most $[W_{t-1,0}]^+ + W_{t-1,1}$ empty batteries can be returned (we assume that empty batteries can be returned only, if a recharged battery is available for replacement). On the other hand, if there are enough batteries to satisfy demand, then for each demanded charged battery an empty one is returned. The transition equation for empty batteries is given by

$$U_t = U_{t-1} - x_{t-1} + \min\left([-W_{t-1,0}]^+ + D_{t-1}, [W_{t-1,0}]^+ + W_{t-1,1}\right).$$

The number of charged batteries on hand at the beginning of period t depends on the number of charged batteries on hand and the demand in the previous period, and also on the number of batteries that finished charging, i.e., $W_{t,0} = W_{t-1,0} - D_{t-1} + W_{t-1,1}$. The transition for the charging batteries is given by $W_{t,n} = W_{t-1,n+1}$ for $n < L$ and $W_{t,L} = x_{t-1}$.

The total number of batteries at a station is constant during its operation, because empty batteries can only be dropped off at the station when a fully charged battery is available as replacement. We denote the total number of batteries at the station by N. It is determined by

$$N(W_t, U_t) = [W_{t,0}]^+ + \sum_{n=1}^{L} W_{t,n} + U_t. \tag{1}$$

In each period, penalty costs for backlogged demand and costs for electric energy to charge batteries are incurred. We denote the per unit backlogging cost by p_t. Unlike previous research (Mak et al. 2013, Avci et al. 2013), we assume that customers are provided with a fully charged battery, if available, and paid a compensation of p_t per period for waiting for a full battery otherwise. Charging costs depend on the current price for electric energy E_t and we model the price as a mean-reverting process (e.g., Kim and Powell 2011, Eydeland and Wolyniec 2003) with cyclic mean and noise variance (e.g., Lucia and Schwartz 2002). We define $E_{t+1} - E_t = (\mu_{t+1} - \mu_t) + \kappa(\mu_t - E_t) + \zeta_t$, where the noise term ζ_t is i.i.d. with distribution $\mathcal{N}(0, \sigma_t^2)$. We only consider cost in our model, because contracts between the customer and operator of the swap station are typically designed to work with either fixed monthly or per swap charges (Tesla Motors 2013, Better Place 2013). In both cases revenue, as well as demand for batteries, is an exogenous parameter in our model that depends only on the number of contracted customers of the operator.

The cost incurred in one period is given by

$$C_t(W_t, x_t, E_t, D_t) = e_t \left(\beta_0 x_t + \sum_{n=1}^{L} \beta_n W_{t,n} \right) + p_t \left[D_t - W_{t,0} \right]^+,$$

where β_n corresponds to the amount of energy in kWh per battery in period n of the charging process.

The decision on how many batteries to start charging is constrained by the number of available uncharged batteries, i.e., $0 \leq x_t \leq U_t$, and by the number of available charging bays at the station K, i.e., $0 \leq x_t \leq K - \sum_{n=1}^{L} W_{t,n}$. Let $S_t = (W_t, U_t, E_t)$ be the state of the station at the beginning of time period t, i.e., before actions are taken and random information is revealed, and let E_t represent the energy price in the previous time period. We denote the set of feasible charging decisions, given the current state of the system and the charging capacity by $\mathcal{X}(S_t, K) = \{x \in \mathbb{Z} : x \geq 0, x \leq U_t, x \leq K - \sum_{n=1}^{L} W_{t,n}\}$.

We do not consider holding costs for batteries, because the capital cost for energy bound in the batteries is negligible compared to penalty and energy costs. However, for each battery and for each charging bays at the station an initial investment of C^N and C^K is necessary. In these investments, we include the expected infinite horizon maintenance or replacement costs for batteries and charging bays, and therefore do not include it into C_t. Our goal is to find the cost minimizing total number of batteries and charging bays at the station.

Without loss of generality, we assume that in $t=0$ all available batteries are charged, i.e., $W_0=[N,\dots,0]$ and $U_0=0$, and denote the initial state including the current price for electric energy E_0 by S_0.

The optimization problem is

$$Z^*=\min_{N\geq 0,K\geq 0}\left\{KC^K+NC^N+\sum_{t=0}^{\infty}\gamma^t\mathbb{E}_{E_t}\left[\min_{x_t\in\mathcal{X}(S_t,K)}\mathbb{E}_{D_t}C_t(W_t,x_t,E_t,D_t)\middle|S_t\right]\right\},\tag{2}$$

where $0\leq\gamma<1$ is a discount factor.

We reformulate the objective function of Problem (2) as a dynamic program and recursively define the optimal value functions for each period of the problem. We denote the state space of the problem by $\mathcal{S}(K)=\mathcal{W}(K)\times\mathcal{U}\times\mathcal{E}$, where $\mathcal{W}(K)=\{W\in\mathbb{Z}^{L+1}:W_n\geq 0,n=1,\dots,L,\sum_{n=1}^{L}W_n\leq K\}$, $\mathcal{U}=\mathbb{Z}_+$, and $\mathcal{E}\subset\mathbb{R}$. We denote the optimal value function of the system at state S, given charging capacity K, in period t by $V_t(S,K)$, which represents the expected infinite horizon cost from period t on, if cost optimal actions are taken in subsequent periods.

The optimality equation for periods $0\leq t<T$ is

$$V_t(S_t,K)=\min_{x_t\in\mathcal{X}(S_t,K)}\left\{\mathbb{E}_{D_t,E_{t+1}}\left[C_t\left(W_t,x_t,E_t,D_t\right)+\gamma V_{\mathrm{mod}(t+1,T)}\left(S_{t+1},K\right)\middle|S_t\right]\right\},\tag{3}$$

where $\mathrm{mod}(t+1,T)=t+1 \bmod T$ and S_{t+1} is defined based on the transition equations for W_t, U_t, and E_t. Note that the expectation in Equation (3) is with respect to D_t and E_{t+1}, because the current price of electricity E_t is known when making the charging decision, while the demand of period t, D_t is a stochastic quantity. It can easily be shown that the value functions in Equation (3), which involve only $V_0,\dots,V_{T-1}$, also solve the infinite time horizon problem in (2), because the input data is periodic. The intuition behind this is to rewrite the optimization problem in a single optimality equation for periods $t=0,\dots,T-1$ and treat the resulting problem as a conventional infinite horizon dynamic program.

Our objective is to compute the cost minimizing number of batteries and charging capacity for a predefined initial state of the system. We define the initial state as $S_0(N)=([N,\dots,0],0,E_0)$. The goal is to solve

$$(N^*,K^*)=\operatorname*{arg\,min}_{N\geq 0,K\geq 0}\left\{KC^K+NC^N+V_0\left(S_0(N),K\right)\right\}.\tag{4}$$

Solving this problem requires calculating the value functions V_t for all $t=0,\dots,T-1$, which - as a byproduct - yields the optimal recharging policy for the station. In the next section we analyze the infinite horizon value function.

3.2. Properties

In this section we analyze the infinite horizon dynamic program for the single station model. We prove a number of properties on which we rely in our solution approach.

A complicating element of the dynamic program is the nonlinear transition function for uncharged batteries. Our first result shows that in general the value function is not coordinatewise convex in S.

PROPERTY 1. *The value function $V_t(S,K)$ is not coordinatewise convex in S on $\mathcal{S}(K)$.*

All proofs are contained in the Appendix.

If $V_t(S,K)$ is not coordinatewise convex in S, it is hard to solve Problem (3) efficiently. Although V_t is not in general coordinatewise convex, we can prove convexity on a subset of the state space that is relevant for optimal solutions. We transform the dynamic program into an equivalent form with a structure that is easier to handle and reformulate $\mathcal{X}(S_t,K)$ as

$$\tilde{\mathcal{X}}(W_t,K,N) = \{x \in \mathbb{Z}_0^+ : x \le N - \sum_{n=0}^{L} W_{t,n}, x \le N - \sum_{n=1}^{L} W_{t,n}, x \le K - \sum_{n=1}^{L} W_{t,n}\}.$$

Using this relation, we can express all quantities in terms of W_t, E_t, K, and N. We define

$$\tilde{V}_t(W_t,E_t,K,N) = \min_{x_t \in \tilde{\mathcal{X}}} \left\{ \mathbb{E}_{D_t,E_{t+1}} \left[C_t\left(W_t,x_t,e_t,D_t\right) + \gamma \tilde{V}_{\mathrm{mod}(t+1,T)}\left(W_{t+1},E_{t+1},K,N\right) \middle| S_t \right] \right\}. \quad (5)$$

The state space of $\tilde{V}_t$ is given by $\tilde{\mathcal{S}}(K,N) = \{(W_t,E_t) | [W_{t,0}]^+ + \sum_{n=1}^{L} W_{t,n} \le N, \sum_{n=1}^{L} W_{t,n} \le K\} \times \mathbb{R}$.

Proposition 1 establishes the relation between V_t and $\tilde{V}_t$.

PROPOSITION 1. *For any t it holds $V_t((W_t,U_t,E_t),K) = \tilde{V}_t(W_t,E_t,K,U_t + [W_{t,0}]^+ + \sum_{n=1}^{L} W_{t,n})$ for all $(W_t,U_t,E_t) \in \mathcal{S}(K)$.*

The proof follows from a simple inductive argument and is omitted.

It directly follows from Proposition 1 that instead of solving Problem (4), we can solve the problem

$$(N^*,K^*) = \operatorname*{arg\,min}_{N \ge 0, K \ge 0} \left\{ KC^K + NC^N + \tilde{V}_0(W_0,E_0,K,N) \right\}. \quad (6)$$

The solution (N^*,K^*) then also solves Problem (4).

We continue by analyzing $\tilde{V}_t$ and the next result provides coordinatewise convexity of $\tilde{V}_t$.

PROPERTY 2. *For $p_t \ge 0$ the value function $\tilde{V}_t(W,E,K,N)$ is coordinatewise convex in all components of W, in K, and in N for all t.*

We say that a function of a single discrete variable is convex if its first difference is increasing. The interpretation of Property 2 is that V_t is coordinatewise convex in S only on a subspace $\mathcal{S}^c(N,K) = \{(W,U,E) : E \in \mathcal{E}, W \in \mathcal{W}(K), [W_0]^+ + \sum_{n=1}^{L} W_n + U = N, K \in \mathbb{Z}_0^+, N \in \mathbb{Z}_0^+\}$ of the state space $\mathcal{S}$. The convex part of the state space corresponds to regions in which the effect on the total number of batteries caused by perturbing the state remains constant. To illustrate this, consider an increase in $W_{t,0}$ from -10 to -9. Here, the number of batteries at the station remains constant, if all other state coordinates remain constant. However, increasing $W_{t,0}$ from 0 to 1 also increases N by 1, if all other state coordinates remain constant. The reformulation of the model forces N to remain constant, when varying a state coordinate.

The next results provides convexity of the charging decision problem in Equation (3). The proof uses $\tilde{V}_t$ and Property 2.

Property 3. *For all K, N, t and $p_t \geq 0$, it holds that*

1. $V_t(S,K)$ *is coordinatewise convex in W and U on $\mathcal{S}^c(N,K)$ and*
2. $V_t(([W_0,\dots,W_L+x],U-x,E),K)$ is convex in x on $\mathcal{S}^c$ for $-W_L \leq x \leq U$.

By the second part of Property 3, we can solve the minimization problem in Equation (3) efficiently, if the state remains in $\mathcal{S}^c$ when transitioning between periods. Because the action space restricts x to $0 \leq x \leq U$, and because C_t is a convex function, the minimization in Equation (3) is a convex problem.

Convexity in the decision variable allows us to solve the dynamic program efficiently and to evaluate $V_t(S_0(N),K)$ for fixed values of K and N. We analyze the behavior of V_t with respect to K and N next. Let $f_t : \mathbb{Z}_0^+ \times \mathbb{Z}_0^+ \to \mathbb{R}$ with $f_t(N,K) = V_t(S_0(N),K)$.

It is well known that for functions defined on the integers, optimality conditions are more complex than in the continuous case. While convexity of the objective and domain is sufficient for finding an optimal solution in the continuous case, a clear definition of convexity does not exist for integer domain problems. However, analogous concepts like $L^\natural$-convexity and $M^\natural$-convexity (Murota 2003), are sufficient for optimality of a locally optimal solution. In this case an optimal solution can be found with a simple greedy approach. Efficient algorithms also exist for optimizing submodular functions on integer domains (e.g., Iwata et al. 2001 and Orlin 2009).

Unfortunately, f_t does not exhibit any of these properties as our next property states.

Property 4. *Function $f_t(N,K)$ is neither $L^\natural$-convex, nor $M^\natural$-convex or submodular.*

Instead, we can show that $f_t(N,K)$ is coordinatewise convex and non-increasing in K and N and we rely on these properties to solve Problem (4) optimally.

Property 5. *Function $f_t(N,K)$ is coordinatewise convex and non-increasing in K and N for all t.*

Because $f_t(N,K)$ is coordinatewise convex and non-increasing, $NC^N + KC^K + f_t(N,K)$ is also coordinatewise convex in N and K. Coordinatewise convexity does not suffice for a local optimum to be globally optimal, but it allows us to develop an optimal solution algorithm based on coordinatewise convexity and a lower bound, which we introduce next.

3.3. Lower Bound

In this section, we present lower bounds on $f_t(N,K)$. We rely on these bounds in our solution approach to restrict the solution space. The bounds are computed by solving an infinite horizon dynamic program of lower dimension than the exact model.

Global Lower Bound. By Property 5, $f_t(N,K)$ is non-increasing in N and K. As a result, $f_t(N,K) \geq \lim_{\tilde{N}\to\infty,\tilde{K}\to\infty} f_t(\tilde{N},\tilde{K})$ for all t, N, and K. For an infinite number of batteries and infinite charging capacity, there is no upper limit on the number of batteries we can charge and the action space simplifies to $x_t \geq 0$. As a result, we do not need to keep track of the number of uncharged batteries U at the station and the state reduces to (W,E).

Because we allow for backlogging of unsatisfied demand for batteries, the dynamic program without the upper limit on the charging decision can be transformed into an equivalent dynamic program of lower dimension (e.g., Zipkin 2000).

Let $\bar{W}_t = \sum_{n=0}^{L} W_{t,n}$, which represents the inventory position of charged batteries, i.e., the number of batteries on hand plus the number of batteries currently charging, at the station at the beginning of time period t. We define

$$\bar{C}_t(\bar{W}_t, x_t, E_t) = x_t\left(\beta_0 E_t + \mathbb{E}\left[\sum_{\tau=1}^{L}\gamma^\tau \beta_\tau E_{\mathrm{mod}(t+\tau,T)}\right]\right) + \gamma^L p_{\mathrm{mod}(t+L,T)}\mathbb{E}\left[(D_{t,\mathrm{mod}(t+L,T)} - \bar{W}_t)^+\right]$$

and

$$\bar{V}_t(\bar{W}_t, E_t) = \mathbb{E}_{E_{t+1},D_t}\left[\min_{x_t\geq 0}\left\{\bar{C}_t(\bar{W}_t, x_t, E_t) + \gamma \bar{V}_{\mathrm{mod}(t+1,T)}(\bar{W}_{\mathrm{mod}(t+1,T)}, E_{\mathrm{mod}(t+1,T)})\right\}\right]. \quad (7)$$

The state transition function for $\bar{W}_t$ is $\bar{W}_{\mathrm{mod}(t+1,T)} = \bar{W}_t - D_t + x_t$. Cost $\bar{C}_t$ incorporates penalty costs from period $t+L$ on only, because penalty costs in periods t to $t+L-1$ cannot be influenced by the charging decision in period t. To obtain a valid lower bound on f_t, we additionally must account for the penalty and charging cost from period t to $t+L$ for W_t, which is given by $F_t(W_t,E_t) = \mathbb{E}[\sum_{\tau=t}^{t+L-1}\gamma^{\tau-t}p_{\mathrm{mod}(\tau,T)}(D_{\mathrm{mod}(\tau,T)} - W_{\mathrm{mod}(\tau,T),0})^+] + E_t\sum_{n=1}^{L}\beta_n W_{t,n} + \mathbb{E}[\sum_{\tau=t+1}^{t+L-1}\gamma^{\tau-t}E_{\mathrm{mod}(\tau,T)}\sum_{n=1}^{L-\tau+t}\beta_n W_{\mathrm{mod}(\tau,T),n}]$. The expected value can easily be evaluated, because we assume that initially all batteries are charged, i.e., $\bar{W}_0 = N$.

Let $\underline{V}_t = \bar{V}_t(W_t,E_t) + F_t(W_t,E_t)$. The following proposition asserts that $\underline{V}_t$ is a global lower bound, i.e., independent of N and K, on $f_t(N,K)$.

PROPOSITION 2. *$\underline{V}_t \leq f_t(N,K)$ for all N, K, and t.*

Note that the state space of this dynamic program has only two dimensions which allows us to solve large problem instances efficiently.

Coordinatewise Lower Bounds. We can obtain tighter coordinatewise bounds by restricting the charging decision depending on the values of N and K.

For N batteries in the system, the inventory position $\bar{W}_t$ cannot exceed N, i.e., $\bar{W}_t \leq N$ for all t. Using this relation, we restrict the charging decision in each period to values $0 \leq x_t \leq N - \bar{W}_t$. The resulting action space is $\bar{\mathcal{X}}^N(\bar{W}_t) = \{x : x \in \mathbb{Z}_0^+, 0 \leq x \leq N - \bar{W}_t\}$. Set $\bar{\mathcal{X}}^N$ neglects capacity restrictions, but models the upper limit on the charging decision caused by the number of batteries correctly for $W_{t,0} \geq 0$. For $W_{t,0} < 0$, the maximum feasible charging decision is $-W_{t,0}$ units higher than in the exact model. We denote the coordinatewise lower bound for given N by $\underline{\mathrm{V}}_t^N$.

Similarly, for given capacity K, the charging decision cannot exceed K, i.e., $x_t \leq K$ for all t. While the number of batteries in the system is neglected, for $L = 1$ the action space is exact with respect to the restriction imposed by the charging capacity. For $L > 1$, the maximum charging capacity is too large compared to the exact model whenever $\sum_{n=1}^{L} W_{t,n} \geq 0$. The resulting action space is given by $\bar{\mathcal{X}}^K = \{x : x \in \mathbb{Z}_0^+, 0 \leq x \leq K\}$ and we denote the coordinatewise lower bound for given K by $\underline{\mathrm{V}}_t^K$.

To calculate the value of the coordinatewise lower bounds, we solve dynamic program (7) with action space $\bar{\mathcal{X}}^N$ for coordinate N and with action space $\bar{\mathcal{X}}^K$ for coordinate K. As for the global lower bound, we additionally need to account for penalty costs from period t to $t+L$, which are not included in dynamic program (7). The resulting lower bounds are $\underline{\mathrm{V}}_t^K = \bar{V}_t(W_t, E_t)^K + F_t(W_t, E_t)$ and $\underline{\mathrm{V}}_t^N = \bar{V}_t(W_t, E_t)^N + F_t(W_t, E_t)$ for given N and K, respectively. We rely on the following result when developing our solution strategy in Section 5.

Proposition 3. *$\underline{V}_t \leq \underline{V}_t^K \leq f_t(N,K)$ and $\underline{V}_t \leq \underline{V}_t^N \leq f_t(N,K)$ for all N, K, and t.*

In the next section we state a number of properties of the optimal solution to Problem (4) based on the result in Proposition 3.

3.4. Properties of the Optimal Solution

Based on the properties of the value function established in Subsection 3.2 and the lower bounds on the optimal solution from Subsection 3.3, we next derive conditions for the optimality of a solution to Problem (4).

Let (N^*, K^*) denote the optimal solution to Problem (4). The following relations describe properties of the optimal solution in terms of $\underline{\mathrm{V}}_t^N$, $\underline{\mathrm{V}}_t^K$, and $\underline{\mathrm{V}}_t$.

(1) $K^* \leq N^*$: In the optimal solution, there are at least as many batteries in the system as charging capacity is available.

(2) $(N^*, K^*) \notin \{N, K) : f_t(N, K-1) - \underline{\mathrm{V}}_t < C^K\}$: If the gap between $\underline{\mathrm{V}}_t$ and $f_t(N, K-1)$ is smaller than C^K, an additional unit of charging capacity can never amortize its investment.

(3) $(N^*, K^*) \notin \{(N, K) : f_t(N-1, K) - \underline{\mathrm{V}}_t < C^N\}$: If the gap between $\underline{\mathrm{V}}_t$ and $f_t(N-1, K)$ is smaller than C^N, an additional battery can never amortize its investment.

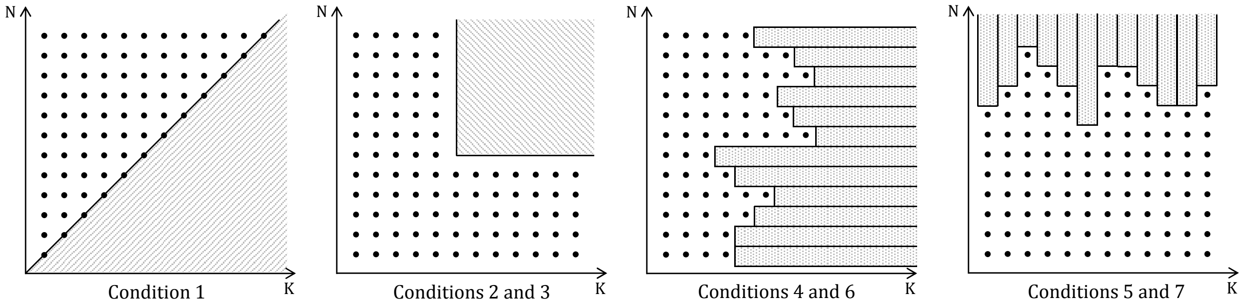

Figure 1 Illustration of Conditions 1 to 7

(4) $(N^*, K^*) \in \{(N,K) : f_t(N,K) - f_t(N,K-1) \le -C^K\}$: If increasing K does not decrease operating cost by at least C^K, then the solution cannot be improved by increasing K.

(5) $(N^*, K^*) \in \{(N,K) : f_t(N,K) - f_t(N-1,K) \le -C^N\}$: If increasing N does not decrease operating cost by at least C^N, then the solution cannot be improved by increasing N.

(6) $(N^*, K^*) \notin \{(N,K) : f_t(N,K-1) - \underline{\mathrm{V}}_t^N < C^K\}$: If the gap between $f_t(N,K-1)$ and $\underline{\mathrm{V}}_t^N$ is smaller than C^K, an additional unit of charging capacity can never amortize its investment.

(7) $(N^*, K^*) \notin \{(N,K) : f_t(N-1,K) - \underline{\mathrm{V}}_t^K < C^N\}$: If the gap between $f_t(N-1,K)$ and $\underline{\mathrm{V}}_t^K$ is smaller than C^N, an additional battery can never amortize its investment.

Condition (1) is intuitive, because a charging bay has no value, if it is never used. Conditions (2) and (3) are similar to Conditions (6) and (7), as both relate the cost of an additional battery or unit of charging capacity to the maximum savings possible by increasing these quantities. The maximum achievable savings are estimated by the gap between $f_t(N,K)$ and $\underline{\mathrm{V}}_t$ in Conditions (2) and (3) and by the gap between $f_t(N,K)$ and $\underline{\mathrm{V}}_t^K$ or $\underline{\mathrm{V}}_t^N$ in Conditions (6) and (7).

Conditions (4) and (5) use the coordinatewise convexity of $f_t(N,K)$ to check whether an additional battery or unit of charging capacity can decrease the operating cost sufficiently to amortize the investment. If for fixed N Condition (4) is violated for K, it is also violated for all $K' > K$, because by Property 5 f_t is coordinatewise convex. Similarly, if for fixed K Condition (5) is violated for N, it is also violated for all $N' > N$.

Figure 1 illustrates the parts of the solution space that are excluded by the formulated conditions.

In Section 5 we develop a solution algorithm that finds the optimal solution to Problem (4) by bounding the solution space based on the above conditions and searching the remaining possible solutions.

4. Network of Stations

In this section, we extend the single station model to a network of swap stations. We consider a set of swap stations $\mathcal{I}$. Each station is described by the single station model introduced in Section 3.1. The stations in the network are interlinked by the possibility to transship charged and uncharged batteries. We use subscript i for variables and parameters whenever we refer to a quantity that can vary across stations.

4.1. Model

We model charging and transshipment decisions as a centralized decision making process, i.e., decisions are made to minimize total cost and not cost of an individual station. In the sequence of events, transshipment decisions are system wide made concurrently with charging decisions after current prices of electric energy have been revealed and before demand is realized. We denote the decision on the number of charged and uncharged batteries to transship from station i to station j by $y_{t,i,j}$ and $z_{t,i,j}$, respectively. We assume that the transshipment time between two stations is deterministic and denote it by $L_{i,j}$. We define $\bar{L}_i = \max_j L_{j,i}$. For the transshipment, fixed cost $c^f_{t,i}$ per dispatch and variable cost $c^v_{t,i,j}$ per battery is incurred.

The number of uncharged batteries at station i is a vector with $\bar{L}_i$ coordinates, because we have to capture uncharged batteries that are in transit to the station. The first coordinate, i.e., $U_{t,i,0}$, refers to the number of uncharged batteries on hand. Vector $W_{t,i,j}$ captures not only charging batteries at the station, but also charged batteries in transit to the station. We introduce an additional state variable capturing the number of batteries currently being charged at station i and denote it by $R_{t,i}$. This additional component is necessary, because we cannot calculate the number of charging batteries from $W_{t,i}$, but we must constrain the number of charging batteries to be less than K_i. We denote the state of a single station in the network by $S_{t,i} = (W_{t,i}, U_{t,i}, R_{t,i}, E_{t,i})$.

The number of charged batteries that become available at the beginning of period $t+n$ in the network model additionally depends on the transshipment decisions at all other stations, i.e., $W_{t,i,n} = W_{t-1,i,n+1} + \sum_{j\neq i} \delta(L_{j,i}, n) y_{t-1,j,i}$ for $n \neq L$ and $W_{t,i,n} = W_{t-1,i,n+1} + x_{t-1,i} + \sum_{j\neq i} \delta(L_{j,i}, n) y_{t-1,j,i}$ for $n = L$, where $\delta(x,y) = 1$ for $x = y$ and $\delta(x,y) = 0$ otherwise and $n = 0, \ldots, \max(L, \bar{L}_i)$. The number of charged batteries on hand depends on outgoing transshipments from the station and is given by $W_{t,i,0} = W_{t-1,i,0} - D_{t-1,i} + W_{t-1,i,1} - \sum_{j\neq i} y_{t-1,i,j}$.

Likewise, the number of uncharged batteries that become available at the beginning of period $t+n$ is given by $U_{t,i,n} = U_{t-1,i,n+1} + \sum_{j\neq i} \delta(L_{j,i}, n) z_{t-1,j,i}$ for $n = 0, \ldots, \bar{L}_i$ and the number of uncharged batteries on hand is given by

$$U_{t,i,0} = U_{t-1,i,0} + U_{t-1,i,1} - x_{t-1,i} - \sum_{j\neq i} z_{t-1,i,j} + \min\left([-W_{t-1,i,0}]^+ + D_{t-1,i}, [W_{t-1,i,0}]^+ + W_{t-1,i,1}\right).$$

The total number of batteries at a station in the network model is not constant, but the total number of batteries at all stations or in transit is constant. We denote the number of batteries at or in transit to station i by

$$N_i(W_{t,i}, U_{t,i}) = [W_{t,i,0}]^+ + \sum_{n=1}^{\max(L,\bar{L}_i)} W_{t,i,n} + \sum_{n=0}^{\bar{L}_i} U_{t,i,n}. \tag{8}$$

The total cost incurred at station i in period t is given by

$$\begin{aligned} C_{t,i}(W_{t,i}, R_{t,i}, x_{t,i}, y_{t,i}, z_{t,i}, E_{t,i}, D_{t,i}) =& E_{t,i}\left(\beta_0 x_{t,i} + \sum_{n=0}^{L-1} \beta_n R_{t,i,n}\right) + p_{t,i}\left[D_{t,i} - W_{t,i,0}\right]^+ \\ &+ \sum_{j\neq i} c^f_{t,i}\sigma(y_{t,i,j} + z_{t,i,j}) + c^v_{t,i,j}(y_{t,i,j} + z_{t,i,j}), \end{aligned} \tag{9}$$

where $\sigma(x) = 0$ for $x \leq 0$ and $\sigma(x) = 1$ otherwise.

The charging and transshipment decisions are constrained by $x_{t,i} + \sum_{j\neq i} z_{t,i,j} \leq U_{t,i,0}$, $\sum_{j\neq i} y_{t,i,j} \leq [W_{t,i,0}]^+$, and $x_{t,i} \leq K_i - \sum_{n=0}^{L-1} R_{t,i,n}$. We denote the set of feasible charging and transshipment decisions, given the current state and the charging capacity of the station by $\mathcal{X}_i(S_{t,i}, K_i)$.

The optimization problem for the network model is

$$\begin{aligned} &(N_0^*, \ldots, N_I^*, K_0^*, \ldots, K_I^*) = \\ &\operatorname*{arg\,min}_{\substack{N_0,\ldots,N_I \\ K_0,\ldots,K_I}} \sum_{i\in\mathcal{I}} N_i C^N + K_i C^K + \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{E_{t,i}}\left[\min_{x_{t,i},y_{t,i},z_{t,i}} \mathbb{E}_{D_{t,i}} C_{t,i}(W_{t,i}, R_{t,i}, x_{t,i}, y_{t,i}, z_{t,i}, E_{t,i}, D_{t,i}) \Big| S_t\right]. \end{aligned}$$

Again, we reformulate the problem as a dynamic program and recursively define the optimal value functions for each period of the problem as

$$\begin{aligned} V_t(S_t, K_1, \ldots, K_I) = \mathbb{E}_{E_{t+1},D_t}\Bigg[&\min_{x_t,y_t,z_t} \sum_{i\in\mathcal{I}} C_{t,i}(W_{t,i}, R_{t,i}, x_{t,i}, y_{t,i}, z_{t,i}, E_{t,i}, D_{t,i}) \\ &+ \gamma V_{\mathrm{mod}(t+1,T)}\left(S_{t+1}, K_1, \ldots, K_I\right) \Big| S_t\Bigg]. \end{aligned} \tag{10}$$

For the network model, our goal is to solve

$$(N_0^*, \ldots, N_I^*, K_0^*, \ldots, K_I^*) = \operatorname*{arg\,min}_{\substack{N_0,\ldots,N_I \\ K_0,\ldots,K_I}} \sum_{i=0}^{I} N_i C^N + K_i C^K + V_0\left(S_0(N_1, \ldots, N_I), K_1, \ldots, K_I\right), \tag{11}$$

where $S_0 = [S_{0,0}, \ldots, S_{0,I}]$, $W_{0,i} = [N_i, 0, \ldots, 0]$, $R_{0,i} = [0, \ldots, 0]$, and $U_{0,i} = [0, \ldots, 0]$.

4.2. Properties

The cost function $C_{t,i}$ includes a fixed cost component for transshipments and is non-convex for $c^f > 0$. As a result, V_t is also non-convex for $c^f > 0$ and the properties of the single station model do not carry over to the network model in this case. Our further analyses therefore assume $c^f = 0$.

In the single station model, we showed coordinatewise convexity of $V_t(S, K)$ on subspace $\mathcal{S}^c$ by using a reformulated model, in which we can omit the nonlinear transition function for U. For the network model, the constraint $\sum_{j\neq i} y_{t,i,j} \leq [W_{t,i,0}]^+$ causes the action space to be non-convex.

However, for a slightly restricted model we can again show coordinatewise convexity. If we disallow transshipments of charged batteries, i.e. set $y_{t,i,j} = 0$, we can express the action space as a polyhedron as follows.

To keep track of the number of batteries at station i in period t, let $N_{t+1,i} = N_{t,i} + \sum_{j\neq i}(y_{t,j,i} - y_{t,i,j} + z_{t,j,i} - z_{t,i,j})$. The action space is then given by

$$0 \le x_{t,i} + \sum_{i\neq j} z_{t,i,j} \le N_{t,i} - \sum_{n=0}^{\max(L,\bar{L}_i)} W_{t,i,n} - \sum_{n=1}^{\bar{L}_i} U_{t,i,n}$$

$$0 \le x_{t,i} + \sum_{i\neq j} z_{t,i,j} \le N_{t,i} - \sum_{n=1}^{\max(L,\bar{L}_i)} W_{t,i,n} - \sum_{n=1}^{\bar{L}_i} U_{t,i,n},$$

and

$$0 \le x_{t,i} \le K_i - \sum_{n=0}^{L} R_{t,i,n}.$$

Coordinatewise convexity can then be shown by considering a convex continuous extension of the value function, which takes the same function values at integer points.

PROPERTY 6. *If transshipments of charged batteries are not allowed and $c^f = 0$, then the value function of the network model is coordinatewise convex in W, K_i and N_i for all i.*

In our solution approach, we approximate the exact value function V_t by a convex function of lower dimensionality, because the restricted model is convex. Note, however, that the complete model is not necessarily convex, though in all numerical experiments we could not find a counterexample.

5. Solution Approach

In this section we present our solution approaches for solving the models presented in Sections 3.1 and 4.1. The model for a single station can be solved optimally for small instances within reasonable computational time, but larger instances and the network model, because it grows exponentially in the number of stations, are computationally intractable. We present an optimal solution algorithm for the single station model and an approximation for the network model, and report optimality gaps for small instances and lower bounds for instances of relevant size in Section 6.

5.1. Single Station

Our goal is to find $(N^*, K^*) = \arg\min_{N,K} f_0(N,K) + NC^N + KC^K$. From Properties 4 and 5 we know that f_t is coordinatewise convex, but neither $L^\natural$-convex, nor $M^\natural$-convex or submodular. As a result, local optimality of a point does not guarantee global optimality (Murota 2003) and a pure greedy search algorithm is not guaranteed to find an optimal solution. Instead, we combine the lower bounds from Section 3.3 and greedy search to develop an efficient solution algorithm.

Our solution approach consists of two phases. In the first phase, we use the global lower bound from Section 3.3 to bound the solution space. In the second phase, we use the coordinatewise lower bounds and coordinatewise convexity to search the remaining solution space for the optimal solution.

In our algorithm, we have to compute the minimum expected infinite horizon cost for given N, K, and $S_0(N)$. We do so by solving the dynamic program in Problem (4) using value iteration and denote the value function after iteration k by V_t^k. We use MacQueen-Porteus error bounds (Bertsekas 2007) to accelerate convergence and also to check for optimality of the current solution. This strategy allows us to stop evaluating the dynamic program, before its optimal solution has been calculated.

Let

$$\bar{b}_t^k(N,K) = V_t^k(S_0(N),K) + \frac{\gamma^L}{(1-\gamma^L)} \sup_S \{V_t^k(S,K) - V_t^{k-1}(S,K)\}$$

and

$$\underline{b}_t^k(N,K) = V_t^k(S_0(N),K) + \frac{\gamma^L}{(1-\gamma^L)} \inf_S \{V_t^k(S,K) - V_t^{k-1}(S,K)\}.$$

For all k, the relation $\bar{b}_t^k(N,K) \le f_t(N,K) \le \underline{b}_t^k(N,K)$ holds.

We introduce both phases of our solution algorithm next.

Bounding the solution space. We first calculate an upper bound on K by increasing K and N along the diagonal $K = N$ through the solution space until further increases of K and N cannot amortize the initial investment, i.e., until $f_0(N,K) \le \underline{V}_t + \min(C^N, C^K)$. We set $K = N$, because according to Condition (1) in Section 3.4, $N^* \ge K^*$ holds. For each point (N,K), we compute the value of f_0 by solving the dynamic program in Problem (4) using value iteration. In our approach we use the upper bound on value f_0 after k iterations, $\bar{b}_0^k$, instead of f_0, because we can stop the value iteration algorithm as soon as the criterion $\bar{b}_0^k(N,K) \le \underline{V}_0 + \min(C^N, C^K)$ is met (this corresponds to Conditions (2) and (3) from Section 3.4). At this point, adding more batteries or charging bays will not reduce expected operating costs enough to justify the initial investment. Algorithm 1 summarizes the proposed procedure and returns an upper bound on the optimal charging capacity that we denote by $\bar{K}$.

Finding the optimal solution. Given $\bar{K}$, the remaining solution space to search for the optimal solution is $\{N \ge K \ge 0, K \le \bar{K}\}$ (see Figure 2 on the left). We set $N = K = 1$, calculate the coordinatewise lower bound $\underline{V}_0^K$, and increase N until either $f_0(N,K) + C^N > f_0(N-1,K)$ (Condition (5)) or $f_0(N,K) < \underline{V}_0^K + C^N$ (Condition (3)) to obtain $N^*(K)$ and the corresponding expected cost. We then increment K, calculate $\underline{V}_0^K$, and increase N until either $f_0(N,K) + C^N > f_0(N-1,K)$ (Condition (5)) or $f_0(N,K) < \underline{V}_0^K + C^N$ (Condition (3)), or we decrease N until $f_0(N,K) + C^N < f_0(N-1,K)$ (Condition (5)) to obtain $N^*(K)$. We then again increment K and repeat the procedure until $K = \bar{K}$. Algorithm 2 summarizes the procedure. Algorithm 2 uses $\underline{b}_0^k$ and $\bar{b}_0^k$ instead of f_0 when applying Conditions (3) and (5) in steps 5, 13, and 15 to exploit the possibility to terminate the computation of $f_0(N,K)$ early. In each step of the algorithm, we store

Algorithm 1 Upper Bound on K^*

1: Set $K=1$, $N=1$, $k=0$ and $\underline{b}_0^k(N,K)=-\infty$, $\bar{b}_0^k(N,K)=\infty$ for all $N>0$, $K>0$.
2: Compute $\underline{V}_0$ by using standard dynamic programming.
3: **while** $\bar{b}_0^k(N,K)>\underline{V}_0+\min(C^N,C^K)$ **do**
4: $\quad k \leftarrow k+1$
5: $\quad$ Compute V_T^k by solving Problem (3) for all $S\in\mathcal{S}^c(N,K)$ with V_0^{k-1} in the right hand side.
6: $\quad$ **for** $t=T-1\rightarrow 0$ **do**
7: $\quad\quad$ Compute V_t^k by solving Problem (3) for all $S\in\mathcal{S}^c(N,K)$ with V_{t+1}^k in the right hand side.
8: $\quad$ **end for**
9: $\quad$ Compute $\bar{b}_0^k(N,K)$ and $\underline{b}_0^k(N,K)$.
10: $\quad$ **if** $\underline{b}_0^k(N,K)\geq\underline{V}_0+\min(C^N,C^K)$ **then**
11: $\quad\quad$ Set $K=K+1$, $N=N+1$, $k=0$, and go to Step 3.
12: $\quad$ **end if**
13: **end while**
14: $\bar{K}\leftarrow K-1$

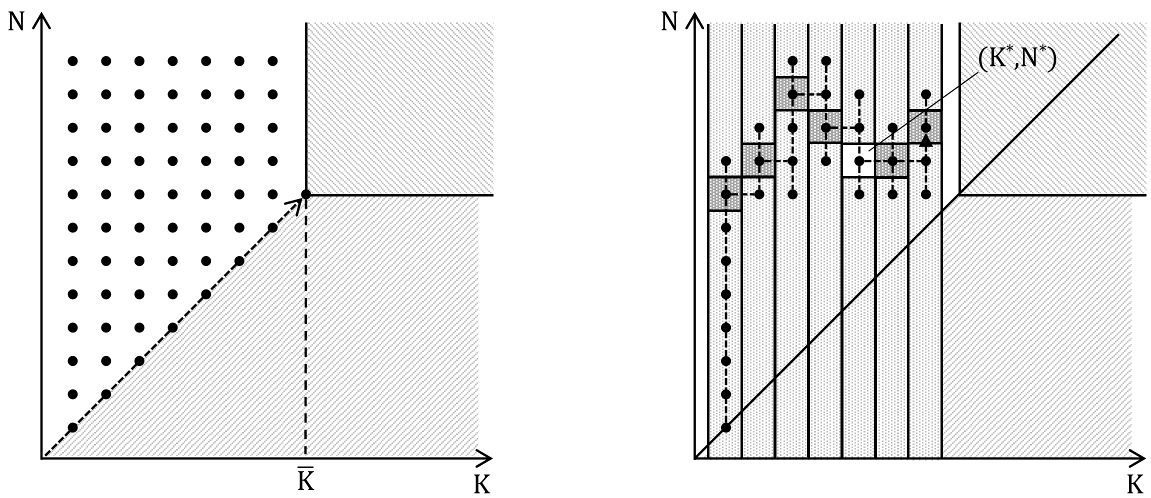

Figure 2 Phase 1 (left) and Phase 2 (right) of the Solution Algorithm

and update the current best solution. Figure 2, right picture, illustrates a sample search path of the algorithm. Dots indicate pairs (N,K) for which $f_0(N,K)$ was evaluated. Shaded areas were excluded from the search because they violated an optimality condition. Dark shaded fields are coordinatewise optimal, but a better solution (N^*,K^*) was found.

5.2. Network of Stations

When considering a network of swap stations, the dynamic program in Equation (10) becomes computationally intractable and we cannot compute the system cost exactly within reasonable computational time. Instead, we rely on a solution heuristic to solve the problem. We introduce an approximate dynamic programming (ADP) algorithm that approximately solves Problem (11)

Algorithm 2 Finding (N^*, K^*)

Set $K = K^* = 0$, $N = N^* = 0$, $k = 0$, $t = 0$, $C^* = \infty$, and tolerance $\epsilon > 0$.
while $K \le \bar{K}$ **do**
  Set $\underline{b}_0^0(N,K) = -\infty$, $\bar{b}_0^0(N,K) = \infty$ for all N.
  Compute $\underline{V}_0^K$ by standard dynamic programming.
  while $\bar{b}_0^k(N,K) > \underline{V}_0^K + C^N$ and $|\bar{b}_0^k(N,K) - \underline{b}_0^k(N,K)| > \epsilon$ and $|\bar{b}_0^k(N-1,K) - \underline{b}_0^k(N-1,K)| > \epsilon$ **do**
    Compute $V_T^k(N,K)$ by solving Problem (3) for all $S \in \mathcal{S}^c(N,K)$ with $V_0^{k-1}(N,K)$ in the right hand side.
    Compute $V_T^k(N-1,K)$ by solving Problem (3) for all $S \in \mathcal{S}^c(N-1,K)$ with $V_0^{k-1}(N-1,K)$ in the right hand side.
    for $t = T-1 \to 0$ **do**
      Compute $V_t^k(N,K)$ by solving Problem (3) for all $S \in \mathcal{S}^c(N,K)$ with $V_{\mathrm{mod}(t+1,T)}^k(N,K)$ in the right hand side.
      Compute $V_t^k(N-1,K)$ by solving Problem (3) for all $S \in \mathcal{S}^c(N-1,K)$ with $V_{\mathrm{mod}(t+1,T)}^k(N-1,K)$ in the right hand side.
    end for
    Compute $\bar{b}_0^k(N,K)$, $\underline{b}_0^k(N,K)$, $\bar{b}_0^k(N-1,K)$, and $\underline{b}_0^k(N-1,K)$.
    if $\underline{b}_0^k(N-1,K) \ge \bar{b}_0^k(N,K) + C^N$ **then**
      Set $\underline{b}_0^0(N,K) = \underline{b}_0^{k-1}(N,K)$, $\bar{b}_0^0(N,K) = \bar{b}_0^{k-1}(N,K)$, $N = N+1$, and $k = 0$.
    else if $\bar{b}_0^k(N-1,K) \le \underline{b}_0^k(N,K) + C^N$ **then**
      Set $N = N-1$, $\underline{b}_0^0(N,K) = \underline{b}_0^{k-1}(N,K)$, $\bar{b}_0^0(N,K) = \bar{b}_0^{k-1}(N,K)$, and $k = 0$.
    end if
    $k \leftarrow k+1$
  end while
  if $\frac{\underline{b}_0^k(N,K) + \bar{b}_0^k(N,K)}{2} + NC^N + KC^K \le C^*$ **then**
    Set $N^* = N$, $K^* = K$, and $C^* = \frac{\underline{b}_0^k(N,K) + \bar{b}_0^k(N,K)}{2} + NC^N + KC^K$
  end if
  $K \leftarrow K+1$
end while

by approximating the value functions of the dynamic program using a analytical function whose parameters we estimate based on Monte-Carlo simulation.

ADP for the Network of Stations. We approximate the exact value functions V_t by replacing their table lookup form by an analytical form that is defined by a smaller number of parameters (Powell 2007). The approximation is chosen to be of sufficiently low dimensionality and we estimate its parameters by sampling the stochastic information process and updating the parameters using the information of the incumbent solution. Often, problems exhibit special structure suggesting a certain form for the approximating function. We use a separable convex piece-wise linear approximation, because our problem has integer decisions and it is coordinatewise convex for a mildly restricted model. Additionally, convexity of a separable piece-wise linear function can easily be maintained by simple projection algorithms, e.g., the SPAR algorithm (Powell 2007).

We introduce our approximate value function next and then provide details on how to iteratively update the approximation and concurrently optimize the charging capacity and number of batteries at each station.

Separable piece-wise linear approximation. The state space of Problem (11) is too large to enumerate, because even small problem instances with 24 periods, 5 stations, and 100 batteries have state spaces with far more than $24 * 100^5$ states. To reduce the number of parameters of the value function, we replace the exact value function by a separable piece-wise linear approximation $\hat{V}_t(S_t)$.

Let Π_i be the set of all coordinates of W, U, and R corresponding to station i and let $\Pi = \cup_{i\in I}\Pi_i$ be the joint set over all stations. For every station i, coordinate $j \in \Pi_i$, and energy price interval $e \in \mathcal{E}_{t,i}$, we define a separable piece-wise linear function $g_{t,i}^{e,j}(S_j)$, where S_j is the battery count of coordinate j. As an example, $g_{2,4}^{5,j}(15)$ estimates the value of having $S_j = 15$ charged batteries (i.e., the value of coordinate j which can correspond to $W_{4,3}$ is 15) that become available in 3 time periods at station 4 in period 2, if the current price of electric energy is in interval 5. We denote the slope vector of $g_{t,i}^{e,j}$ by $v_{t,i}^{e,j}$.

The value function approximation for a single station is given by

$$\hat{V}_{t,i}(S_{t,i}) = \sum_j g_{t,i}^{E_{t,i,j}}(S_{t,i,j}), \tag{12}$$

where $g_{t,i}^{e,j}(S_{t,i,j}) = \sum_{m=0}^{S_{t,i,j}} v_{t,i}^{e,j}(m)$. We assume without loss of generality that $g_{t,i}^{e,j}(0) = 0$ and thus the value of the piece-wise linear function at a point is the sum of its slopes up to that point. The value function approximation for the complete network model is given by

$$\hat{V}_t(S_t) = \sum_{i=0}^{I} \hat{V}_{t,i}(S_{t,i}).$$

To approximate Problem (11) we replace the exact value function $V_t(S_t, K_0, \ldots, K_I)$ by $\hat{V}_t(S_t)$. We show how to iteratively update the slope vectors $v_{t,i}^{e,j}$ next.

Updating the Approximation. The value function approximation has a lower number of parameters that must be estimated, because it assumes a separable structure. However, the number of parameters is still too large to enumerate and the expected value in Equation (13) is computationally expensive to evaluate.

We circumvent this problem by iteratively updating the parameters using solution information obtained from solving Problem (11) using the current value function approximation $\hat{V}_{\text{mod}(t+1,T)}$ and sampling random variables E_t and D_t.

First, we replace the exact value function $V_t(S_t)$ by the approximate value function $\hat{V}_t(S_t)$. The optimality equation for period t then reads

$$\tilde{V}_t(S_t) = \mathbb{E}_{E_{t+1},D_t}[\hat{Y}_t(S_t, E_{t+1}, D_t, \hat{V}_{\text{mod}(t+1,T)})|S_t], \tag{13}$$

where

$$\hat{Y}_t(S_t, E_{t+1}, D_t, \hat{V}_{\text{mod}(t+1,T)}) =$$
$$\min_{x_t,y_t,z_t} \sum_{i=0}^{I} C_{t,i}(W_{t,i}, R_{t,i}, x_{t,i}, y_{t,i}, z_{t,i}, E_{t,i}, D_{t,i}) + \hat{V}_{\text{mod}(t+1,T)}\left(S_{\text{mod}(t+1,T)}\right).$$

Second, instead of computing the expected value in Equation (13) explicitly, we approximate it by sampling E_{t+1} and D_t. Let k be the current iteration of our procedure. We denote the value function approximation after its last update by $\hat{V}_t^{k-1}$ and the samples of the energy prices and the demands in iteration k by $e_{t,i}^k$, $d_{t,i}^k$, respectively.

Given the current state, S_t^k, to obtain next period's value function approximation, we solve

$$(\hat{x}_t^k, \hat{y}_t^k, \hat{z}_t^k) = \underset{x_t,y_t,z_t}{\arg\min} \sum_{i=0}^{I} C_{t,i}(W_{t,i}^k, R_{t,i}^k, x_{t,i}, y_{t,i}, z_{t,i}, e_{t,i}^k, d_{t,i}^k) + \hat{V}_{\text{mod}(t+1,T)}^{k-1}\left(S_{\text{mod}(t+1,T)}^k\right). \tag{14}$$

Third, to update the slope vectors, we perturb the current state and calculate numerical derivatives in direction of all coordinates $j \in \Pi_i$ for all i. Let $S_{t,i,j}^k$ denote the perturbed state obtained by setting $S_{t,i,j}^k = S_{t,i}^k + \mathbb{1}_j$, where $\mathbb{1}_j$ corresponds to the unit vector in direction of coordinate j. For all $j \in \Pi_i$ and all i, we calculate $\Delta_{t,i}^{j,k} = \hat{Y}_t(S_{t,i,j}^k, e_{t,i}^k, d_{t,i}^k, \hat{V}_{\text{mod}(t+1,T)}^{k-1}) - \hat{Y}_t(S_t^k, e_{t,i}^k, d_{t,i}^k, \hat{V}_{\text{mod}(t+1,T)}^{k-1})$ and update the slope vectors by setting

$$v_{t,i}^{e_{t,i}^k,j,k}(S_{t,i,j}^k) = (1-\alpha^k) v_{t,i}^{e_{t,i}^k,j,k-1}(S_{t,i,j}^k) + \alpha^k \Delta_{t,i}^{j,k} \tag{15}$$

for some step size α^k. Here, $v_{t,i}^{e_{t,i}^k,j,k}$ is the slope in iteration k.

After the update step in Equation (15), the piece-wise linear functions $g_{t,i}^{e,j}$ are not necessarily still convex. We thus complete the update by executing the SPAR algorithm (Powell 2007) on the updated slopes to maintain convexity and obtain the value function approximation $\hat{V}_t^k$ in period t of iteration k.

The final step is to use actions $(\hat{x}_t^k, \hat{y}_t^k, \hat{z}_t^k)$ and samples $e_{t,i}^k$ and $d_{t,i}^k$ to transition from state S_t^k into state $S_{\text{mod}(t+1,T)}^k$. Then the procedure is repeated for the next time period and we set $k = k+1$ when revisiting time period $t = 0$.

Algorithm 3 summarizes the update process for the value function approximation.

Optimization of Charging Capacity and Number of Batteries. The update process for the value function described in the previous section assumes fixed charging capacities and numbers of batteries. To optimize these quantities, we adjust all K_i and N_i after a fixed number of iterations of Algorithm 3 were performed based on the following logic.

The slopes of the value function correspond to the marginal value of an additional battery, i.e., $V_t(S_t + \mathbb{1}_j) - V_t(S_t)$, represents the infinite horizon value of an additional battery with characteristics corresponding to coordinate j of the state space, if the state is S_t in period t. As an example, if

Algorithm 3 ADP Algorithm for fixed K_i, N_i

Initialize $v_t^{e,j}$ for all $e \in E_t$, $j \in \Pi$, and $t = 0, \ldots, T-1$ such that $g_t^{e,j}$ are convex functions.
Set $k \leftarrow 0$, $t \leftarrow 0$ and choose $\alpha_0 > 0$.
while stopping criterion is not met **do**
  if $t = 0$ **then**
    Set $k \leftarrow k+1$, and choose new step size $\alpha^k > 0$.
  end if
  Obtain $e^k_{\mathrm{mod}(t+1,T),i}$ and $d^k_{t,i}$ by sampling $E_{\mathrm{mod}(t+1,T),i}$ and $D_{t,i}$ for all i.
  Obtain $(\hat{x}^k_t, \hat{y}^k_t, \hat{z}^k_t)$ by solving Problem (14).
  for all $j \in \Pi$ **do**
    Set $S^{\pi,k}_t = S^k_t + \mathbb{1}_j$.
    Calculate $\Delta^k_t(j, S^k_t) = \hat{Y}_t(S^{\pi,k}_t, e^k_{\mathrm{mod}(t+1,T)}, d^k_t, \hat{V}^{k-1}_{\mathrm{mod}(t+1,T)}) - \hat{Y}_t(S^k_t, e^k_{\mathrm{mod}(t+1,T)}, d^k_t, \hat{V}^{k-1}_{\mathrm{mod}(t+1,T)})$.
    Set $v_t^{E^k_{t,i},j,k}(S^k_{t,j}) = (1-\alpha^k) v_t^{E^k_{t,i},j,k-1}(S^k_{t,j}) + \alpha^k \Delta^k_t(j, S^k_t)$.
    Execute SPAR algorithm on $v_t^{E^k_{t,i},j,k}$.
  end for
  Compute $S^k_{\mathrm{mod}(t+1,T)}$ from S^k_t, $(\hat{x}^k_t, \hat{y}^k_t, \hat{z}^k_t)$, and $e^k_{\mathrm{mod}(t+1,T)}$ and d^k_t.
  Set $t \leftarrow \mathrm{mod}(t+1, T)$
end while

j corresponds to R_2, then $V_t(S_t + \mathbb{1}_j) - V_t(S_t)$ represents the infinite horizon value of an additional battery (compared to state S_t) that is currently charging and becomes available in 2 periods.

Similarly, the slopes of the value function approximation $v^{e,j}_{t,i}$ estimate the value of an additional battery with characteristic corresponding to coordinate j. We can thus use the approximation $v^{e,j,k}_{t,i}$ in iteration k to estimate the optimal values of N_i and K_i.

To estimate the optimal number of batteries at each station we solve the problems

$$N_i^* = \arg\min_{N_i} \{N_i C^N + \hat{V}_{t,i}(S_{t,i})\} \tag{16}$$

for all i subject to $W_{t,i,0} \le N_i$, $W_{t,i,n} = 0$ for $n > 0$, $U_{t,i,n} = 0$, and $R_{t,i,n} = 0$ for all n. We allow only charged batteries to be "purchased", because we use the convention that initially all batteries are charged.

Similarly, to estimate the optimal charging capacity we balance the cost of a unit of charging capacity against the estimated infinite horizon value of charging batteries. We solve the problems

$$K_i^* = \arg\min_{K_i} \{K_i C^K + \hat{V}_{t,i}(S_{t,i})\} \tag{17}$$

for all i subject to $\sum_{n=1}^{L} R_{t,i,n} \le K_i$, $W_{t,i,n} = R_{t,i,n}$ and $U_{t,i,n} = 0$ for all n.

Setting new values for N_i and K_i leads the ADP algorithm to explore new parts of the state space and the value function approximation only gradually adapts to the new parameters. Thus, we do not solve Problems (16) and (17) in each iteration, but optimize over N_i and K_i only every

$\bar{k}$ iterations of Algorithm 3 instead. This strategy admits "enough time" to the ADP algorithm to estimate the value function for the newly explored parts of the state space, before readjusting capacities and the number of batteries.

Algorithm 4 illustrates how to integrate the optimization over K_i and N_i into the approximate dynamic programming algorithm.

Algorithm 4 Optimizing N_i and K_i

```
1:  Execute Steps 1-2 of Algorithm 3.
2:  Set initial K_i ≥ 0 and N_i ≥ K_i for all i = 1,...,I.
3:  while stopping criterion is not met do
4:      while mod(k, k̄) > 0 do
5:          Execute Steps 4-16 of Algorithm 3.
6:      end while
7:      for all i = 1,...,I do
8:          Obtain new value for N_i from solving Problem (16).
9:          Obtain new value for K_i from solving Problem (17).
10:     end for
11: end while
```

Calculation of Expected Infinite Horizon Operating Cost. Algorithm 4 returns an estimate of the optimal number of batteries N^* and charging capacity K^* along with an approximate value function. We use these data to calculate the infinite horizon operation cost of the system by Monte-Carlo simulation.

We start in state $S_0(N_1^*,\dots,N_I^*)$ and run the simulation for a given number of ζ iterations and compute the operating cost C_k in each iteration. We then run the simulation for another ζ iterations and use the average operating cost in these ζ iterations as an estimate for the average per period operating costs. For discount factor γ, we estimate the total expected infinite horizon operating cost by $\hat{V}_0^m(S_0(N_1^*,\dots,N_I^*)) = \sum_{k=0}^{\zeta}\gamma^k C_k + \frac{\gamma}{\zeta(1-\gamma)}\sum_{k=\zeta}^{2\zeta} C_k$.

This procedure is repeated $M = 100$ times and the mean over all $\hat{V}_0^m(S_0(N_1^*,\dots,N_I^*))$ serves as the expected infinite horizon operating cost.

6. Computational Results

We apply our solution approach to a potential network of swap stations in the Bay Area in California. We chose this region for our scenario, because EV adoption rates here are highest in California and EV sales in California account for nearly 25 percent of EV sales in the United States (Center for Sustainable Energy California 2013). We first consider a single station and show the influence of different parameters on the expected optimal operating cost and the optimal solution for N

and K. We then extend our scenario to a network of stations. We compare the performance of our heuristic solution approach for the network to the optimal solution for a small test instance to show that our approximation yields near-optimal results. We then assess the value of allowing transshipments in a battery swap network by applying our approach to problem instances based on actual data. The algorithms were implemented in C++ and the integer problems of Algorithm 3 and Algorithm 4 were solved using CPLEX 12.5 under Linux on a single Westmere Hexa-Core Xeon X5650 processor with 2.66 GHz.

6.1. Data and Parameters

Our model requires data about demand for batteries, prices of electric energy, cost of batteries and charging capacity, station locations, and transshipment cost and times. To obtain the data, we rely on a number of sources and we pre-process data when necessary. All data used in the computational study can be obtained from the authors' website[1].

For all experiments we use a period length of one hour and a planning horizon of one week ($T = 168$). We set the per-period discount factor to $\gamma = 0.99999$ corresponding to an annual cost of capital of about 8%.

Demand for Batteries. We use data from the distribution of trips by time of day (US Department of Transportation 2003; Table A-12) and from the distribution of daily trips by day of the week (US Department of Transportation 2003; Table A-13) to estimate the relative distribution of demand for batteries by hour over the planning horizon. Figure 3 on the right shows the relative estimated demand rates over a day, i.e., $\frac{\lambda_t}{\sum_\tau \lambda_\tau}$, where λ_t is the parameter of the Poisson distribution in period t.

The absolute demand for batteries is hard to estimate, as it depends mainly on the EV adoption rate, which in turn is determined by many factors (e.g., prices for EVs, gasoline prices, marketing spent of the system operator, etc.). Because we do not have reliable real world data for the absolute demand, we perform an extensive sensitivity analysis.

Electric Energy Prices. Historical locational marginal prices for electric energy of the California ISO region are available from California ISO OASIS (2011). We assumed the station operator to purchase electric energy based on a market-based rate tariff. We therefore used the hourly prices from Monday 12 a.m. to Sunday 12:00 p.m. of the years 2009 to 2013 to estimate the mean price and the noise variance for each hour of a typical week and added a margin of 10% for distribution charges. We estimated the mean price and the noise variance quantities for each swap station separately, as energy prices may vary by location in the locational marginal pricing system employed

[1] http://www.scenos.de/research/swapping/data.zip

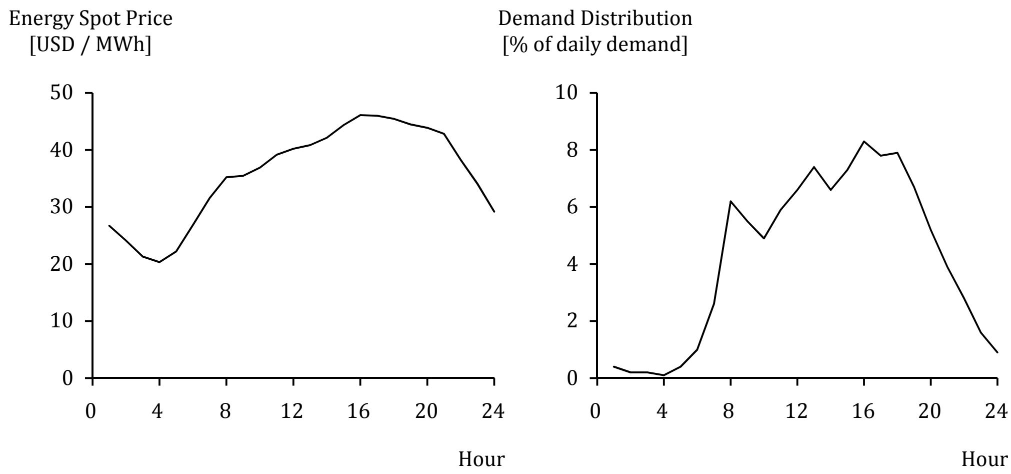

Figure 3 **Energy Spot Price and Demand Distribution Profile for a Sample Day**

by the California ISO. We use the estimated mean and noise variance to parameterize the mean-reverting price process as outlined in Section 3.1. We then discretized the energy prices into eleven intervals of equal probability around their means for each time period for the computations. Figure 3 on the left shows average energy spot prices for Mondays at the San Diego Gas and Electric Advanced Pricing Node.

Cost of Batteries and Charging Capacity. The price of a complete automotive lithium-ion battery pack in 2012 amounted to about USD 450 per kilowatt hour (kWh) (Hensley et al. 2012). The battery pack built in the Renault Fluence Z.E., the only car currently available for use with swap stations, has a capacity of 22 kWh amounting to a total cost of roughly USD 10,000 per battery. With an expected lifespan of 8 years, expected reductions in battery cost Hensley et al. (2012), and cost for remanufacturing of a pack, we estimated the discounted infinite horizon cost of a battery to be USD 12,500 (see Appendix A for details).

The cost of installing a unit of charging capacity depends on the available connection to the power grid. While charging a few batteries in parallel does not put too much strain on the grid, large charging capacities require enough available energy distribution capacities. Cost for installation of a charging bay is very hard to estimate, because it highly depends on the existing energy distribution infrastructure and used automation hardware of the swap station. Considering the fact that a mass-produced consumer charging station with manual control currently costs around USD 1,700, which does not include installation cost and possible required extensions to the energy distribution network, we believe a cost of $C^K = 25,000$ to be of the right magnitude for the overall infinite horizon cost of a fully automated charging bay. In any case, we perform a parameter sweep and our results show that the optimal number of charging bays is very robust with respect to C^K.

Station Locations, Transshipment Cost and Transshipment Times. We chose ten locations for swap stations along the bay area freeway network and did not consider intracity streets.

This is justified by the fact that battery swapping will mainly be needed for long-distance travel, while charging for shorter trips is likely to be performed at a charging station at home (Mak et al. 2013).

We calculated travel times between any two stations using Microsoft Map Point 2012. We rounded the times up to the next full period and added another period for arrival of the transshipment shuttle. We assume transshipment cost to be charged by battery and distance. As a proxy, we used taxi rates of San Francisco, CA, which we believe to be an upper bound on the cost for transshipment services. The taxi rate in San Francisco is USD 2.75 per mile plus a fixed cost of USD 3.50 for the first 1/5 mile. We set the variable transshipment cost to be USD 2.75 per battery and mile and the fixed cost to USD 3.50 per trip.

Table 1 summarizes parameters that have common values in all our experiments.

Parameter	Symbol	Value
Number of periods	T	168
Charging time	L	7
Discount Factor	γ	0.99999
Penalty Cost	p	100
No. of Energy Cost Intervals	$\|E\|$	11
Unit Cost Battery	C^N	12,500
Unit Cost Charging Bay	C^K	25,000
Mean Reversion Factor	κ	0.50

Table 1 Parameter Values in all Experiments

6.2. Single Station

We first consider a single station to identify the fundamental effects of different parameters on the optimal solution. Because of the huge state space of the exact model, we can solve the problem optimally only for a single station and for low demand rates.

6.2.1. Base Case. In our base case for the single station model we normalize the absolute mean demand for batteries over all $T = 168$ modeled periods to equal $T\sum_t \lambda_t = 168$, i.e., on average 168 batteries are requested in a week. The relative mean demand rates λ_t in each period are chosen according to the profile shown in Figure 3. Additionally, we discretized the energy price state space into eleven intervals of equal probability around the mean in each period. Figure 4 shows the optimal expected operating cost and equipment cost for increasing charging capacity. The minimum expected total cost is incurred for $N = 27$ batteries and $K = 11$ charging bays. The infinite horizon expected operating cost is USD $151,120$ for these values and total cost amounts to USD $763,280$. While lower operating costs could be achieved for higher values of K, the reduction in operating costs is not sufficient to make the expenses for additional charging bays profitable.

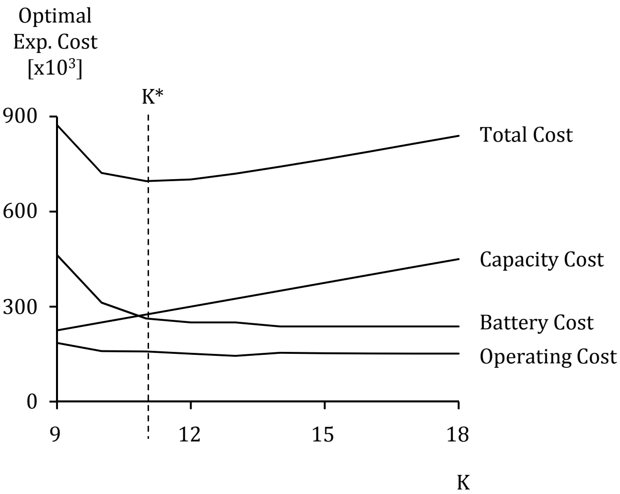

Figure 4 **Optimal Exp. Cost**

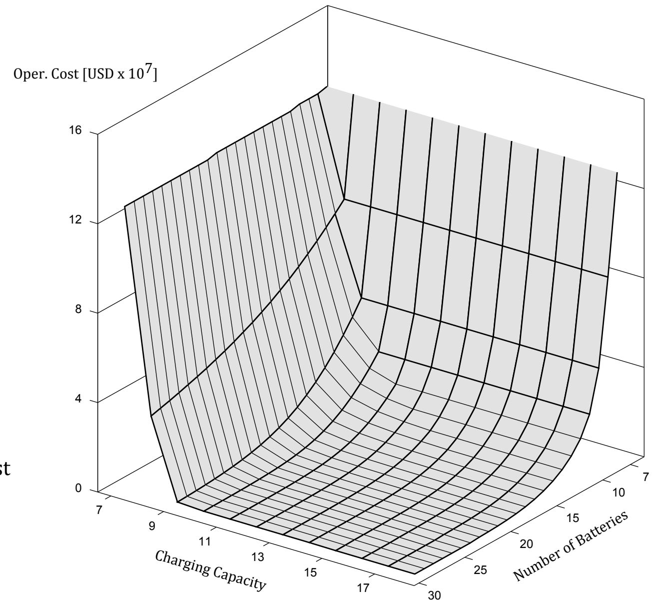

Figure 5 **Operating Cost**

6.2.2. Sensitivity. The base case assumes the parameter values summarized in Table 1. Some of the parameters were estimated and are subject to variation over time (in case of, e.g., battery cost, energy cost) or depend on the specific contract between the operator and customer (for penalty costs in case of stockouts). To quantify the effects of these exogenous parameters on the solution, we next perform a sensitivity analysis.

There is an intuitive relation among all cost factors. Keeping all other factors constant, lower cost for batteries and charging bays will result in higher optimal values for N and K; higher penalty costs for stockouts will have a similar effect. Total expected cost will in general increase, when increasing any of the cost parameters and decrease otherwise. Of special interest are the relations of the components between each other. For example, for increasing energy prices and lower battery cost, exploiting price arbitrage over the day will get more attractive compared to our base case, leading to a higher optimal value for N.

Effect of Energy Price Volatility. To assess the effect of variable energy prices, we computed the optimal values of N and K for three different price scenario. In the first scenario, energy prices are constant, deterministic and equal to the daily mean energy price, which refers to a typical retail tariff. In the second scenario, energy prices are deterministic, but the hourly rate varies over the 168 periods according to the profile in Figure 3. This represents a Time-Of-Usage contract, where prices depend on the hour of the day. In the third scenario, we model energy prices to be stochastic and use the estimated values for mean and variance as described. The last scenario corresponds to purchasing electricity from the spot market at real time prices and we model the price process as a mean reverting process (see Section 3.1).

To isolate the effect of price volatility, we calculated the optimal values of N and K for a constant demand rate of $\lambda_t = 1.0$ for all $t = 0, \ldots, T$.

As a reference point, Table 2 reports the optimal values for N and K along with the expected total cost for the analyzed scenarios.

The first observation is that the expected operating cost is almost identical for variable deterministic and stochastic prices. This is an intuitive result, because we assume stationary distributions and the average energy procurement costs equal their mean in each period in the long run.

The second key observation is that for constant demand rates the expected operating cost is also almost identical for variable and constant energy prices. This effect is surprising at first, because one would expect that lower energy purchasing costs are possible by exploiting price variations. While technically this is possible, we cannot observe this effect due to the high costs for batteries and charging bays. Both N^* and K^* do not vary between variable and constant, and deterministic and stochastic energy price scenarios, because the lower operating costs achievable by exploiting energy price variations are not sufficient to balance cost of additional batteries at a cost of USD 12,500 per battery, i.e., at the assumed ratio of energy cost to battery cost. In other words, it is not beneficial to delay charging batteries to periods with lower expected energy prices if this results in an increased number of batteries, because the savings achievable do not justify the investment in additional batteries that are required to prevent stockouts.

For variable demand rates, the operating cost is a bit lower for variable energy prices. This is caused by the fact that energy prices tend to be high, when demand for batteries is high, but charging times shift the procurement of energy for recharging to periods with lower average prices (see Figure 3).

Effect of Demand Volatility. Similarly, to quantify the effect of demand volatility over the day on the optimal values of N and K, we solve the problem for constant demand rates of $\lambda_t = 1.0$ to demand rates that vary over the week. In both scenarios, the expected total demand over the planning horizon equals $T \sum_t \lambda_t = 168.0$. Comparing the two columns of Table 2, it can be seen that in general volatile demand requires considerably more batteries in the system than constant demand, which shows that neglecting the variability of demand leads to an underestimation of expenses for batteries.

Effect of Infrastructure Cost. Especially the cost of batteries is changing rapidly (see, e.g., Boston Consulting Group 2010). To quantify effects on the optimal solution, we perform a sweep over the cost for batteries and charging bays and solve instances for C^N between USD 5,000 and USD 25,000 and instances for C^K between USD 10,000 and USD 40,000. Table 3 summarizes the results of our computations.

Energy Price		Demand constant λ	Demand variable λ
variable stochastic	N^*	21	**27**
	K^*	11	**11**
	cost	696,832	**763,280**
variable deterministic	N^*	21	27
	K^*	11	11
	cost	696,002	762,640
constant deterministic	N^*	21	27
	K^*	11	11
	cost	695,981	765,951

Table 2 Optimal N and K and Minimum Expected Cost in USD

C^K		C^N 5,000	10,000	12,500	15,000	20,000	25,000
10,000	N^*	29	27	26	26	26	25
	K^*	11	12	12	12	12	12
	Cost	389,223	528,164	595,263	660,263	790,263	916,384
20,000	N^*	29	27	27	27	26	25
	K^*	11	11	11	11	11	12
	Cost	499,223	640,140	707,640	775,140	906,910	1,036,384
25,000	N^*	29	27	**27**	27	26	26
	K^*	11	11	**11**	11	11	11
	Cost	554,223	695,140	**763,280**	830,140	961,910	1,091,910
30,000	N^*	31	27	27	27	26	26
	K^*	10	11	11	11	11	11
	Cost	599,651	750,140	817,640	885,140	1,016,910	1,146,910
40,000	N^*	32	30	29	27	26	26
	K^*	10	10	10	11	11	11
	Cost	699,356	852,382	926,376	995,140	1,126,910	1,256,910

Table 3 Optimal N and K and Minimum Expected Cost in USD

It can be observed that in general the optimal values for N and K are very robust with respect to varying equipment cost. The optimal number of batteries remains between 25 and 32 for all combinations of C^N and C^K, while the optimal number of charging bays ranges between 10 and 12. Our results also show the intuitive result that batteries and charging bays are a substitute for each other to a certain extent.

Effect of Penalty Cost to Energy Price Ratio. For the relation of average energy prices to penalty costs, we perform calculations for per unit per period penalty costs from USD 40 to USD 200 in steps of USD 20. Table 4 summarizes the results and shows that total cost increases for higher penalty costs.

p	40	60	80	100	120	140	160	180	200
N^*	25	26	27	**27**	27	28	28	28	28
K^*	11	11	11	**11**	11	11	11	11	11
Op. Cost	155,739	151,688	145,794	**151,120**	154,436	145,272	147,678	150,077	152,447
Tot. Cost	743,239	751,688	758,294	**763,280**	766,936	770,272	772,678	775,077	777,447

Table 4 Optimal N and K and Minimum Expected Cost in USD

Again the optimal values of N and K are very robust with respect to the relation of the penalty cost to energy prices. In general, lower penalty costs lead to slightly lower optimal number of batteries, and, for our values of C^N and C^K, the optimal number of charging bays remains constant.

6.3. Network of Stations

For the network of stations, we cannot solve Problem (11) exactly and instead we approximate the solution. We first validate our approximation approach and then analyze effects on the operating cost, battery and charging capacity expenses induced by the network structure.

6.3.1. Algorithm Validation. To validate the performance of Algorithms 3 and 4 that are used to solve the network model, we perform a number of tests on problem instances that can be solved optimally by Algorithm 2 and compare the solutions. However, we can solve the problem optimally only for a single station and low demand rates. We therefore must rely on the comparison of the results obtained from the approximate algorithms (Algorithm 3 and 4) to the results obtained from the optimal solution algorithm (Algorithm 2) for a single station.

For the first test, we optimized the number of batteries for a fixed charging capacity, i.e., we applied Algorithm 4 for fixed values of K. For charging capacities lower than the average demand rate, the system always operates at an inventory position below zero and at full utilization of the charging bays. In this case the optimal number of batteries equals the charging capacity, because the value of an additional battery is zero, as it can never be charged.

Figure 6 shows the optimal number of batteries for increasing charging capacity up to $K = 19$ along with the best solutions found by Algorithm 4 for values $K \geq 8$. We cannot compute the optimal solution for values of $K > 19$, because the problem becomes too large for the exact algorithm and we must rely on our approximation algorithm.

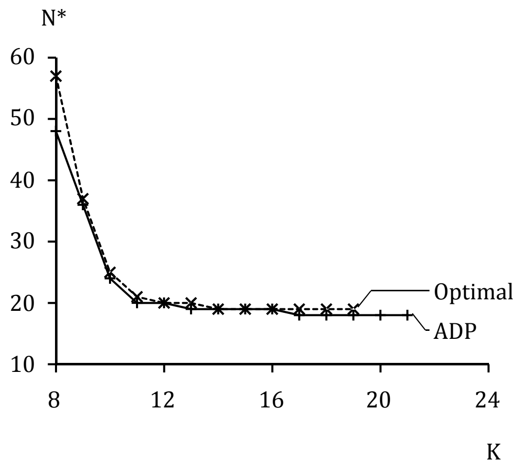

Figure 6 **Values of N^* for fixed K ADP vs. Optimal**

The solutions found by the approximation are either equal to the optimal solutions or slightly underestimate the optimal number of batteries for higher values of K. For lower values of K the algorithm takes more iterations to converge and we had to stop the simulation early, before convergence sets in.

We then concurrently optimized the number of batteries and charging bays for a single station. Figures 7 and 8 compare the performance of the ADP algorithm to the optimal values. Figure 7 shows the values of N and K found by Algorithm 4 and Figure 8 shows the expected operating cost resulting from simulating the current value function approximation in the corresponding iteration along with confidence intervals. The dashed lines show the upper and lower boundary of the 95% confindence interval for 100 replications of the simulation procedure outlined in Section 5.2.

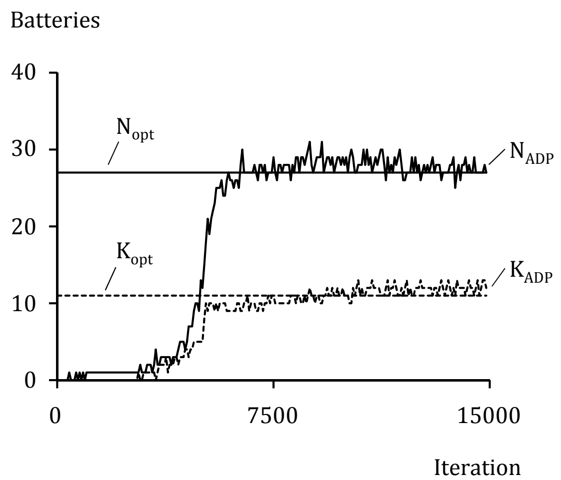

Figure 7 Values of N^* and K^* ADP vs. Optimal

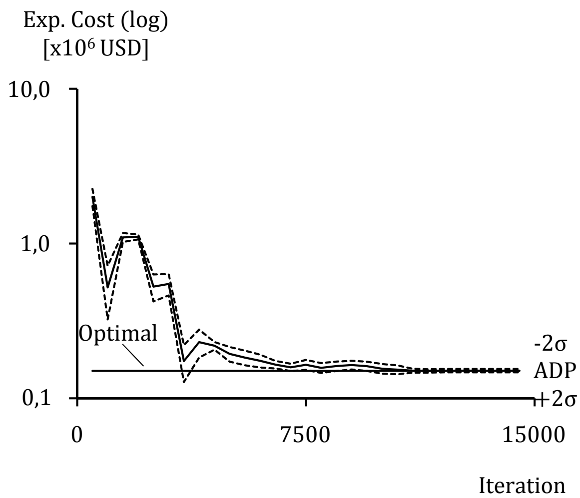

Figure 8 Exp. Operating Cost ADP vs. Optimal

The upper bound of the confidence interval for the expected operating cost is at USD 151,683 after 11,000 iterations, which corresponds to an optimality gap of 0.80% for this instance. The upper bound of the confidence interval for total cost is at USD 789,183 for the ADP solution compared to USD 763,280 in the optimal solution. The difference is mainly caused by the fact that the ADP solution had $K = 12$ charging bays in its last iteration vs. $K = 11$ for the optimal solution. For practical problems, one can easily simulate a number of points around the optimal solution and implement the lowest cost solution.

In the second test we solved a test instance consisting of a network of swap stations with identical parameters to compare it to the optimal results for ten single stations that are operating isolated from each other. Doing this we can check whether the solution obtained from the approximate algorithms for the network instance shows the expected properties, e.g., total expected cost must be less than or equal to the solution for isolated stations. Additionally, we can quantify the effect that operating swap stations in a network has on the optimal equipment of a station with batteries and charging bays.

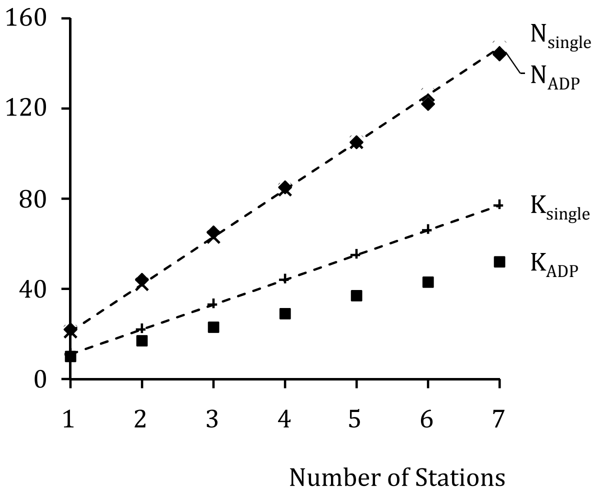

Figure 9 N **and** K **for increasing No. of Stations**

Figure 10 Swap Station Locations in the Bay Area

Figure 9 shows the intuitive result that in a network of stations lower investment in equipment is necessary. However, this is especially true for charging bays, whereas the number of batteries in the system is not considerably lower. This can be explained by the relatively long transshipment lead times of between four and six periods in our test scenario. In this case, the advantage of a network based system lies especially in sharing charging bays, while transshipment of charged batteries cannot reduce the required safety stock considerably.

6.3.2. Base Case. Our base case considers ten battery swap stations spread out over the bay area in California (see Figure 10). We chose this region for our scenario, because the EV adoption rates here are highest in California. We chose prominent locations near highways for the stations, because battery swapping is mainly required for longer distance travel. We used energy price data of ten different advanced pricing nodes from CAISO and assumed equal demand distribution parameters for all stations.

Table 5 compares the solutions for the network model with and without transshipments. For total cost we can only state minimum achievable savings, because the solution for the model with transshipments was calculated by the ADP algorithm and is an upper bound on the optimal cost.

The solution shows minimum potential savings of USD 473,872 (6.2%) when transshipments are allowed between battery swap stations. If no transshipments are allowed, the optimal expected total cost amounts to USD 7,606,504 vs. USD 7,132,632, with transshipments. The savings stem not only from lower expected operating cost, but also from lower investment in batteries and charging infrastructure. Interestingly, mainly the number of required charging bays is lower when transshipments are allowed (90 vs. 110), while the total number of batteries does not differ much in both cases (268 vs. 270).

	Base Case	with Transshipments	abs. Difference	rel. Difference
N	270	268	-2	-0.7%
K	110	90	-20	-18.2%
Oper. Cost [10^6]	1,533	1,360	-173	-11.3%
Tot. Cost [10^6]	7,607	7,133	-473	-6.2%

Table 5 Results Network Model Base Case vs. with Transshipments

6.3.3. Sensitivity. So far, all exact calculations and validation scenarios as well as the base case assumed an average demand rate of $\frac{1}{T}\sum_t \lambda_t = 1.0$ per station. Additionally, the relevant factor in determining the benefit of operating a network of swap stations compared to operating them separately is the transshipment cost. This cost depends highly on the specific transshipment process and contractual details. To capture the effects of both factors, we analyze the solution sensitivity with respect to demand rate and transshipment cost. As mentioned, we can solve instances for higher demand rates only approximately using Algorithms 3 and 4.

Effect of Demand Rate. We performed experiments for average demand rates of $\frac{1}{T}\sum_t \lambda_t$ between 1.0 and 10.0. A single battery swap takes about 2-3 minutes, which allows a maximum of 20 swaps per hour. For an average demand rate of 10.0, the peak demand rate is 19.84 (see Figure 3), and we therefore use an average demand rate of 10.0 as an upper bound on the average acceptable demand rate.

Our results in Table 6 show that the total number of batteries in the system grows sublinearly and the number of charging bays in the system grows approximately linearly in the demand rate. The behavior with respect to batteries can be explained by the fact that we model the demand for batteries in each period to follow a Poisson distribution. In this case, the standard deviation of the arrivals equals the square root of the demand rate and, as a result, an increasing demand rate causes the safety stock to increase approximately proportionally to the standard deviation of demand. Table 6 summarizes the results for varying the demand rate.

Avg. Demand Rate	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
N	**268**	482	682	881	1,073	1,269	1,455	1,648	1,836	2,016
K	**90**	178	271	360	453	538	622	714	809	898
Oper. Cost [10^6]	**1,533**	3,021	4,569	6,129	7,593	9,287	10,703	11,932	13,547	15,252
Tot. Cost [10^6]	**7,133**	13,490	19,875	26,142	32,335	38,599	44,446	50,376	56,716	62,897

Table 6 Cost and Optimization Results for varying Demand Rate

Transshipment Cost. The assumed transshipment costs were calculated based on taxi rates in the bay area. Most likely, transshipment service can be purchased at a lower price from dedicated service providers. We therefore perform calculations for transshipment costs from 10% to 90% of the estimated cost.

Our results show the intuitive result that for lower transshipment costs, total operating costs decrease. Additionally, the optimal numbers of batteries and charging bays also decrease slightly, because the pooling effect can be achieved at a lower cost than providing more batteries and spare charging capacity for the risk of stocking out. Of course, in any instance the total average demand rate multiplied by the charging time is a lower bound on the optimal number of charging bays. Table 7 summarizes the results for varying transshipment cost.

% of Base Cost	10	20	30	40	50	60	70	80	90	100
N	240	245	246	247	250	253	255	260	264	**268**
K	78	78	80	81	82	84	86	89	90	**90**
Oper. Cost [10^6]	1,564	1,546	1,527	1,512	1,500	1,487	1,499	1,506	1,520	**1,533**
Tot. Cost [10^6]	6,514	6,558	6,602	6,625	6,675	6,749	6,836	6,981	7,070	**7,133**

Table 7 Cost and Optimization Results for varying Transshipment Cost

7. Conclusion

In this paper, we have modeled and analyzed the problem of operating and equipping a network of battery swap stations with charging bays and batteries. The goal is to calculate the optimal equipment configuration of a single station and of a network of swap stations, while considering the uncertainty in demand for swap service and prices of electric energy. To solve our model, we have developed an optimal solution algorithm for small problems and an approximating algorithm based on dynamic programming and Monte-Carlo-Sampling for instances of relevant size. Our approximation provides near-optimal results for single station problems and its results for the network problem provide a lower bound on the savings achievable by allowing lateral transshipments of batteries between swap stations.

To identify relevant parameters influencing the optimal equipment choice of a station operator, we have implemented our solution approach and conducted an extensive numerical study based on publicly available data. Our implementation is a decision making tool for the equipment and charging decisions in a battery swap network and can be readily applied by system operators. Our results show that equipment costs account for a large portion of total infinite horizon costs of a battery swap station, especially during the launch phase when demand rates are low. As a result, identifying the optimal quantities of batteries and charging bays is a crucial task, and our tool is capable of performing it.

We also derived important managerial implications for operating a network of battery swap stations. Our calculations show that expenses for equipment can be substantially reduced, if transshipments between stations are allowed. We also identified unleveled demand over the day as a key driver of equipment costs, which suggests incentivizing customers to shift their demand for swapping service to off-peak hours. Another key finding with implications beyond battery swap stations is that at the current cost of batteries, it is not profitable to purchase more batteries for exploitation of arbitrage due to the differences in electricity costs over the day.

We also hope to spark interest in further investigating the characteristics of battery swap stations and networks to help increase success of EVs. Especially integrating battery recharging decisions with dispatch of intermittent energy sources seems to be a field of research worth investigating.

Acknowledgments

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Appendix A: Estimation of C^N and C^K

The price of a complete automotive lithium-ion battery pack in 2012 amounted to about USD 450 per kilowatt hour (kWh) (Hensley et al. 2012). The battery pack built in the Renault Fluence Z.E., the only car currently available for use with swap stations, has a capacity of 22 kWh. We therefore used a current price for a 22 kWh battery of USD 10,000. Using the hourly discount factor of $\gamma = 0.99999$ and an expected life span of 8 years (this is the warranty period for the battery of a BMW i3), the cost for remanufacturing/replacing for the first time is multiplied by $\gamma_1 = 0.50$ and for the second time by $\gamma_1^2 = 0.25$ and so forth. Assuming 25% of purchasing cost for remanufacturing of a battery, this results in infinite horizon cost of $C^N = 10,000 + \frac{\gamma_1}{1-\gamma_1} 2500 \approx 12,500$.

Cost for installation of a charging bay is very hard to estimate, because it highly depends on the existing energy distribution infrastructure and used automation hardware of the swap station. Considering the fact that a mass-produced consumer charging station with manual control currently costs around USD 1,700, which does not include installation cost and possible required extensions to the energy distribution network, we believe a cost of $C^K = 25,000$ to be of the right magnitude for the overall infinite horizon cost of a fully automated charging bay. In any case, our results show that the optimal number of charging bays is very robust with respect to C^K.

Appendix B: Notation

$\underline{b}_t^k$	lower bound on optimal value in iteration k
$\bar{b}_t^k$	upper bound on optimal value in iteration k
β_n	amount of energy absorbed by the battery in period n of the charging process
C_t	cost contribution in period t
$\bar{C}_t$	cost contribution in period t of lower bounding model
C^K	infinite horizon cost per charging bay
C^N	infinite horizon cost per battery
D_t	demand for batteries in period t
e_t	price of electric energy in period t
E_t	price of electric energy in period prior to period t
f_t	infinite horizon operating cost from period t
$\mathcal{E}$	space of possible values for E
γ	discount factor
K	number of charging bays
κ	mean reversion parameter
λ_t	demand rate in period t
L	battery charge time
N	number of batteries
μ_t	mean of energy cost distribution in period t
p_t	per period, per unit penalty cost for stockouts
Π	set of all coordinates of state space
$R_{t,i}$	number of charging batteries in period t that become available in i periods
σ_t	standard deviation of energy cost distribution in period t
S_t	state
$\mathcal{S}$	state space
$\mathcal{S}^c$	convex subspace of state space
T	planning horizon, i.e., number of periods
U_t	number of available uncharged batteries in period t
$\mathcal{U}$	space of feasible values for U
$W_{t,i}$	number of charged batteries in period t that become available in i periods
$\bar{W}_t$	inventory position of charged batteries in period t
V_t	value function
$\underline{V}_t$	global lower bound
$\underline{V}_t^K$	coordinatewise lower bound for K charging bays
$\underline{V}_t^N$	coordinatewise lower bound for N batteries
$\tilde{V}_t$	value function of transformed model
$\hat{V}_t$	value function approximation
$\bar{V}_t$	value function of lower bounding model
$\mathcal{W}$	space of feasible values for W
x_t	number of batteries to start charging in period t
$\mathcal{X}$	action space
$\tilde{X}$	action space of transformed model
$\bar{X}$	action space of lower bounding model
$y_{t,i,j}$	number of uncharged batteries transshipped from station i to j starting in t
$z_{t,i,j}$	number of charged batteries transshipped from station i to j starting in t
ζ_t	normally distributed noise term of energy price in period t

Appendix C: Proofs of Statements

All properties from Sections 3.2 and 4.2 are proven for finite horizon versions of the dynamic program. In this dynamic program the fixed planning horizon of T periods is cycled through I times, resulting in a total of $\hat{T} = IT$ periods. The corresponding results for the infinite horizon case follow by taking the limit $I \to \infty$ (see, for instance, Bertsekas 2007, Chapter 1).

C.1. Proof of Property 1

We prove the property by a counterexample.

We consider a deterministic model with $L = 1$, $D_t = 10$, $E_t = 3$, and $p_t = 100$ over a horizon of $\bar{T} = 2$ periods, i.e., all cost is zero from the third period on. Without loss of generality, we set $K = M$, where M is a big number. We omit dependence of V_t on K.

We assume $V_{\bar{T}+1} = 0$ and $U_0 = 0$.

In $t = \bar{T} = 2$ it is always optimal not to start charging batteries, because the terminal value $V_{\bar{T}+1} = 0$ and energy prices $E_t > 0$. Then $V_2(S_2, 100) = 100[10 - W_{2,0}]^+ + 2([W_{2,0}]^+ + U_2)$ and V_2 is convex in S_2. Function V_2 takes its minimum at $(W_{2,0}, U_2) = (10, 0)$.

For $L = 1$, S_t is a function of S_{t-1}, x_{t-1}, and D_{t-1}. It is defined by $W_{t,0} = W_{t-1,0} + x_{t-1} - D_{t-1} = W_{t-1,0} + x_{t-1} - 10$ and $U_t = U_{t-1} - x_{t-1} + \min([-W_{t-1,0}]^+ + D_{t-1}, [W_{t-1,0}]^+ + x_{t-1}) = U_{t-1} - x_{t-1} + \min([-W_{t-1,0}]^+ + 10, [W_{t-1,0}]^+ + x_{t-1})$.

Since $U_0 = 0$, we have $x_0 = 0$. As a result, $V_0(-11, 0) = 2100 + V_1(-21, 0)$ and $V_0(-10, 0) = 2000 + V_1(-20, 0)$, $V_0(0, 0) = 1000 + \gamma V_1(-10, 0)$, $V_0(1, 0) = 902 + \gamma V_1(-9, 1)$, $V_0(20, 0) = 40 + V_1(10, 10)$ and $V_0(21, 0) = 42 + V_1(11, 10)$.

Consider $V_1(S_1)$. It is given by

$$V_1(S_1) = 100[10 - W_{1,0}]^+ + 2([W_{1,0}]^+ + U_1) + \min_{x \le U_1} \left\{ 3x + \gamma(100[10 - W_{2,0}]^+ + 2([W_{2,0}]^+ + U_2)) \right\}.$$

We obtain $V_1(-10, 0) = 2000 + \gamma \cdot 3000$, $V_1(-9, 1) = 1902 + 3 + \gamma \cdot 2802$, $V_1(-21, 0) = 3100 + \gamma \cdot 4100$, $V_1(-20, 0) = 3000 + \gamma \cdot 4000$, $V_1(10, 10) = 40 + 30 + \gamma \cdot 40$ and $V_1(11, 10) = 42 + 27 + \gamma \cdot 42$.

Let us assume $\gamma = 0.9$. The first difference of V_0 is then not increasing for all S_0, because

$$V_0(-10, 0) - V_0(-11, 0) = 2000 - 2100 + \gamma(3000 - 3100) + \gamma^2(4000 - 4100) = -100(1 + \gamma + \gamma^2) = -271$$

$$V_0(1, 0) - V_0(0, 0) = 902 - 1000 + \gamma(1905 - 2000) + \gamma^2(2802 - 3000) = -98 - 95\gamma - 198\gamma^2 = -343.88$$

and

$$V_0(21, 0) - V_0(20, 0) = 42 - 40 + \gamma(69 - 70) + \gamma^2(42 - 40) = 2 - \gamma + 2\gamma^2 = 2.72.$$

This completes the counterexample and shows that $V_t(S_t, K)$ is not coordinatewise convex in $S_t \in \mathcal{S}$.

C.2. Proof of Property 2

We prove coordinatewise convexity of $\tilde{V}_t$ in W, K, and N by analyzing a continuous extension $\bar{V}_t$. We first show that $\bar{V}_t$ is convex in W, K, and N. We then show that $\tilde{V}_t$ is the restriction of $\bar{V}_t$ to integer values for W, K, and N. Coordinatewise convexity then follows from the fact that $\bar{V}_t$ is convex.

We define:

- $\bar{\mathcal{X}}(W, K, N) = \textbf{conv } \tilde{\mathcal{X}}(W, K, N)$
- $\bar{\mathcal{W}}(K) = \textbf{conv } \mathcal{W}(K)$.
- $J_t(x, E_t, W_t, E_{t+1}, D_t, K, N) = C_t(W_t, x, E_t, D_t) + \gamma \bar{V}_{t+1}(W_{t+1}, E_{t+1}, K, N)$

- $x_t^*(E_t, W_t, E_{t+1}, D_t, K, N) = \arg\min_{x \in \bar{\mathcal{X}}} J_t(x, E_t, W_t, E_{t+1}, D_t, K, N)$
- $\bar{J}_t(x, E_t, W_t, E_{t+1}, D_t, K, N) = J_t(\lfloor x \rfloor, E_t, W_t, E_{t+1}, D_t, K, N)(\lceil x \rceil - x) + J_t(\lceil x \rceil, E_t, W_t, E_{t+1}, D_t, K, N)(x - \lfloor x \rfloor)$
- $\bar{V}_t(W_t, E_t, K, N) = \mathbb{E}_{E_{t+1}, D_t}\left[\min_{x \in \bar{\mathcal{X}}(W_t,K,N)} \left\{\bar{J}_t\left(x, E_t, W_t, E_{t+1}, D_t, K, N\right)\right\} \Big| E_t\right]$

Note that:

1. J_t is convex, if C_t and $\bar{V}_{t+1}$ are convex and W_{t+1} is an affine function of W_t and x.
2. $\bar{J}_t$ is convex, if J_t is convex, because it is the linear interpolation of J_t between integer points in x.

The state transition function is given by $W_{t+1}(W_t, D_t, x) = [W_{t,0} + W_{t,1} - D_t, W_{t,2}, \ldots, W_{t,L}, x]$, and it is clearly an affine function of W_t and x. We also use the standard trick to turn a constrained convex problem into an unconstrained convex problem via the set characteristic function.

We denote the domain of J_t for given E_t, E_{t+1}, and D_t by $\mathcal{J}(E_t, E_{t+1}, D_t)$ and the domain of $\bar{V}_{t,i}$ for given E_t by $\mathcal{V}(E_t)$. We have

$$\mathcal{J}(E_t, E_{t+1}, D_t) = \{(x, E_t, W_t, E_{t+1}, D_t, K, N) : K \in \mathbb{R}_0^+, N \in \mathbb{R}_0^+, W_t \in \bar{\mathcal{W}}(K), x \in \bar{\mathcal{X}}(W_t, K, N)\}$$

and

$$\mathcal{V}(E_t) = \{(W_t, E_t, K, N) : K \in \mathbb{R}_0^+, N \in \mathbb{R}_0^+, W_t \in \bar{\mathcal{W}}(K), x \in \bar{\mathcal{X}}(W_t, K, N)\}.$$

Sets $\mathcal{J}(E_t, E_{t+1}, D_t)$ and $\mathcal{V}(E_t)$ are polyhedra and therefore convex.

We show convexity of $\bar{V}_t(W_t, E_t, K, N)$ by induction. First note that $C_t(W_t, x, E_t, D_t)$ is convex in x and W_t for all E_t, D_t, and t when $p_t \geq 0$.

Base Case.

Let us assume $\bar{V}_{\bar{T}+1} = 0$. Then $J_{\bar{T}}(x, E_{\bar{T}}, W_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}}, K, N) = C_{\bar{T}}(W_{\bar{T}}, x, E_{\bar{T}}, D_{\bar{T}})$. Set $\mathcal{J}(E_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}})$ is convex and nonempty. Additionally, $J_{\bar{T}}$ and $\bar{J}_{\bar{T}}$ are convex. Then $\min_{x \in \bar{\mathcal{X}}(W_t,K,N)} \left\{\bar{J}_{\bar{T}}\left(x, E_{\bar{T}}, W_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}}, K, N\right)\right\}$ is convex in $W_{\bar{T}}$, K, and N, because it is the minimum of a convex function over a nonempty convex set (Boyd and Vandenberghe 2004, Section 3.2.5). Taking the expected value with respect to e preserves convexity and we conclude that $\bar{V}_{\bar{T}}(W_{\bar{T}}, E, K, N)$ is convex in $W_{\bar{T}}$, K, and N.

Induction Step.

Now assume that $\bar{V}_{t+1}(W_{t+1}, E_{t+1}, K, N)$ is convex in W_{t+1}, K, and N on $\mathcal{V}(E_{t+1})$.

Then, J_t and $\bar{J}_t$ are convex in W_t, K, and N, because C_t is convex and W_{t+1} is an affine mapping of W_t and x. Then, because $\mathcal{V}(E)$ is convex and nonempty, and because taking the expected value preserves convexity, we conclude that $\bar{V}_{t,i}(W, E, K, N)$ is convex in W, K, and N on $\mathcal{V}(E)$.

This concludes the first part of the proof.

It remains to show, that $\tilde{V}_t(W, E, K, N) = \bar{V}_t(W, E, K, N)$ for all $(E, W, K, N) \in \mathcal{E} \times \{W \in \mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\} \subseteq \mathcal{E} \times \mathcal{V}(E)$. We show this by induction. First, note that for $(E, W, K, N) \in \mathcal{E} \times \{W \in$

$\mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$ we have $\max\{x : x \in \bar{\mathcal{X}}(W,K,N)\} = \max\{x : x \in \tilde{\mathcal{X}}(W,K,N)\} \in \mathbb{Z}_0^+$ and $\min\{x : x \in \bar{\mathcal{X}}(W,K,N)\} = \min\{x : x \in \tilde{\mathcal{X}}(W,K,N)\} \in \mathbb{Z}_0^+$.

Base Case.

Assume $\bar{V}_{\bar{T}+1} = 0$. Function $\bar{J}_{\bar{T}}$ is piece-wise linear in x and has integer breakpoints. Then it holds

$$\begin{aligned}
&\min_{x \in \bar{\mathcal{X}}(W_{\bar{T}},K,N)} \bar{J}_{\bar{T}}(x, E_{\bar{T}}, W_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}}, K, N) \\
&= \min\{\bar{J}_{\bar{T}}(\lfloor x^*_{\bar{T}} \rfloor, E_{\bar{T}}, W_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}}, K, N), \bar{J}_{\bar{T}}(\lceil x^*_{\bar{T}} \rceil, E_{\bar{T}}, W_{\bar{T}}, E_{\bar{T}+1}, D_{\bar{T}}, K, N)\} \\
&= \min_{x \in \tilde{\mathcal{X}}(W_{\bar{T}},K,N)} C_{\bar{T}}(W_{\bar{T}}, x, E_{\bar{T}}, D_{\bar{T}}).
\end{aligned}$$

It follows that $\tilde{V}_{\bar{T}}(W_{\bar{T}}, E_{\bar{T}}, K, N) = \bar{V}_{\bar{T}}(W_{\bar{T}}, E_{\bar{T}}, K, N)$ for all $(E_{\bar{T}}, W_{\bar{T}}, K, N) \in \mathcal{E} \times \{W_{\bar{T}} \in \mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$.

Induction Step.

The induction assumption is $\tilde{V}_{t+1}(W_{t+1}, E_{t+1}, K, N) = \bar{V}_{t+1}(W_{t+1}, E_{t+1}, K, N)$ for all $(E_{t+1}, W_{t+1}, K, N) \in \mathcal{E} \times \{W_{t+1} \in \mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$.

From the induction assumption, it follows that $C_t(W_t, x, E_t, D_t) + \gamma \tilde{V}_{t+1}(W_{t+1}, E_{t+1}, K, N) = C_t(W_t, x, E_t, D_t) + \gamma \bar{V}_{t+1}(W_{t+1}, E_{t+1}, K, N) = \bar{J}_t(x, E_t, W_t, E_{t+1}, D_t, K, N)$ for all $(E_{t+1}, W_{t+1}, K, N) \in \mathcal{E} \times \{W_{t+1} \in \mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$.

We have that $(E_{t+1}, W_{t+1}, K, N) \in \mathcal{E} \times \{\mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$, where W_{t+1} is a function of W_t, x_t, and D_t, if the following conditions hold:

1. $x_t \in \tilde{\mathcal{X}}(W_t, K, N)$
2. $D_t \in \mathbb{Z}_0^+$
3. $(E_t, W_t, K, N) \in \mathcal{E} \times \{\mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$.

Condition 2 holds by definition and condition 3 holds, if $W_0 \in \mathbb{Z}_+$ and $x_t \in \mathbb{Z}_+$ for all $t = 0, \ldots, t-1$. This is true since $W_0 = (N, 0, \ldots, 0)$ and $N \in \mathbb{Z}_+$ and all x^*_t and D_t are integer.

We show that $x^*_t \in \tilde{\mathcal{X}}(W_t, K, N)$. Since $\bar{J}_t$ is piece-wise linear in x with integer breakpoints, $\arg\min_{x \in \bar{\mathcal{X}}(W_t,K,N)} \bar{J}_t(x, E_t, W_t, E_{t+1}, D_t, K, N)$ is integer, if condition 3 holds (for the case $\bar{J}_t(\lfloor x^*_t \rfloor, E_t, W_t, E_{t+1}, D_t, K, N) = \bar{J}_t(\lceil x^*_t \rceil, E_t, W_t, E_{t+1}, D_t, K, N)$, we use the convention $\arg\min_x \bar{J}_t(x, E_t, W_t, E_{t+1}, D_t, K, N) = \lfloor x^*_t \rfloor$). This means that $x^*_t \in \tilde{\mathcal{X}}(W_t, K, N)$.

This establishes that $(E_{t+1}, W_{t+1}, K, N) \in \mathcal{E} \times \{\mathcal{W}(K), K \in \mathbb{Z}_{0,+}, N \in \mathbb{Z}_{0,+}\}$ and this in turn yields, by the induction assumption, that $\min_{x \in \bar{X}} \bar{J}_t = \min_{x \in \tilde{X}} J_t$.

It directly follows that $\tilde{V}_t(W_t, E_t, K, N) = \bar{V}_t(W_t, E_t, K, N)$ and we conclude that $\tilde{V}_{t,i}(W, E, K, N)$ is coordinatewise convex in W, K, and N.

C.3. Proof of Property 3

By Property 2, we know that $\tilde{V}_t$ is coordinatewise convex in W for W, K, and N integer. Additionally, for any W by Proposition 1 equation $V_t((W_t, U_t, E_t), K) = \tilde{V}_t(W_t, E_t, K, U_t + [W_{t,0}]^+ + \sum_{n=1}^{L} W_{t,n})$ holds. The statements then follows by noting that $\tilde{V}_t(W_t, E_t, K, U_t + [W_{t,0}]^+ + \sum_{n=1}^{L} W_{t,n})$ is coordinatewise convex on $\{(W, U) : W \in \mathcal{W}(K), [W_0]^+ + \sum_{n=1}^{L} W_n + U = N\}$.

C.4. Proof of Property 4

It is easy to construct counterexamples showing that f_t does not inherit any of the mentioned properties in general.

C.5. Proof of Property 5

We show coordinatewise convexity of $f_t(N,K)$ in N and K first.

Because $f_t(N,K) = \tilde{V}_t(W_0(N), E_0, K, N)$ and $W_0(N) = (N, 0, 0, \ldots, 0)$ for all N and K, we can rely on the proof of Property 2 to show coordinatewise convexity of f_t. From the proof we know that $\tilde{V}_t(W,E,K,N) = \bar{V}_t(W,E,K,N)$ for integer values of W, K, and N. Because $W_0(N)$ is integer, $\tilde{V}_t(W_0(N),E,K,N) = \bar{V}_t(W_0(N),E,K,N)$. Additionally, $\bar{V}_t(W_0(N),E,K,N)$ is convex and coordinatewise convexity of $\tilde{V}_t(W_0(N),E,K,N)$ in N follows. Coordinatewise convexity of $f_t(N,K)$ in K, follows from similar arguments.

We show that $f_t(N,K)$ is non-increasing in N and K next.

The following relations are easily verified:

1. $\mathcal{W}(K_1,N) \subseteq \mathcal{W}(K_2,N)$ for $K_2 > K_1$
2. $\mathcal{W}(K,N_1) \subseteq \mathcal{W}(K,N_2)$ for $N_2 > N_1$
3. $\tilde{\mathcal{X}}(W,K,N_1) \subseteq \tilde{\mathcal{X}}(W,K,N_2)$ for $N_2 > N_1$ and $W \in \mathcal{W}(K,N_1)$
4. $\tilde{\mathcal{X}}(W,K_1,N) \subseteq \tilde{\mathcal{X}}(W,K_2,N)$ for $K_2 > K_1$ and $W \in \mathcal{W}(K_1,N)$

We show that $f_t(N,K) = \tilde{V}_t(W_0(N),E,K,N)$ is non-increasing in K by induction on t first. Let $K_2 > K_1 \geq 0$.

Base Case.

We show $\tilde{V}_{\bar{T}}(W_{\bar{T}},E_{\bar{T}},K_1,N) \geq \tilde{V}_{\bar{T}}(W_{\bar{T}},E_{\bar{T}},K_2,N)$ for all $W_{\bar{T}} \in \mathcal{W}(K_1,N)$. By relation 4,

$$\min_{x \in \tilde{\mathcal{X}}(W_{\bar{T}},K_1,N)} C_{\bar{T}}(W_{\bar{T}},x,E_{\bar{T}},D_{\bar{T}}) \geq \min_{x \in \tilde{\mathcal{X}}(W_{\bar{T}},K_2,N)} C_{\bar{T}}(W_{\bar{T}},x,E_{\bar{T}},D_{\bar{T}})$$

and, as a result,

$$\tilde{V}_{\bar{T}}(W_{\bar{T}},E_{\bar{T}},K_1,N) \geq \tilde{V}_{\bar{T}}(W_{\bar{T}},E_{\bar{T}},K_2,N).$$

The statement of the property follows for the base case by noting that $W = W_0(N) \in \mathcal{W}(K_1,N)$.

Induction Step.

We assume that $\tilde{V}_{t+1}(W_{t+1},E_{t+1},K_1,N) \geq \tilde{V}_{t+1}(W_{t+1},E_{t+1},K_2,N)$ for all $W_{t+1} \in \mathcal{W}(K_1,N)$, E_{t+1}, and N. Note that from $W_t \in \mathcal{W}(K_1,N)$ it follows $W_{t+1} \in \mathcal{W}(K_1,N)$, if $x \in \tilde{\mathcal{X}}(W_t,K_1,N)$. It immediately follows that

$$\begin{aligned}
&\min_{x \in \tilde{\mathcal{X}}(W_t,K_1,N)} C_t(W_t,x,E_t,D_t) + \tilde{V}_{t+1}(W_{t+1},E_{t+1},K_1,N) \\
&\qquad \geq \min_{x \in \tilde{\mathcal{X}}(W_t,K_1,N)} C_t(W_t,x,E_t,D_t) + \tilde{V}_{t+1}(W_{t+1},E_{t+1},K_2,N) \\
&\qquad \geq \min_{x \in \tilde{\mathcal{X}}(W_t,K_2,N)} C_t(W_t,x,E_t,D_t) + \tilde{V}_{t+1}(W_{t+1},E_{t+1},K_2,N),
\end{aligned}$$

where the first inequality is by the induction assumption and the second inequality is by relation 4. This implies $\tilde{V}_t(W_t, E_t, K_1, N) \geq \tilde{V}_t(W_t, E_t, K_2, N)$ for $W_t \in \mathcal{W}(K_1, N)$, E_t, and N.

The statement of the property then follows, because $W_0(N) \in \mathcal{W}(K_1, N)$.

The proof that $\tilde{V}_t(W_0(N), E_0, K, N)$ is non-increasing in N follows similar arguments for two number $N_2 > N_1$ and is omitted.

C.6. Proof of Property 6

The proof is essentially identical to the proof of Property 2 and thus omitted.

C.7. Proof of Proposition 2

The proof of Proposition 2 is a byproduct of the proof of Proposition 3.

C.8. Proof of Proposition 3

We prove the statement by induction.

Base Case. We want to show that $\underline{V}_{\bar{T}} \leq \underline{V}^N_{\bar{T}} \leq f_{\bar{T}}(N,K)$ and $\underline{V}_{\bar{T}} \leq \underline{V}^K_{\bar{T}} \leq f_{\bar{T}}(N,K)$. Because for all $t > \hat{\bar{T}}$, $V_t = \underline{V}_t = 0$ and $p_t = E_t = D_t = 0$, it suffices to show that

$$\min_{x_{\bar{T}} \geq 0} \{x_{\bar{T}} E_{\bar{T}}\} \leq \min_{0 \leq x_{\bar{T}} \leq N - \bar{W}_{\bar{T}}} \{x_{\bar{T}} E_{\bar{T}}\} \leq \min_{x_{\bar{T}} \in \mathcal{X}(S_{\bar{T}}, K)} \{x_{\bar{T}} e_{\bar{T}}\}$$

and

$$\min_{x_{\bar{T}} \geq 0} \{x_{\bar{T}} E_{\bar{T}}\} \leq \min_{0 \leq x_{\bar{T}} \leq K} \{x_{\bar{T}} E_{\bar{T}}\} \leq \min_{x_{\bar{T}} \in \mathcal{X}(S_{\bar{T}}, K)} \{x_{\bar{T}} E_{\bar{T}}\}$$

for all $\bar{W}_{\bar{T}} = \sum_{n=0}^{L} W_{\bar{T}}$. The relations hold obviously by definition of $\mathcal{X}(S_{\bar{T}}, K)$.

Induction Step. Let us assume that $\underline{V}_{t+1} \leq \underline{V}^N_{t+1} \leq f_{t+1}(N,K)$ and $\underline{V}_{t+1} \leq \underline{V}^K_{t+1} \leq f_{t+1}(N,K)$ for all $\bar{W}_{t+1} = \sum_{n=0}^{L} W_{t+1,n}$.

We show relation $\underline{V}_t \leq f_t(N,K)$. The other relations are obtained similarly.

Lower bound $\underline{V}_t$ is defined by

$$\begin{aligned}\underline{V}_t =& \bar{V}_t(\bar{W}_t, E_t) + F_t(W_t, E_t) \\ =& \mathbb{E}\left[\min_{x_t \geq 0} \left\{\bar{C}_t(\bar{W}_t, x_t, E_t) + \gamma \bar{V}_{\text{mod}(t+1,T)}(\bar{W}_{\text{mod}(t+1,T)}, E_{\text{mod}(t+1,T)}) + F_t(W_t, E_t)\right\}\right].\end{aligned}$$

By noting that

$$\begin{aligned}F_t(W_t, E_t) = \gamma F(W_{t+1}, E_{t+1}) + & p_t \mathbb{E}[(D_t - W_{t,0})^+] - \gamma^L p_{t+L} \mathbb{E}[(D_{t+L} - W_{t+L-1,0} + x_t)^+] \\ & + e_t \sum_{n=1}^{L} \beta_n W_{t,n} - x_t \left(\mathbb{E}\left[\sum_{\tau=1}^{L} \gamma^\tau \beta_\tau E_{t+\tau} | E_{t+\tau-1}\right]\right),\end{aligned}$$

this simplifies to

$$\begin{aligned}\underline{V}_t = \mathbb{E}\Bigg[& p_t (D_t - W_{t,0})^+ + e_t \left(\beta_0 x_t + \sum_{n=1}^{L} \beta_n W_{t,n}\right) + \gamma^L p_{t+L}((D_{t,t+L} - \bar{W}_t)^+ - (D_{t+L} - W_{t+L,0})^+) \\ & + \min_{x_t \geq 0} \left\{\gamma \bar{V}_{t+1}(\bar{W}_{t+1}, E_{t+1}) + \gamma F_{t+1}(W_{t+1}, E_{t+1})\right\}\Bigg].\end{aligned}$$

Because $\mathbb{E}[(D_{t,t+L} - \bar{W}_t)^+] = \mathbb{E}[(D_{t+L} - W_{t+L,0})^+]$, this is the same as

$$\underline{V}_t = \mathbb{E}\left[\min_{x_t \geq 0} \left\{C_t(W_t, x_t, E_t, D_t) + \gamma \bar{V}_{t+1}(\bar{W}_{t+1}, E_{t+1}) + \gamma F_{t+1}(W_{t+1}, E_{t+1})\right\}\right]. \tag{18}$$

The relation $\underline{V}_t \leq f_t(N,K)$ then follows from the induction assumption, comparing Equation (18) to the definition of $f_t(N,K)$ and noting that the feasible set of actions $x_t \geq 0$ always includes all feasible $x \in \mathcal{X}(N,K)$.

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