Optimal Emissions Reduction Investment under Green House Gas Emissions Regulations

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Abstract

Emissions intensive firms, like utility companies with coal-fired power plants, can invest in process improvement projects at their existing facilities to reduce carbon emissions. An example of such an investment is to retrofit coal-fired power plants with the carbon capture and sequestration technology. Under different carbon regulations, their investment decisions result in a change of the production cost structure and/or impose a constraint on production quantity. We study joint production capacity and investment decisions under command-and-control and market-based regulations (including carbon tax and cap-and-trade) for an emissions intensive company that faces stochastic demand. We analytically compare the company's performance along four dimensions, including profit, total emissions, investment amount, and investment timing. We find that the company can perform better under either command-and-control and cap-and-trade along any of these four dimensions, depending on the company's investment cost and emissions regulation parameters.

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1 Introduction

Firms have long recognized the importance of controlling Green House Gas (GHG) emissions due to the threat of potential carbon emissions regulations and the intention of establishing a good public image. In the United States, progressive firms are investing to reduce their GHG emissions on a voluntary basis without established carbon emissions regulations. They participate in certain environmental leadership programs, like the Climate Leaders of the U.S. Environmental Protection Agency. Emission intensive manufacturers and utility companies are among these environmental leaders, such as the world's leading producer of aluminum Alcoa, and the well known environmental steward American Electric Power (AEP).

Investment in process improvement projects is crucial for firms with a large existing capital investment to stay competitive or even viable in a carbon constrained world. Such firms usually cannot completely forgo their existing expensive plants and equipments, which is the reality that many existing plants are facing. Due to large capital investment, a firm's only option is to improve its existing technology instead of switching to a completely new technology. One such example is the investment by Alcoa in a new smelting control algorithm to reduce GHG emissions during the aluminum production process (Hoffman (2006)). Another example is the investment by AEP in the Carbon Capture and Storage (CSS) technology for coal-fired power plants. Using CSS, coal-fired power plants capture CO_2 emitted during the generation process through pipelines and store it underground permanently. Process improvement investment can be a practical cost-effective way to control carbon emissions. This type of investment is different from a fundamental technology change investment, under which a firm abandons its existing technology and switches to a new one. One example of the latter is the investment in new power plants with renewable clean energy, like wind farms, by utility companies that mainly operate coal-fired plants.

Although progressive firms in the United States are investing in carbon emissions reduction projects without the existence of emissions regulations, government regulations on carbon emissions seem a matter of time. However, the outlook is still quite vague in terms of the eventual form of an emissions regulation. A possible regulation is cap-and-trade (Tietenberg (2006)), under which a regulated firm is allocated with some amount of initial emissions allowances either for free (i.e. grand-fathering) or through auctions. If the firm emits less than its initial allowances, it can sell the extra allowances to the emissions trading market. Otherwise, it needs to purchase extra allowances to cover its excessive emissions. A large-scale emissions cap-and-trade system was first implemented in the United States for SO_2 emissions as required by the 1990 Clean Air Act Amendment. The most well known emissions trading system for carbon emissions is the European Union Emissions Trading System (EU-ETS). Another type of a regulation is carbon tax, under which a regulated firm needs to pay tax per unit emissions. Both cap-and-trade and carbon tax regulations are market-based regulations, which impose a price on carbon. In contrast, there are command-and-control based regulations, one important type of which is to limit the total emissions of a firm during a certain time period.[1] This type of a command-and-control regulation is referred to as emission standards in Montero (2002a,b)[2]. An example of emission standards in practice is the Volatile Organic Compound (VOC) regulations under the Clean Air Act Amendment.[3] Emission standards have received much attention among economists (Helfand (1991), Milliman & Prince (1989), Montero (2002a,b), Dietz & Michaelis (2004), and Requate (2005)).[4] We refer to it by the command-and-control regulation. The effect of a firm's investment depends on the type of emissions regulation the firm is subjected to. Therefore, it is important for firms to evaluate their possible investments to reduce carbon emissions under different types of carbon emissions regulations.

The effect of investment also depends on a firm's production capacity. If the firm can produce more, then it can benefit more from its investment. Once the firm has installed CCS, the more it can produce, the more emissions allowances it can save and sell to the emissions trading market under cap-and-trade. Hence interaction exists between investment and production capacity decisions under emissions regulations. In this paper, we study the joint production capacity and investment problem for an emissions-intensive company that operates several plants under cap-and-trade, carbon tax and command-and-control regulations. It turns out that the cap-and-trade and carbon tax models are very similar and do not require separate analyses. We characterize optimal investment in process improvement projects and production capacity under these regulations with uncertain demand. We analytically compare the expected profit, total emissions, and investment amount using a single period model under cap-and-trade and command-and-control. Contrary to the preference for the cap-and-trade regulation by economists, we find out that the company can be better off with regard to any comparison criterion under the command-and-control regulation. For example, we derive sufficient and necessary conditions for the company to have higher expected profit under command-and-control. Furthermore, we compare the investment timing under cap-and-trade and command-and-control using a two-period model, and derive sufficient conditions for the company to invest earlier under either regulation. The company should not always invest earlier under the cap-and-trade regulation as claimed by economists (Tietenberg (2006)). Our results show different tendencies and insights due to the stochastic demand in our models and the flexibility of production quantity adjustments. The economics literature assumes that the company do not adjust production after a regulation. In our model the company makes an optimal production decision.

[1]See Helfand (1991) for five different types of command-and-control regulations.

[2]Another type of a command-and-control regulation, performance standards, is also studied in Montero (2002b). Performance standards, also known as concentration standards, limit the emissions per unit output instead of total emissions. Since emission standards bear greater similarity with the cap-and-trade regulation, we choose to compare emission standards and cap-and-trade regulations, which makes the comparison more relevant.

[3]Such regulations limit the emissions of VOC in order to reduce smog formation. Exact requirements vary by state, but generally include obtaining a permit allowing a specific amount of VOC emissions from all sources within a facility.

[4]In Requate (2005)), an emissions standard is referred to as an absolute standard while a performance standard is referred to as a relative standard.

All of the comparisons are mainly from the company's perspective. The comparisons can inform the company which regulation is better for it based on its own strategic standpoints and parameters so that it could support and lobby for that regulation. For example, the company may prefer a regulation under which it has a higher expected profit and requires less upfront investment. Our analyses identify conditions for such a preference. Throughout the paper, a numerical example with parameters estimated from published industry and research data is used to illustrate the analytical results.

The contributions of our work are as follows. First, we introduce demand uncertainty into single period models about investment in emissions reduction technologies. Deterministic models are found in the economics literature regarding technology innovations induced by emissions regulations, see Montero (2002b), Dietz & Michaelis (2004) for examples.[5] Demand uncertainty is an important operational issue, and derivation of an optimal investment strategy becomes much more involved by introducing this aspect. Second, using the single period models, we provide a thorough analytical comparison of cap-and-trade and command-and-control regulations from the company's perspective with demand uncertainty. We compare the two regulations in multiple dimensions, including expected profit, total emissions and investment amount of the company, while the above mentioned economics literature only compares the incentive[6] of the company to invest from a policy maker's point of view. Contrary to the conventional wisdom, we observe that it is possible for the company to have both a higher profit and lower total emissions under cap-and-trade. In addition, adding demand uncertainty makes our comparisons very different from the economics literature. The most important contribution is the development of a two period model to analytically compare investment timing of the company under the cap-and-trade and command-and-control regulation. To the best of our knowledge, our work is the first one to perform such a timing comparison for investment in process improvement projects while taking into account the interaction between investment and production capacity decisions. We find that the company should not always invest earlier under cap-and-trade regulation as claimed by economists.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature and positions our work. Section 3 introduces the single period models under cap-and-trade and command-and-control regulations for comparisons of the expected profit, total emissions and investment amount in Section 4. Section 5 introduces the two period models under cap-and-trade and command-and-control, and compares the investment timing. The production cost of the company may change as a result of an investment to reduce emissions. Sections 3, 4 and 5 assume that the production cost does not change with the investment to reduce emissions. Section 6 discusses the impact of a change of the production cost on our results. Section 7 presents managerial insights that can be drawn from our study and discusses possible extensions.

[5] In this stream of economics literature, an emissions reduction technology is referred to as an abatement technology.

[6] Except in Montero (2002b), incentive to innovate is defined as the profit difference of a firm after and before possible innovation. In Montero (2002b), it is defined as the optimal investment amount in innovation for some given reduction level of abatement cost.

2 Literature Review

Our work is closely related to papers that study incentives for technology innovations induced by emissions regulations in the economics literature, see Downing & White (1986), Milliman & Prince (1989), Jung et al. (1996), Requate (1998, 2003, 2005), Requate & Unold (2001, 2003), Montero (2002a,b), Fischer et al. (2003), and Dietz & Michaelis (2004) among others. These papers compare the incentive for a firm to innovate in an emissions reduction technology under various regulations, including command-and-control or market-based regulations. Our work differs from these economics papers in three ways. First, we consider the investment decision by a firm that does not yet possess an existing abatement technology, while all these economics papers focus on firms that do already possess some abatement technology. Firms do not have incentives to acquire an abatement technology without emissions regulation. When an emissions regulation is in place, firms face the investment decision to reduce emissions without abatement earlier than the investment decisions to innovate on their existing abatement decisions. Second, our work explicitly models demand uncertainty while these papers assume deterministic demand. Demand uncertainty is an important operational issue, and makes the analysis more involved than when the demand is deterministic. Third, our work is mainly from a firm's perspective while these papers are from a policy maker's perspective. We compare a firm's expected profit, total emissions, investment amount and timing under command-and-control and cap-and-trade regulations, while these papers only compare a firm's incentive to innovate.

The economics papers that study investment under uncertainty are also relevant to our study, see Leahy (1993), Insley (2003), Zhao (2003), and Krysiak (2008) among others. These papers explicitly take uncertainty into account when modeling firms' investment decisions, and employ real options approaches. The book by Dixit & Pindyck (1994) provides a nice overview of this topic. Among these papers, Zhao (2003) and Krysiak (2008) compare the investment incentives of firms under emissions tax and cap-and-trade regulations, and bear the most similarity to our work. However, they leave out the firms' production decision and ignore the interaction between the abatement investment and production quantity, while we capture this interaction. In addition, their study only compares the firm's investment incentive while we have performed a much more thorough comparison from a firm's perspective, covering in addition the expected profit, total emissions, and investment timing. Finally, their comparison is between emissions tax and cap-and-trade regulations, while ours is between the cap-and-trade and command-and-control regulation. The studies by Leahy (1993) and Zhao (2003) touch on the firms' investment timing problem, and state the conditions for investment in terms of the realization of some random variables (e.g. price for the output good as in Leahy (1993)). However, they do not compare under which regulation a firm is optimal to invest earlier as in the current study.

Recently, there has been a surge of papers in the operations research and management science field that study the implications of carbon regulations on firm's emissions reduction investment.

In a two-stage model, Drake et al. (2010) study the technology choice and capacity investment decisions under two regulations, namely cap-and-trade and carbon tax for power plants and mature capital intensive manufacturers like cement and steel. At the first stage, the firm chooses the capacities in two technologies when demand is uncertain. After demand uncertainty is resolved, the firm chooses production quantities subject to capacity constraints. Technology shares, expected profit, expected production and total emissions are compared under these two regulations. This paper is about selecting technologies and capacity investment for new plants. However, our work is about investment in process improvement projects for existing plants. Furthermore, it focuses on cap-and-trade and carbon tax regulations while we also consider the command-and-control type of regulation. Subramanian et al. (2004) study firm's compliance strategies under the permit auction regulation on emissions. Under this regulation, firms need to purchase permits through an auction to cover their emissions. Possible compliance strategies include investing in an abatement technology, purchasing permits in the auction, and changing the production quantity. Krass et al. (2010) analyze the effects of using carbon taxes in motivating the choice of emissions-reducing technologies by a profit-maximizing monopolistic firm. There is a finite number of technologies for the firm to choose from, which vary in the up-front investment, variable operating cost and emissions reduction effectiveness. Subramanian et al. (2004) and Krass et al. (2010) only study a single type of regulation while we compare two types of regulations (i.e. cap-and-trade and command-and-control).

Studies on investment in process improvement projects trace back to the 1980's in the context of just-in-time and lean supply chains in manufacturing. Porteus (1985) considers the investment to reduce setup cost in the EOQ model with a one-time investment opportunity. He jointly optimizes the investment and lot size decisions. Porteus (1986) further captures the relationship between quality and lot size, and considers the investment to improve the production quality as well as to reduce setup costs. Fine & Porteus (1989) study a dynamic version of the work by Porteus (1986), allowing an investment opportunity in each time period with random investment effects. More recently, Zhu et al. (2007) examines investment for quality improvement for both buyers and suppliers in a supply chain. Our work employs similar ideas from these papers to model the investment needed to reduce the unit emissions level. To achieve a certain level l of the production factor to improve (e.g. setup cost or defective probability in these papers), the money needed to invest is a function $f(l)$ of the factor level. Our production factor to improve is the unit emissions level, which has different implications on the production cost or flexibility than to reduce setup cost or improve production quality. In addition, our work captures various regulations while the previous papers do not.

Next we present our models about investment and production capacity decisions for an emissions intensive firm under GHG regulations. For expositional simplicity, we assume the production cost does not change with the investment to reduce emissions in the following models. We discuss the impact of a change of production cost on our results in Section 6.

3 Single Period Models

We consider a company operating several plants subject to certain carbon regulations, including cap-and-trade, carbon tax and command-and-control. We assume that the emissions trading price under cap-and-trade regulation is given exogenously. The demand is stochastic with gross margin profit p per unit, which is equal to price minus production cost. The demand has continuous cumulative distribution function F. The cost for preparing the production capacity is constant c per unit. For example, the preparation cost can be the unit cost for materials purchased before the actual production. Note that the gross margin profit only occurs for quantity produced and sold while the cost for preparing capacity is accounted for the production capacity even if the company does not produce up to the capacity.

As of current, the firm does not possess an abatement technology. The initial unit emissions level is e_0 per unit production. However, the firm can invest in process improvement projects to reduce the unit emissions level to e per unit production with a technologically possible lower bound $\underline{e}$. Let c_e denote the unit emissions cost, which can be the expected emissions trading price under the cap-and-trade regulation or the unit tax under the carbon tax regulation. We assume that the firm's profit margin $p-c-c_e e_0$ is positive (otherwise, the firm is optimal to produce nothing). The investment needed to reach unit emissions level e is $f(e)$, which decreases in e with $f(e_0) = 0$. The investment to reduce emissions can be through either adopting an existing abatement technology or improving production process. For example, a utility company with coal-fired power plants can invest in the CCS abatement technology.

We do not use the same way to model the effect of investment as in the economics papers on induced technology innovation due to emissions regulations (e.g. Dietz & Michaelis (2004)) for two reasons. On the one hand, although the model in these economics papers can be adapted for a firm without any abatement technology to adopt an existing one, it is not easy to specify the abatement cost function such that the firm's optimal abatement level before innovation is zero.[7] Our model is more natural for investment in an existing abatement technology. On the other hand, we model investment in a similar way as existing models for investment in process improvement projects in the literature (e.g. Porteus (1985)). We often assume that the investment function $f(e)$ is linear, and it is equal to $a(e_0 - e)$. Here a is the unit investment cost needed to reduce the unit emissions level by one unit. An example of such investment is a utility company operating several coal-fired power plants that needs to decide how many plants to retrofit with the CCS technology to reduce carbon emissions.

Throughout the paper, we use a numerical example with parameters estimated from published industry and research data to illustrate our results. We introduce the example in the following before further discussing the details of our models.

[7]Note that for firms without any abatement technology, the abatement level should be zero before investing in innovation.

Example: American Electric Company (AEP) is one of the largest utilities in the U.S. and an environmental leader that actively engages in GHG emissions reduction activities. We use a possible investment by AEP in CCS to retrofit its existing pulverized coal-fired power plants as an example to illustrate our results. We point out that as of year 2012, there is no commercial scale operation of CCS at any existing power plant in the US. The CCS technology under consideration is post-combustion with chilled ammonia (see McKinsey (2008) for more about different types of CCS). The investment cost to retrofit an existing power plant varies with the type of the plant. Table 1 presents AEP's model parameters.

Table 1: AEP's Model Parameters

Parameter	Description	Value	Unit	Source
μ	AEP's annual demand mean	140,474	10^6 kWh	AEP (2011a), AEP (2011b)
σ	AEP's annula demand standard deviation	4,120	10^6 kWh	AEP (2011a), AEP (2011b)
c	Capacity preparation cost	2.42	cents/kWh	EIA (2012b), AEP (2011b)
p	Profit margin with capacity preparation cost	6.4	cents/kWh	EIA (2012a), AEP (2011a)
e_0	Initial unit emissions level	0.976	tonne/MWh	NETL (2007a)
$\underline{e}$	Unit emissions lower bound	0.0976	tonne/MWh	NETL (2007a)
a	Unit investment cost to reduce unit emissions level by 1 tonne/kWh	1,071,399	million dollars	NETL (2007a)

We use the historical sales data of AEP to estimate its demand. According to American Electric Power (AEP) (2011a,b), the total retail electricity sales by coal fired plants are 143,654 (year 2011), 141,948 (year 2010), and 135,819 (year 2009) million kilowatt hours. If we assume that demand follows the normal distribution, then the mean and standard deviation are 140,474 and 4,120 million kilowatt hours, respectively. The cost for preparing production capacity is the cost of coal needed to generate electricity. According to the Energy Information Administration (EIA) (2012b), 1,942 kilowatt hours of electricity is generated by a ton of coal. The average cost of coal is \$47 per ton based on American Electric Power (AEP) (2011b). Therefore, the cost of coal per kWh is 2.42 ($= 47/1,942 \times 100$) cents. According to EIA (2012a), the average retail electricity price for year 2011 is 9.55 cents/kWh. The electricity price is expected to increase under emissions regulations. The electricity price increased on average 25% based on German forward prices for 2006 under EU-ETS (Hyvarinen (2006)). Islegen & Reichelstein (2011) conclude that the upper bound for the increase in the electricity price under emissions regulations is around 30% in the United States. Therefore we assume that the electricity price will increase 30% under an emissions regulation in the United States. In addition, the profit margin is 10% of the revenue generated from the sales of electricity based on American Electric Power (AEP) (2011a). With these assumptions, the marginal profit p per kWh is equal to 6.40 ($=9.55 \times 1.3 - 9.55 \times 0.9 + 2.42$) cents. For AEP, as long as the unit emissions cost c_e is smaller than 41.25 dollars, we have positive marginal profit $p - c - c_e e_0$ without any investment.

According to the National Energy Technology Laboratory (NETL) (2007a), the total investment I_{Ohio} needed to retrofit AEPs Conesville, Ohio, Unit #5 plant to reduce its CO_2 by 90% is 400 million dollars. The initial emissions e_0 of this plant are 0.976 tonne/MWh, and hence the emissions lower bound $\underline{e}$ is 0.0976 tonne/MWh. For AEP, the investment cost across its entire coal-fired fleet is approximately linear in the unit emissions level under the following two assumptions: (1) the retrofit cost is linear in terms of the retrofitted capacity; and (2) the long-run capacity utilization at each power plants is almost constant. The calculation of the unit investment cost is illustrated in Figure 1. We denote the generation capacity of the AEPs Conesville, Ohio, Unit #5

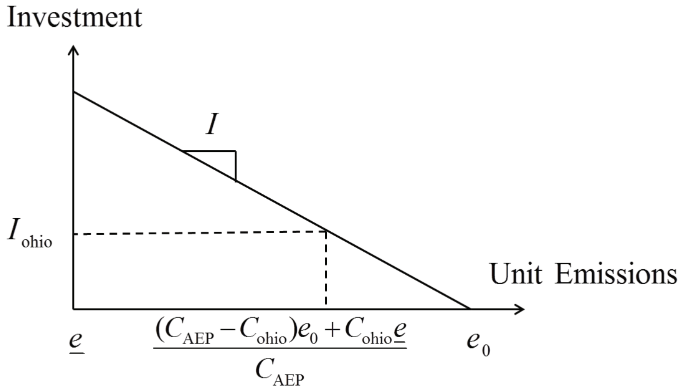


Figure 1: Unit Investment Cost

plant by C_{Ohio}, and the total generation capacity for coal-fired power plants of AEP by C_{AEP}. Note that the total emissions of AEP after applying CCS at Ohio are equal to $(C_{\text{AEP}} - C_{\text{Ohio}})e_0 + C_{\text{Ohio}}\underline{e}$, which on the other hand are equal to $C_{\text{AEP}} \times$ unit emissions. Hence for an investment of I_{Ohio}, the unit emissions are equal to $\dfrac{(C_{\text{AEP}} - C_{\text{Ohio}})e_0 + C_{\text{Ohio}}\underline{e}}{C_{\text{AEP}}}$. We know that C_{Ohio} is 450 MW based on National Energy Technology Laboratory (NETL) (2007a). According to American Electric Power (AEP) (2011a), C_{AEP} is 25,725 MW. The investment cost to reduce unit emissions I, which is the slope in Figure 1, is equal to \$26,032 million per tonne/MWh ($= \dfrac{I_{\text{Ohio}}}{C_{\text{Ohio}}/C_{\text{AEP}}(e_0 - \underline{e})}$). In perspective, it costs AEP around 8,000 millions to reduce its current unit emission level by 30% for its entire coal-fired fleet. We amortize the investment cost over $n = 20$ years to obtain investment per year. Discount rate δ is assumed to be 2%.[8] The total amount I is timed with the annuity factor $\dfrac{\delta}{1 - (1 + \delta)^{-n}}$ to get the investment per period. Therefore, the unit investment cost a to reduce unit emissions by 1 tonne/kWh is 1,071,399 millions. □

[8]Discount rate δ is between 2 to 3 percents based on the return for treasury bond on bankrate.com in May of 2012.

3.1 Model under Cap-and-Trade

Under cap-and-trade, the emissions trading price is stochastic with expected value c_e, and is exogenously determined through the emissions trading market. The sequence of events for the model under cap-and-trade is shown in Figure 2. At the beginning of a period before demand and emissions trading price under the cap-and-trade are realized, the firm decides the production capacity x and emissions level e per unit production. During the period, uncertain demand D and emissions trading price are resolved. At the end of the period, the firm chooses to produce the optimal amount, which is $\min(D, x)$. The optimal production quantity is $\min(D, x)$ because it is never optimal to produce more than the demand. In addition, the firm can not produce more than its prepared capacity. In addition, the firm sells or buys extra emissions allowances if it emits less or more than its initial allowances A in the emissions trading market.

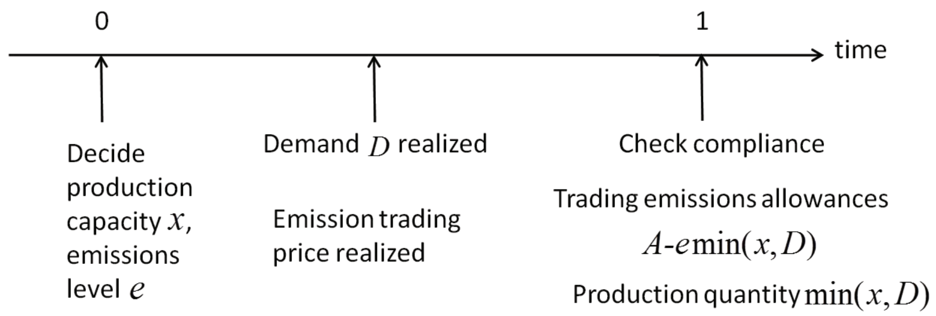


Figure 2: Sequence of Events under Cap-and-Trade Regulation

The firm chooses the optimal production capacity x and unit emissions level e to maximize its expected profit as follows:

$$\max_{x\geq 0, \underline{e}\leq e\leq e_0} \quad p\mathbb{E}[\min(D, x)] - cx - c_e(e\mathbb{E}[\min(D, x)] - A) - f(e) \tag{1}$$

where the first term is the expected marginal profit from selling; the second term is the cost to prepare the production capacity; the third term is the expected cost or revenue from selling or buying extra allowances; and the last term is the investment cost. After uncertain demand is realized, if $e\text{min}(D, x) \geq A$, the firm has to purchase $(e\min(D, x) - A)$ amount of allowances. Otherwise, it sells $(A - e\min(D, x))$ amount of allowances.

There are two variations of the cap-and-trade regulation based on the allocation scheme of initial allowances. One allocation scheme is named "grand-fathering," under which the initial allowances are given to regulated firms free of charge. The model under this allocation scheme is shown in (1). The other allocation scheme is by auction, under which initial allowances are purchased by firms through an auction held by a regulating entity. If we assume that the expected auction price for the initial allowances is the same as the expected emissions trading price, then the model under this allocation scheme is a special case of (1) with $A = 0$.

The model under the carbon tax regulation is the same as the model under the cap-and-trade regulation with auctioned initial allowances. If c_e represents the tax rate under the carbon tax regulation, then (1) models the carbon tax regulation. Since the cap-and-trade and carbon tax regulations yield the same model, we herein refer to both regulations as the cap-and-trade regulation.

To facilitate the presentation, before deriving an optimal solution under the cap-and-trade regulation, we define some additional variables. In these definitions, the superscript *ct* denotes cap-and-trade. The superscript *nv* denotes the classic news vendor, and the superscript *cc* denotes command-and-control, defined later.

$x^{nv}(p)$: the news vendor production quantity with unit marginal profit p which satisfies $F(x^{nv}(p)) = \frac{p-c}{p}$;
$\pi^{nv}(x,p)$: the news vendor profit function with production capacity x and purchase cost c; Note that $x^{nv}(p)$ is its corresponding optimal ordering quantity, and we have

$$\pi^{nv}(x,p) = p\mathbb{E}(D) - p\int_x^{+\infty}(d-x)dF(d) - cx. \tag{2}$$

x^{ct} or x^{cc}: the optimal production capacity under cap-and-trade or command-and-control;
e^{ct} or e^{cc}: the optimal unit emissions level per unit production under cap-and-trade or command-and-control;
$\pi^{ct}(x,e)$ or $\pi^{cc}(x,e)$: the expected profit of the firm under cap-and-trade or command-and-control with production capacity x and unit emissions level e per unit production;
$\pi^{ct}_{\max}$ or $\pi^{cc}_{\max}$: the optimal expected profit under cap-and-trade or command-and-control. Quantity $\pi^{ct}_{\max}$ equals (1) while $\pi^{cc}_{\max}$ will be defined later.

The optimal solution under cap-and-trade is given in the following theorem. The condition about the demand CDF $F(\cdot)$ is a technical condition needed in the proof of the theorem, and it is satisfied by any continuous distribution with finite support, by normal, log-normal, Gamma (with Erlang and exponential distributions as special cases), and Weibull distributions.

Proposition 1 *Under the linear investment function $f(e) = a(e_0 - e)$, if the demand CDF $F(\cdot)$ satisfies $\lim_{x\to+\infty} x(1-F(x)) = 0$, then for a given unit emissions cost c_e, there exists a unit investment cost threshold*

$$a^{ct}(c_e) = \frac{\pi^{nv}(x^{nv}(p-c_e\underline{e}), p-c_e\underline{e}) - \pi^{nv}(x^{nv}(p-c_e e_0), p-c_e e_0)}{e_0 - \underline{e}}$$

such that the optimal solution is

$$(x^{ct}, e^{ct}) = \begin{cases} (x^{nv}(p - c_e\underline{e}), \underline{e}) & a \leq a^{ct}(c_e), \\ (x^{nv}(p - c_e e_0), e_0) & \textit{otherwise}, \end{cases}$$

with the maximum expected profit

$$\pi^{ct}_{\max} = \begin{cases} \pi^{nv}(x^{nv}(p - c_e\underline{e}), p - c_e\underline{e}) - a(e_0 - \underline{e}) & a \leq a^{ct}(c_e), \\ \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0) & \textit{otherwise}. \end{cases}$$

In addition, the investment threshold $a^{ct}(c_e)$ increases in c_e, and is equal to 0 when $c_e = 0$.

All proofs are presented in the appendix. The optimal investment under cap-and-trade is illustrated in Figure 3. There exists a unit investment threshold a^{ct} such that for a given unit emissions cost c_e, if the unit investment cost a is higher than the threshold, then the firm should not invest to reduce the unit emissions level. Instead, it should reduce its prepared production capacity to accommodate the increase in the unit production cost due to carbon emissions. Otherwise, the firm should invest to reduce the unit emissions level as much as possible. The unit investment threshold a^{ct} is an increasing function of the unit emissions cost c_e. It is zero when the unit emissions cost is zero. As the unit emissions cost c_e increases, the investment to reduce carbon emissions is more likely to bring higher profit for a firm.

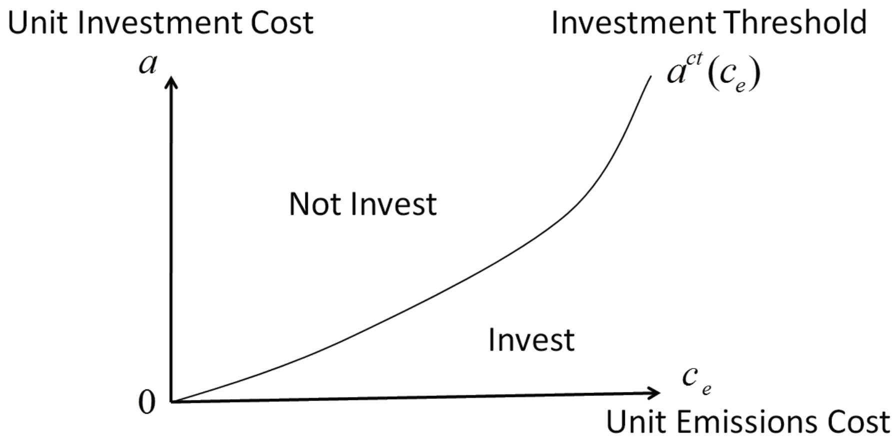


Figure 3: Optimal Investment under Cap-and-Trade Regulation

3.2 Model under Command-and-Control

Under the command-and-control regulation, firms are subject to the strict emissions cap B. We assume that the total emissions of a firm cannot exceed B. The command-and-control regulation can be implemented by imposing high penalties on firms for exceeding their emissions caps. In practice, firms may not want to violate the regulation for other reasons, including (1) to avoid frequent monitoring and scrutiny of the government, (2) to obtain additional negotiation power with the government in hope of less restrictive regulation in the future, or (3) to establish a good corporate image by being a socially responsible corporation (Downing (1982)).

The sequence of events for the model under command-and-control is shown in Figure 4. At the beginning of the period before the demand is realized, the firm decides the production capacity x and the unit emissions level e for unit production. During the period, uncertain demand is realized. At the end of the period, the firm chooses to produce the optimal amount, which is $\min(D, x)$. In addition, the total emissions of the firm cannot exceed its emissions cap B.

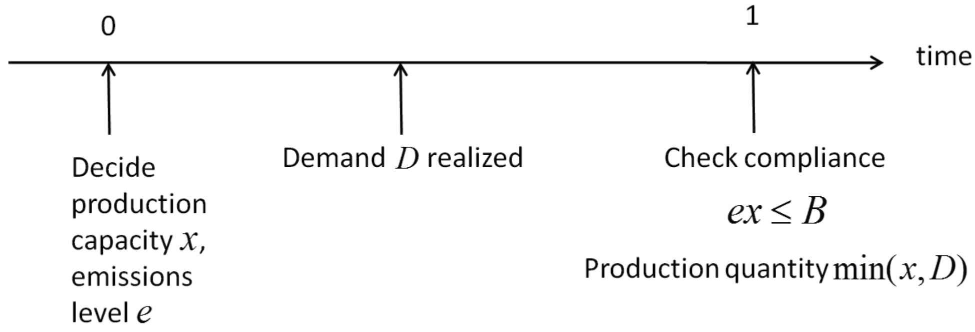


Figure 4: Sequence of Events under Command-and-Control Regulation

The firm chooses the optimal production capacity x and unit emissions level e to maximize its expected profit as follows:

$$\begin{aligned} \max_{x \geq 0, e} \quad & p\mathbb{E}[\min(D, x)] - cx - f(e) \\ \text{s.t.} \quad & ex \leq B \\ & \underline{e} \leq e \leq e_0 \end{aligned}$$

Note that the firm is not allowed to exceed the emissions cap for any production level. Therefore, we impose the constraint that the firm cannot exceed the emissions cap when producing at its capacity.

The optimal solution under command-and-control is characterized in the following theorem.

Proposition 2 *Under the linear investment function $f(e) = a(e_0 - e)$, when the emissions cap is* *not tight, i.e.* $x^{nv}(p)e_0 \leq B$*, the optimal solution is*

$$(x^{cc}, e^{cc}) = (x^{nv}(p), e_0);$$

otherwise, there exists an investment lower bound $\underline{a}^{cc} > 0$ *and upper bound* $\bar{a}^{cc} > 0$ *such that the optimal solution is*

$$(x^{cc}, e^{cc}) = \begin{cases} (B/e_0, e_0) & a \geq \bar{a}^{cc}, \\ (B/\bar{e}, \bar{e}) & a < \underline{a}^{cc}, \\ (B/\tilde{e}, \tilde{e}) & \underline{a}^{cc} \leq a < \bar{a}^{cc}, \end{cases}$$

where $\tilde{e} \in (\max\{B/x^{nv}(p), \underline{e}\}, e_0]$ *and* $\bar{e} \in (\max\{B/x^{nv}(p), \underline{e}\}, e_0)$*.*

Under the command-and-control regulation, a firm should invest to reduce the unit emissions level if $x^{nv}(p)e_0 > B$ and $a < \underline{a}^{cc}$, i.e. when the emissions cap is tight, and the unit investment cost is not too high. Based on Proposition 2, a firm should not invest to reduce the unit emissions level when the emissions cap is not tight, or the unit investment cost is too high. When the investment cost is between $\underline{a}^{cc}$ and $\bar{a}^{cc}$, it is not clear whether the firm should invest or not. In addition, even if it is optimal for the firm to invest, it may not be optimal to reduce the unit emissions as much as possible as under cap-and-trade. It suffices to reduce the unit emissions level to the point where the firm satisfies the emissions cap. There is no incentive for a further investment.

Example: We assume the unit emissions cost c_e to be 20 dollars per tonne,[9] and AEP's initial emissions allowances and emissions bound to be 69.18 million metric tons, which is 50% of the total emissions without any emissions regulations.[10] With these assumptions, we get the investment threshold values for AEP, shown in Table 2. Figure 5(a) and 5(b) present AEP's optimal

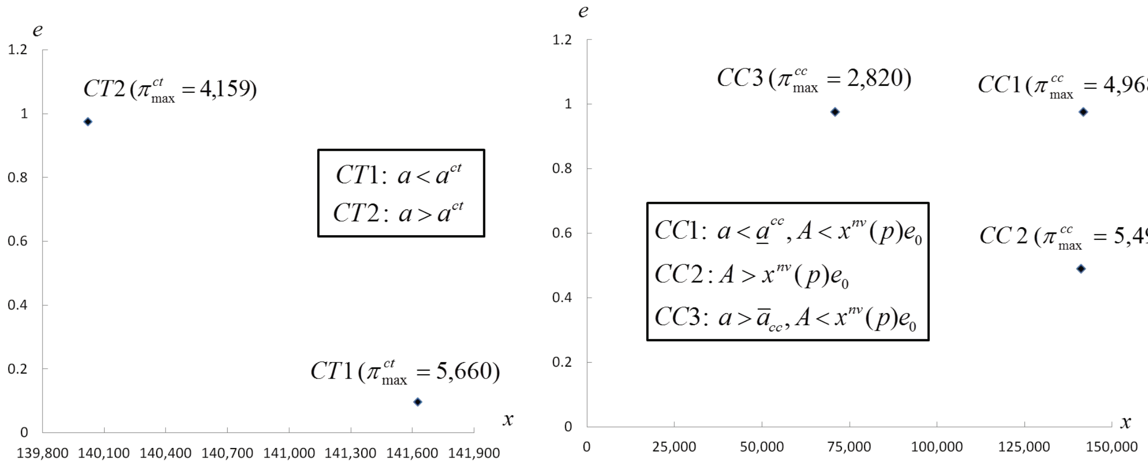
Table 2: AEP's Investment Threshold Values

million dollars			million metric tons
a^{ct}	$\underline{a}^{cc}$	$\bar{a}^{cc}$	$x^{nv}(p)e_0$
2,783,626	2,893,295	9,711,189	138.35

production capacities and unit emissions levels under the cap-and-trade and command-and-control regulations, respectively.

[9]The carbon price under the EU ETS system varies from 0 to 47 dollars a tonne. See `http://www.pointcarbon.com/` for the historical trading price of carbon.

[10]According to Eilperin & Shear (2009), the White House announced on 25 November 2009 that President Barack Obama is offering a U.S. target for reducing greenhouse gas (GHG) emissions in the range of 42% below 2005 levels by 2030. Due to political oppositions in the Congress, we suspect the cap is more likely to be 50%.



(a) Cap-and-Trade (b) Command-and-Control

Figure 5: AEP's Optimal Solutions

The unit emissions levels are shown on the vertical axis with unit being tonne/kWh, and the production capacities shown on the horizontal axis with unit being millions of kWh. The maximum expected profits are shown in parentheses with unit being millions of dollars. In case $CT1$, we have AEP's current unit investment cost being 1,071,399 millions of dollars, smaller than a^{ct}. In case $CT2$, we suppose AEP's unit investment cost to be 306,180 ($= 1.1a^{ct}$) millions of dollars, greater than a^{ct}. In case $CC1$, we have AEP's current investment cost smaller than $\underline{a}^{cc}$, and current emissions bound smaller than $x^{nv}(p)e_0$. In case $CC2$, AEP's emissions bound is assumed to be any value greater than $x^{nv}(p)e_0$. In case $CC3$, we suppose AEP's unit investment cost to be 10,682,307 ($= 1.1\bar{a}^{cc}$) millions of dollars, greater than $\bar{a}^{cc}$. $\square$

4 Comparisons of Expected Profit, Total Emissions and Investment

All of the following comparisons are carried out under the assumption that the initial allowance amount A for cap-and-trade is the same as the emissions cap B for command-and-control. In this section, we use A to denote both the initial allowances and emissions cap. All the results consider a firm's maximum expected profit, and optimal production capacity and investment decisions.

4.1 Expected profit

There are two variations of cap-and-trade regulations, one with grandfathered and the other one with auctioned initial allowances. Recall that the initial allowances are allocated to regulated firms for free with grandfathering, and are purchased by regulated firms through an auction held by a regulating entity with auction. We first compare the expected profit of a firm under command-and-control and cap-and-trade with grandfathered initial allowances.

Theorem 3 *For a general investment function $f(e)$, a firm has higher expected profit under cap-and-trade with grandfathered allowances than under command-and-control.*

Since a firm may obtain extra revenue by selling the unused allowances under cap-and-trade, and has the freedom to exceed its emissions cap, the firm has a higher expected profit under cap-and-trade. This results justify why a firm would prefer cap-and-trade with free initial allowances than command-and-control with regard to the expected profit of the firm.

To compare the expected profit of a firm under command-and-control and cap-and-trade with auctioned initial allowances, we assume that the expected auction price of initial allowances is the same as the expected trading price, which is denoted by c_e. Let $\pi^{cc}_{\max}(A)$ denote the maximum expected profit of the firm under command-and-control as a function of the emissions cap A, and let $\pi^{ct}_{\max}(c_e)$ denote the maximum expected profit of the firm under cap-and-trade with auctioned initial allowances as a function of the unit emissions cost c_e. The following lemma states how these maximum profits change with the corresponding parameters.

Lemma 4 *Function $\pi^{cc}_{\max}(A)$ is nondecreasing in A, and function $\pi^{ct}_{\max}(c_e)$ is nonincreasing in c_e.*

The maximum expected profit of a firm under command-and-control is nondecreasing in the emissions cap A since the firm has higher production flexibility with a larger emissions cap. The maximum expected profit of a firm under cap-and-trade with auctioned initial allowances is nonincreasing in the unit emissions cost c_e since the firm has to pay for all of its emissions, and the payment is higher with larger unit emissions cost.

Theorem 5 *For a given c_e, there exists $A^*(c_e)$ such that*
(1) if the emissions cap for command-and-control $A < A^*(c_e)$, then $\pi^{ct}_{\max}(c_e) > \pi^{cc}_{\max}(A)$;
(2) if $A \geq A^*(c_e)$, then $\pi^{ct}_{\max}(c_e) \leq \pi^{cc}_{\max}(A)$;
(3) furthermore, $A^*(c_e)$ is nonincreasing in c_e and $A^*(0) = x^{nv}(p)e_0$.

Theorem 5 is illustrated in Figure 6. For a given unit emissions cost c_e, there is an emissions cap threshold such that a firm has higher expected profit under command-and-control than under cap-and-trade with auctioned initial allowances if and only if the emissions cap of the firm is larger than the threshold. The maximum profit of a firm under command-and-control increases as the emissions cap increases while the maximum profit under cap-and-trade does not change with the emissions cap. Therefore, the maximum profit under command-and-control is larger than the one under cap-and-trade when the emissions cap is high. In addition, the emissions cap threshold is nonincreasing in the unit emissions cost c_e. Under auctioned initial emissions allowances, a firm has to pay for all of its emissions, and the payment is larger when the unit emissions cost is higher. Hence, its maximum profit under cap-and-trade decreases with the unit emissions cost. The firm does not pay anything for its emissions under command-and-control. Hence, its maximum profit under command-and-control does not change with the unit emissions cost. Therefore, the firm is less likely to have a higher profit under cap-and-trade than under command-and-control when the unit emissions cost is high.

For fixed unit emissions cost, when the emissions cap is large enough such that it has no effect under command-and-control, a firm will achieve the maximum possible expected profit without any form of carbon regulations. In this case, the maximum expected profit under command-and-control must be greater than the maximum expected profit under cap-and-trade with auctioned initial allowance for any positive expected carbon price. This is because the firm has to pay to emit under cap-and-trade no matter what the emissions amount is. Hence, with a larger emissions cap, the firm is more likely to have higher expected profit under command-and-control than under cap-and-trade with auctioned initial allowances. For a fixed emissions cap, the larger the emissions price, the higher a firm has to pay for its emissions, while the expected profit under command-and-control does not change with carbon price. Hence the firm is more likely to have higher expected profit under command-and-control. Although cap-and-trade is perceived as a regulation with lower compliance cost, a firm may actually have a lower expected profit under cap-and-trade with auctioned initial allowances than under command-and-control with a large emissions cap. It could also be the case that a firm may have a lower expected profit under cap-and-trade with auctioned initial allowances than under command-and-control if the unit emissions cost is high. When interpreting the unit emissions cost c_e as the unit tax and $\pi^{ct}_{\max}(c_e)$ as the maximum expected profit under carbon tax, Theorem 5 can also be understood as a comparison of the expected profit of a firm under command-and-control and carbon tax.

4.2 Total Emissions and Total Investment

The comparisons of total emissions and investment amount of a firm do not change with regard to different allocation schemes of the initial allowances. In the following theorems, the cap-and-trade regulation can have either allocation scheme.

Theorem 6 *Under the linear investment function $f(e) = a(e_0 - e)$, a firm has lower total emissions under command-and-control if and only if $a > a^{ct}$ and $A < x^{nv}(p - c_e e_0)e_0$, or $a \leq a^{ct}$ and $A < x^{nv}(p - c_e \underline{e})\underline{e}$.*

This theorem states that a firm is likely to have larger total emissions under cap-and-trade if and only if the emissions cap is tight and low. When the emissions cap is tight, the firm's total

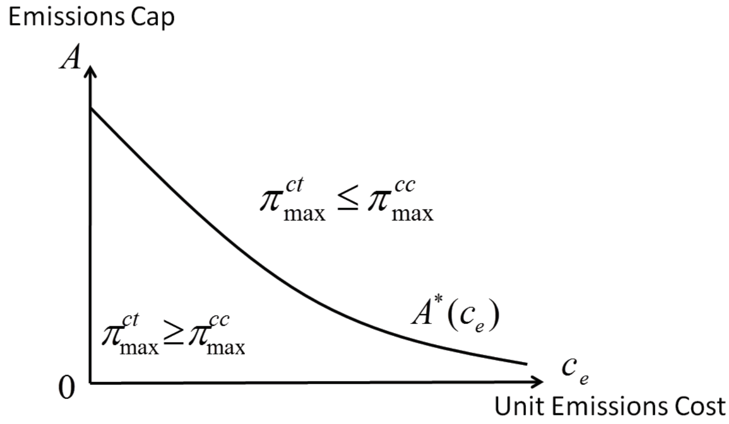


Figure 6: Comparison of Maximum Expected Profit

emissions under command-and-control equal the emissions cap. When the emissions cap is low, the firm is better off exceeding the cap under cap-and-trade. Hence the firm will have higher total emissions under cap-and-trade.

From Theorems 3 and 6, we observe that it is possible for a firm to have both higher profit and less emissions under the cap-and-trade system than under the command-and-control system. This is contradicting the conventional wisdom of more profit means more emissions, e.g. the case of coal-fired power plants versus nuclear power plants.

In terms of the investment amount, the following result outlines both possible cases.

Theorem 7 *Under the linear investment function $f(e) = a(e_0 - e)$, a firm invests more to reduce unit emissions under command-and-control if $A < x^{nv}(p)e_0$, $\underline{a}^{cc} > a > a^{ct}$ and $\underline{a}^{cc} > a^{ct}$, which holds for any small enough c_e. On the other hand, a firm invests more to reduce unit emissions under cap-and-trade if $a < a^{ct}$.*

A firm may have a larger total investment under command-and-control if the carbon trading price is small compared to the unit investment cost, and the emissions cap is tight. In this case, the extra revenue from selling the extra carbon allowances cannot offset the investment cost under cap-and-trade. Hence the firm has no incentive to invest under cap-and-trade. On the other hand, the firm is more profitable to invest under command-and-control to bring the production level up under a very tight cap. Contrary to the claim that firms invest more to reduce emissions under cap-and-trade than under command-and-control in the economics literature, when the carbon trading price is low and the emissions cap is tight, the firm may invest more under command-and-control.

When the unit investment cost is small comparing to the carbon price, the firm will invest more under cap-and-trade no matter if the emissions cap is tight or not, because the firm has no incentive to invest beyond the carbon cap under command-and-control, while it always has the incentive to invest under cap-and-trade as long as the unit investment cost is small enough.

Example: As in the previous example, we assume $c_e = 20$ dollars/tonne, and $A = B = 69.18$ millions of metric tons unless otherwise indicated. Figure 7 presents a comparison between AEP's maximum expected profits, $\pi^{ct}_{\max}$ under cap-and-trade and $\pi^{cc}_{\max}$ under command-and-control, with different initial emissions allowances or bounds. The numbers in parentheses are emissions bounds under command-and-control when comparing with cap-and-trade with auctioned initial allowances. With free initial allowances, as in Theorem 3, AEP has higher expected profit under cap-and-trade than under command-and-control. With auctioned initial allowances, we have $A^*(20) = 12.80$ millions of metric tons. As shown in Theorem 5, AEP has higher expected profit under command-and-control if and only if $A \geq A^*(20)$. When the initial emissions bound is 11.52 ($= 0.9 \cdot A^*(20)$) or 14.08 ($= 1.1 \cdot A^*(20)$) millions of metric tons, AEP has lower or higher expected profit under command-and-control.

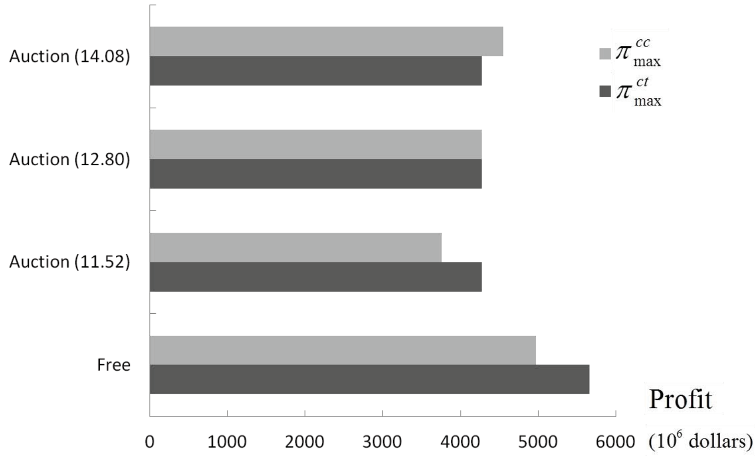


Figure 7: Comparison of AEP's Maximum Expected Profit

Based on Theorem 6, Figure 8 presents a comparison between AEP's total emissions, E^{ct} under cap-and-trade and E^{cc} under command-and-control. The units on the horizontal and vertical axes are millions of dollars and millions of kWh, respectively. AEP has lower (higher) total emissions under command-and-control in the lower right (upper left) region. With our current assumptions about AEP, its unit investment cost a is less than a^{ct}, and the total emissions under cap and trade being 13.82 millions of metric tons are smaller than under command-and-control, being 69.18 millions of metric tons.

AEP's total investments under cap-and-trade and command-and-control are compared for different unit emissions costs in Figure 9, with the unit on the vertical axis being millions of dollars. When $c_e = 20$ dollars/tonne, AEP satisfies the second conditions in Theorem 6, and thus it has a higher total investment under cap-and-trade. Meanwhile, when $c_e = 6$ dollars/tonne, AEP meets the first conditions in Theorem 6, and thus it has a higher total investment under command-and-control. $\square$

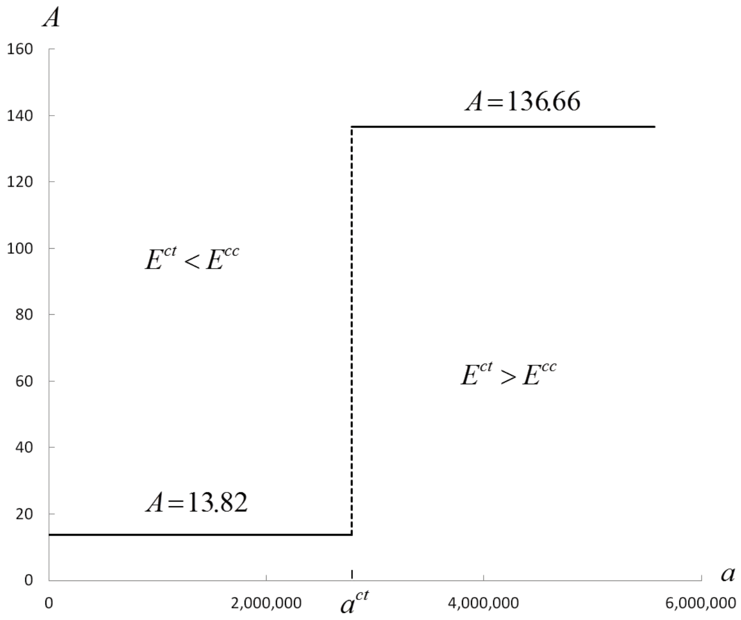


Figure 8: Comparison of AEP's Total Emissions

Figure 9: Comparison of AEP's Total Investment

5 Investment Timing

Economics literature claims that a firm always invests earlier under cap-and-trade than under command-and-control. This assertion is based on fixed production. To the contrary, we derive conditions under which the firm invests earlier either under the cap-and-trade or command-and-control regulation. This is a consequence of adjustable production quantity in our models. To study the investment timing problem faced by the firm, we propose the following two period models with a similar structure as the one period models above. Although the two period models can also be used to analyze the previously studied investment decisions, we believe that the one period models are more appropriate for the more strategic decision concerning whether or not to invest. In addition, these one period models provide valuable insights to analyze the more involved two period models for the investment timing decisions. Besides, one period results are impossible to obtain in the two period setting due to complexity. The comparison of the investment timing of the firm does not change with regard to different allocation schemes of the initial allowances. In the following propositions and theorems, the cap-and-trade regulation can have either allocation scheme.

5.1 Two Period Models

The sequence of events for two period models is shown in Figure 10. At the beginning of period i, the firm chooses its production capacity x_i and unit emissions level e_i for $i = 1, 2$. During a period, demand uncertainty is resolved, and so is the emissions trading price if the firm is under the cap-and-trade regulation. At the end of period i, the firm has to comply with the carbon regulation. Recall that under the cap-and-trade regulation, the firm needs to sell or buy extra emissions allowances if it emits less or more than its initial allowances A_i in the emissions trading market. Under the command-and-control regulation, the total emissions of the firm cannot exceed its emissions cap B_i. Unsatisfied demand at the end of a period is lost. The investment function for period one is $f_1(e_1; e_0)$ with e_0 being the unit emissions level parameter without investment, and it is $f_2(e_2; e_1)$ for period two with e_1 being the unit emissions level parameter without investment. The connection between two periods is through investment in emissions reduction. We assume that the investment cost of the second period is lower than that of the first period due to advancements of technology. Each parameter introduced earlier now has a subscript indicating the time period, i.e., c_i is the cost for preparing the production capacity in period i for $i = 1, 2$.

With these assumptions, the two-period model under cap-and-trade is as follows:

$$\begin{aligned} \max_{x_1,e_1} \quad & p_1\mathbb{E}[\min(D_1, x_1)] - c_1x_1 - c_e^1 e_1\mathbb{E}[\min(D_1, x_1)] - f_1(e_1; e_0) + \delta\pi_2^{ct}(e_1) \\ \text{s.t.} \quad & \underline{e}_1 \le e_1 \le e_0 \end{aligned}$$

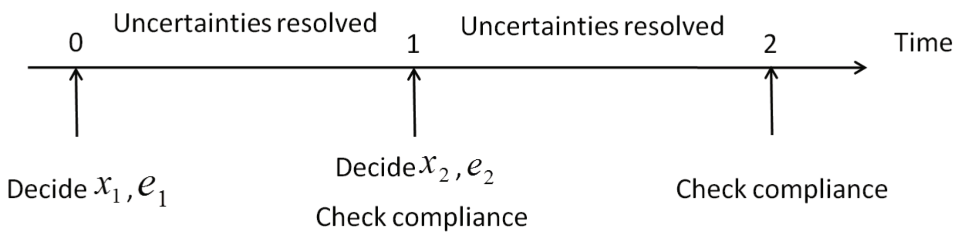


Figure 10: Sequence of Events for Two Period Models

$$x_1 \geq 0.$$

Here δ is the time discount factor, and $\pi_2^{ct}(e_1)$ is the expected profit under the cap-and-trade regulation for period two with the initial initial unit emissions level for period two being e_1. Formally,

$$\begin{aligned}
\pi_2^{ct}(e_1) = \max\nolimits_{x_2,e_2} \quad & p_2\mathbb{E}[\min(D_2,x_2)] - c_2x_2 - c_e^2e_2\mathbb{E}[\min(D_2,x_2)] - f_2(e_2;e_1) \\
\text{s.t.} \qquad & \underline{e}_2 \leq e_2 \leq e_1 \\
& x_2 \geq 0.
\end{aligned}$$

Banking is a possible design feature for cap-and-trade, under which firms can save their current unused emissions allowances for a later period. We do not explicitly model banking in the two period model for two reasons. First, firms may not want to bank their allowances under cap-and-trade since the regulations may change in an unfavorable direction such that the value of banked allowances may drop drastically. This is what happened with the acid-rain cap-and-trade system (Peters (2010)). Second, banking can be interpreted and captured in our model as follows. If the expected emissions trading price in period two is larger than the realized emissions trading price in period one, then it is optimal to bank as many allowances as possible in order to sell them for profit in period two. Otherwise, the firm should not bank them at all.

The two-period model under command-and-control is as follows:

$$\begin{aligned}
\max\nolimits_{x_1,e_1} \quad & p_1\mathbb{E}[\min(D_1,x_1)] - c_1x_1 - f(e_1;e_0) + \delta\pi_2^{cc}(e_1;e_0) \\
\text{s.t.} \qquad & e_1x_1 \leq B_1 \\
& \underline{e}_1 \leq e_1 \leq e_0 \\
& x_1 \geq 0,
\end{aligned}$$

where $\pi_2^{cc}(e_1)$ is the expected profit under the command-and-control regulation for period two when the initial emissions level for period 2 is e_1, and

$$\begin{aligned} \pi_2^{cc}(e_1) = \max_{x_2,e_2} \ & p_2\mathbb{E}[\min(D_2,x_2)] - c_2x_2 - f_2(e_2;e_1) \\ \text{s.t.} \quad & e_2x_2 \leq B_2 \\ & \underline{e}_2 \leq e_2 \leq e_1 \\ & x_2 \geq 0. \end{aligned}$$

5.2 Investment Timing Comparison

When comparing investment timing under the cap-and-trade and command-and-control regulation, for simplicity of exposition, in what follows, we assume that $p_1 = p_2 = p$, $c_1 = c_2 = c$, and $c_e^1 = c_e^2 = c_e$. We also assume that D_1 and D_2 have the same distribution with cumulative distribution function F. Furthermore, we assume that the investment functions are linear with $f_1(e_1) = a_1(e_1 - e_0)$ and $f_2(e_2) = a_2(e_1 - e_2)$. Here a_1, a_2 represents the unit investment cost in period one, two, respectively. As emissions reduction technology advances and matures over time, we expect the unit investment cost to decrease over time. Hence we assume that $a_2 \leq a_1$. Besides, we assume that the technology possible lower bounds are the same for both periods, which means $\underline{e}_1 = \underline{e}_2 = \underline{e}$. For the emissions regulations, we assume that $B_2 \leq B_1$ because regulations are expected to be more stringent in the future.

The following propositions specify sufficient conditions for a firm to invest for sure in period two. Intuitively, this happens when unit investment cost a_2 in period two is small enough. Let α denote a_2/a_1. A sufficient condition for definitely investing in period two under the cap-and-trade regulation is as follows.

Proposition 8 *For any unit investment cost a_1 in period 1, a firm invests for sure in period two under cap-and-trade if $\alpha \leq a_2^{ct}/a_1$, where $a_2^{ct} = c_e \int_0^{x^{nv}(p-c_e e_0)} \bar{d} dF(\bar{d}) + x^{nv}(p - c_e e_0)\dfrac{cc_e}{p - c_e\underline{e}}$.*

Under command-and-control, if $e_0 x^{nv}(p) \leq B_2$, a firm can carry out the production as if there is no emissions regulation in both periods. For this reason the firm never invests in period two no matter how small the unit investment cost is. The following proposition specifies a sufficient condition for a firm to invest in period two if $e_0 x^{nv}(p) > B_2$. Intuitively, this happens when the unit investment cost a_2 in period two is small enough.

Proposition 9 *When $e_0 x^{nv}(p) > B_2$, for any unit investment cost a_1 in period one, there exists $\hat{e} > \dfrac{B_1}{x^{nv}(p)}$ such that a firm invests for sure in period two under command-and-control if $\alpha \leq \underline{a}_2^{cc}/a$, where*

$$\underline{a}_2^{cc} = \begin{cases} \min_{\hat{e} \leq e_1 \leq e_0} -\dfrac{B_2}{e_1^2}(c - p(1 - F(\dfrac{B_2}{e_1}))) & e_0 x^{nv}(p) > B_1, \\ -\dfrac{B_2}{e_0^2}(c - p(1 - F(\dfrac{B_2}{e_0}))) & B_2 < e_0 x^{nv}(p) \leq B_1. \end{cases}$$

In the following propositions and theorems, a_1^{ct}, $\underline{a}_1^{cc}$, and $\bar{a}_1^{cc}$ are respectively equal to a^{ct}, $\underline{a}^{cc}$, and $\bar{a}^{cc}$ from the single period models. Besides, a_2^{ct} and $\underline{a}_2^{cc}$ are as defined in Proposition 8 and 9.

Suppose a firm invests for sure in period 2 under cap-and-trade. A condition for the firm to invest in period one under the cap-and-trade regulation is given in the following proposition.

Proposition 10 *Suppose $\alpha \leq a_2^{ct}/a_1$. A firm invests in period 1 under cap-and-trade if (1) $a_1 \leq a_1^{ct}$, or (2) $a_1^{ct} < a_1 \leq \delta a_2^{ct} + a_1^{ct}$, and $\alpha \geq \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$. A firm does not invest in period 1 if (3) $a_1 > a_1^{ct}$ and $\alpha < \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$.*

Note that the negation of cases (1) and (2) also includes $a_1 > \delta a_2^{ct} + a_1^{ct}$ and $\alpha \geq \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$. However, these conditions cannot hold at the same time with $\alpha \leq a_2^{ct}/a_1$. In (2), the condition $a_1 \leq \delta a_2^{ct} + a_1^{ct}$ is a technical condition for α to exist. Under condition (1), the firm has enough incentive to invest in period one by looking at the benefit of emissions reduction in period one alone. Under condition (2), the firm faces the tradeoff between investing in period one or not, which weighs the benefits of emissions reduction in both periods if investing in period one against the reduction in investment cost if delaying the investment until period two. Comparing to the single period problem under cap-and-trade, a firm invests in period one even when the unit investment cost in period one is greater than the single period threshold value as long as the unit investment cost in period two is not too small. This happens because the firm has higher incentive to invest in period one since it enjoys the benefit of such an investment in both periods.

Suppose a firm invests for sure in period two under command-and-control. The sufficient conditions for a firm to invest or not in period one under the command-and-control regulation is given in the following proposition.

Proposition 11 *Suppose $\alpha \leq \underline{a}_2^{cc}/a_1$, and $B_2 < e_0 x^{nv}(p)$. A firm does not invest in period one if (1) $e_0 x^{nv}(p) \leq B_1$, or (2) $e_0 x^{nv}(p) > B_1$, $a_1 \geq \bar{a}_1^{cc}$, and $\alpha \leq \frac{1}{\delta}(1 - \frac{\bar{a}_1^{cc}}{a_1})$. A firm invests in period one if (3) $e_0 x^{nv}(p) > B_1$, and $a_1 < \underline{a}_1^{cc}$, or (4) $e_0 x^{nv}(p) > B_1$, $\underline{a}_1^{cc} \leq a_1 < \underline{a}_1^{cc} + \delta a_2^{cc}$, and $\alpha > \frac{1}{\delta}(1 - \frac{\underline{a}_1^{cc}}{a_1})$.*

In (4), the condition $a_1 < \underline{a}_1^{cc} + \delta a_2^{cc}$ is a technical condition for α to exist. Comparing to the single period problem under command-and-control, a firm also invests in period one under additional scenarios due to the carryover effect of the investment from period one to period two.

Next we assume the sufficient conditions in Propositions 8 and 9 for a firm to always invest in period two are satisfied. The following two theorems discuss the sufficient conditions for firms to invest earlier under command-and-control or cap-and-trade. In other words, the sufficient conditions under which firms invests in period one only under one regulation but not the other.

Theorem 12 *If $\alpha \leq \min(a_2^{ct}/a_1, \underline{a}_2^{cc}/a_1)$, then a firm invests in period 1 under command-and-control but not cap-and-trade if one of the following conditions hold.*

- *Condition 1:* $e_0 x^{nv}(p) > B_1$, $a_1^{ct} < a_1 < \underline{a}_1^{cc}$, c_e *is small enough such that* $a_1^{ct} < \underline{a}_1^{cc}$,*and* $\alpha < \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$;
- *Condition 2:* $e_0 x^{nv}(p) > B_1$, $\underline{a}_1^{cc} \leq a_1 < \min(\underline{a}_1^{cc} + \delta \underline{a}_2^{cc}, \underline{a}_1^{cc} + \delta a_2^{ct})$, c_e *is small enough such that* $a_1^{ct} < \underline{a}_1^{cc}$, *and* $\frac{1}{\delta}(1 - \frac{\underline{a}_1^{cc}}{a_1}) < \alpha < \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$.

Condition 1 says that when the emissions trading price is small in period one, the unit investment cost in period one is large comparing to the emissions trading price in period one but not too large in absolute value, the emissions cap is tight in period one, and the unit investment cost in period two is much smaller than in period one, the firm will invest in period one only under command-and-control. Under condition 1, when the emissions cap is tight under command-and-control, the firm is better off to invest in period one due to the low unit investment cost. However, the unit investment cost is high comparing to the emissions trading price. Hence the firm does not have enough incentive to invest under cap-and-trade in period one and is better off to delay investment until period two due to the sharp decrease of the unit investment cost in period two.

The condition $a_1 < \min(\underline{a}_1^{cc} + \delta \underline{a}_2^{cc}, \underline{a}_1^{cc} + \delta a_2^{ct})$ is a technical condition for α to exist. Condition 2 says that when the emissions trading price is small in period 1, the emissions cap is tight in period 1, the unit investment cost in period 1 is large, and the unit investment cost in period 2 is small enough to discourage investment in period 1 under cap-and-trade but large enough to encourage investment in period 1 under command-and-control, the firm invests in period 1 only under command-and-control. Under condition 2, when the emissions cap is tight, the firm is better off to invest in period 1 to explore the benefit of investing earlier under command-and-control. Since the unit investment cost in period 1 is large and the unit investment cost in period 2 does not drop sharply enough, there is not much benefit to delay investment until period 2 under cap-and-trade. Hence the firm does not have enough incentive to invest in period 1 under cap-and-trade.

The reverse situation is covered by the following result.

Theorem 13 *If* $\alpha \leq \min(a_2^{ct}/a_1, \underline{a}_2^{cc}/a_1)$, *then a firm invests in period one under cap-and-trade but not command-and-control if one of the following conditions hold.*

- *Condition 3:* $B_2 < e_0 x^{nv}(p) \leq B_1$, *and* $a_1 \leq a_1^{ct}$;
- *Condition 4:* $B_2 < e_0 x^{nv}(p) \leq B_1$, $a_1^{ct} < a_1 \leq \min(a_1^{ct} + \delta a_2^{ct}, a_1^{ct} + \delta \underline{a}_2^{cc})$, *and* $\alpha \geq \frac{1}{\delta}(1 - \frac{a_1^{ct}}{a_1})$;

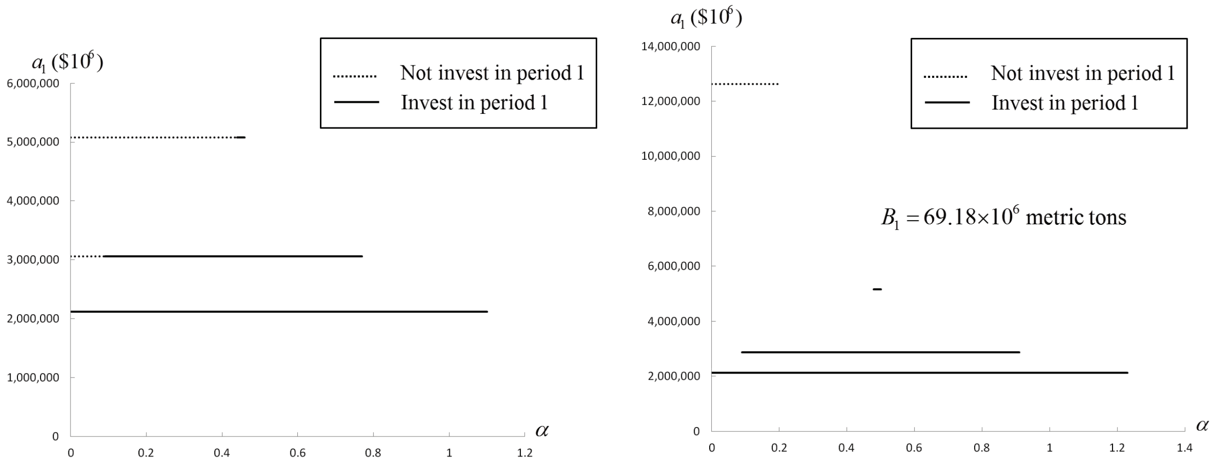
Example: For the two period models, we assume AEP's total investment I_2 to invest in period 2 is 90% of the total investment I_1 to invest in period one. In addition, the numbers of years to amortize the total investment cost are $n_1 = 20$ for period one and $n_2 = 19$ for period two. Because emissions bounds are expected to decrease over time, we also assume that $B_2 = 0.9B_1$. Table 3 lists AEP's model parameter values under these assumptions for this example unless otherwise indicated. From this table, we obtain that AEP invests in period two under cap-and-trade if

$\alpha \leq 1.10$, according to Proposition 8, and under command-and-control if $\alpha \leq 1.23$, according to Proposition 9. Because $\alpha = a_2/a_1 = 0.53$, AEP invests in period two under both regulations.

Table 3: AEP's Parameters for Two-Period Models

million dollars				million metric tons		dollars/tonne
a_1	a_2	a_2^{ct}	$\underline{a}_2^{cc}$	B_1	B_2	c_e
2,121,190	1,131,476	2,340,463	2,601,230	69.18	62.26	12

Figures 11(a) and 11(b) illustrate whether AEP invests in period one under cap-and-trade and under command-and-control for several unit investment costs a_1. The bottom lines correspond to AEP's current situation. Because α is equal to 0.53, AEP invests in period one under both cap-and-trade and command-and-control, according to Proposition 10 condition (1) and 11 condition (3). In Figure 11(a), the unit investment costs for the upper and middle lines are greater than a_1^{ct}, which are within the ranges of a in Proposition 10 conditions (2) and (3). In Figure 11(b), the unit investment cost for the top line is greater than $\bar{a_1}^{cc}$, satisfying Proposition 11 condition (2), and the unit investment costs for the middle two lines are greater than $\underline{a_1}^{cc}$, satisfying Proposition 11 condition (4). Besides what is shown in Figure 11(b), AEP should not invest in period one if $B_1 \geq 138.35$ million metric tons, according to Proposition 11 condition (1).



(a) Cap-and-Trade (b) Command-and-Control

Figure 11: AEP's Investment Decision in Period 1

Although with parameter values listed in Table 3, AEP invests in period one under both cap-and-trade and command-and-trade, there are conditions under which it only invests under one regulation. Figure 12 presents intervals of α's for AEP to invest under only one regulation for different unit investment costs a. The regulation for solid lines is cap-and-trade, and the regulation for dotted lines is command-and-control. The unit emissions cost c_e under cap-and-trade is set to

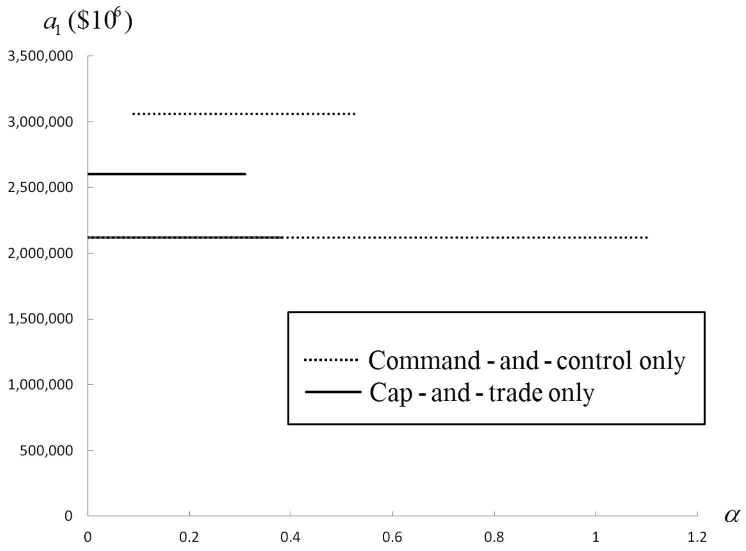


Figure 12: Investment under Only One Regulation for AEP

be six dollars/tonne, and the investment threshold a^{ct} for this c_e is 835,836 millions of dollars. The emissions cap B_1 under command-and-control is set to be 152.19 millions of metric tons, which is greater than $e_0 x^{nv}(p)$. The lower solid line satisfies conditions in Theorem 12 condition (1), the upper solid line Theorem 12 condition (2), the lower dotted line Theorem13 condition (1), and the upper dotted line Theorem13 condition (2). $\square$

6 Change of Production Cost

With investment to reduce emissions, firms' production cost can either increase, for example, due to extra processing of emissions as is the case with CCS, or decrease, for example, due to energy savings and improvement on fuel efficiency. We now consider the impact of a change of production cost on our results.

Let β indicate the increase in the production cost for eliminating one tonne of emissions. The model with change of production cost under cap-and-trade is as follows:

$$\max_{x \geq 0, \underline{e} \leq e \leq e_0} \; p\mathbb{E}[\min(D,x)] - cx - c_e(e\mathbb{E}\min[(D,x)] - A) - \beta(e_0 - e)\mathbb{E}\min[(D,x)] - f(e), \quad (3)$$

where the term $\beta(e_0 - e)\mathbb{E}\min[(D,x)]$ indicates the total additional production cost due to the elimination of emissions. The optimal solutions for the above model can be derived similarly as in Proposition 1.

Corollary 14 *Under the linear investment function $f(e) = a(e_0 - e)$, if the demand CDF $F(\cdot)$ satisfies $\lim_{x\to+\infty} x(1 - F(x)) = 0$, then for a given unit emissions cost c_e, there exists a unit investment cost threshold*

$$a^{ct}(c_e) = \frac{\pi^{nv}(x^{nv}(p - c_e\underline{e} - \beta(e_0 - \underline{e})), p - c_e\underline{e} - \beta(e_0 - \underline{e})) - \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0)}{e_0 - \underline{e}}$$

such that the optimal solution is

$$(x^{ct}, e^{ct}) = \begin{cases} (x^{nv}(p - c_e\underline{e} - \beta(e_0 - \underline{e})), \underline{e}) & a \leq a^{ct}(c_e); \\ (x^{nv}(p - c_e e_0), e_0) & \textit{otherwise}, \end{cases}$$

with the maximum expected profit

$$\pi^{ct}_{\max} = \begin{cases} \pi^{nv}(x^{nv}(p - c_e\underline{e} - \beta(e_0 - \underline{e})), p - c_e\underline{e} - \beta(e_0 - \underline{e})) - a(e_0 - \underline{e}) & a \leq a^{ct}(c_e); \\ \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0) & \textit{otherwise}. \end{cases}$$

In addition, the investment threshold $a^{ct}(c_e)$ increases in c_e, and is equal to 0 when $c_e = \beta$.

The model with change of production cost under command-and-control is as follows:

$$\begin{aligned} \max_{x\geq 0, e} \quad & p\mathbb{E}[\min(D, x)] - cx - \beta(e_0 - e)\mathbb{E}\min[(D, x)] - f(e) \\ \text{s.t.} \quad & ex \leq B \\ & \underline{e} \leq e \leq e_0. \end{aligned}$$

However, due to the complexity incurred as a result of the additional production cost term, we are not able to quantify the optimal solutions.

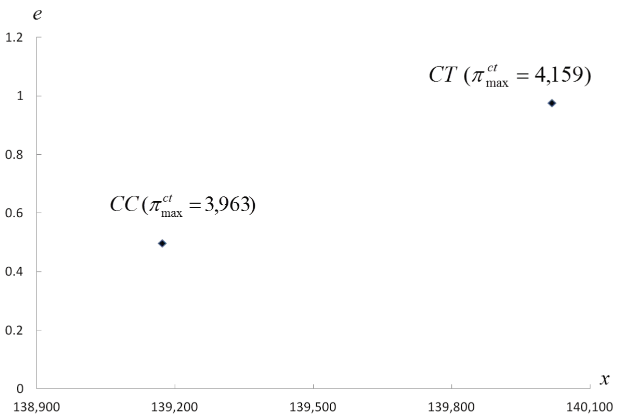
Intuitively, without a change of production cost, the incentive to invest under cap-and-trade is through the emissions charge, while the incentive to invest under command-and-control is through the emissions bound. With a change of production cost, although there are additional cost savings incentives to reduce emissions under both regulations, the original drivers of investment still exist to serve as the main differentiator for these two regulations. Therefore, similar comparison results as those without a change of production cost should hold. Theorem 3, Lemma 4 and Theorem 5 remain the same in this new setting. Slight changes need to be made for the comparison of total emissions and investment amounts, as shown below.

Corollary 15 *Under the linear investment function $f(e) = a(e_0 - e)$, the firm has lower total emissions under command-and-control if and only if $a > a^{ct}$ and $A < x^{nv}(p - c_e e_0)e_0$, or $a \leq a^{ct}$ and $A < x^{nv}(p - c_e\underline{e} - \beta(e_0 - \underline{e}))\underline{e}$.*

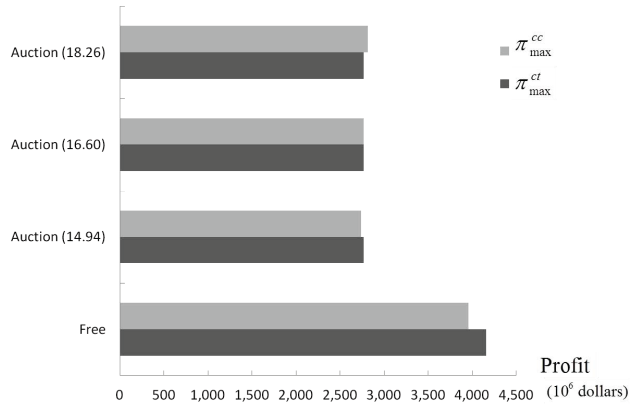
Corollary 16 *Under the linear investment function $f(e) = a(e_0 - e)$,*
(1) a firm invests more to reduce unit emissions under command-and-control if $A < x^{nv}(p)e_0$, a

is small such that the firm invests under command-and-control, and c_e is small enough such that $a > a^{ct}$;
(2) a firm invests more to reduce unit emissions under cap-and-trade if $a < a^{ct}$.

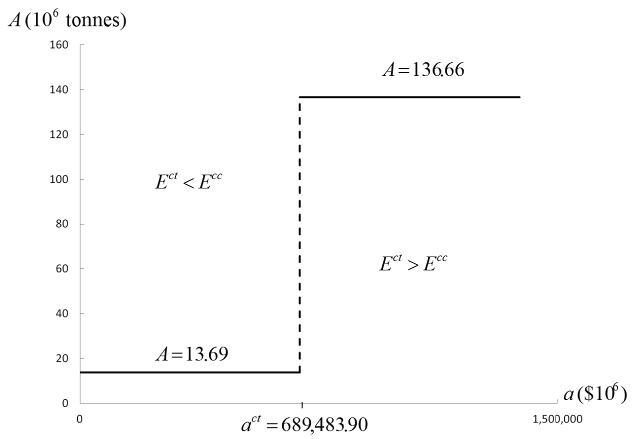
Example: According to the National Energy Technology Laboratory (NETL) (2007b), the additional variable cost v_{Ohio} for running CCS at AEPs Conesville, Ohio, Unit #5 is 1.32 cents/kWh. Therefore, the additional variable cost β is 15.03 ($= \dfrac{v_{\text{Ohio}}}{e_0 - \underline{e}}$) dollars per ton of CO_2 reduced. We assume, as for the case when not considering a change of production cost, the unit emissions cost c_e to be 20 dollars/tonne, and the initial emissions allowance or emissions bound to be 69.18 million metric tons. Figure 13(a) presents the optimal solutions under cap-and-trade and command-and-control for AEP with a change of production cost. We rely on numerical methods to find the optimal solutions under command-and-control. The investment threshold a^{ct} is 689,483 million dollars, which is smaller than AEP's unit investment cost a, being 1,071,399 million dollars.



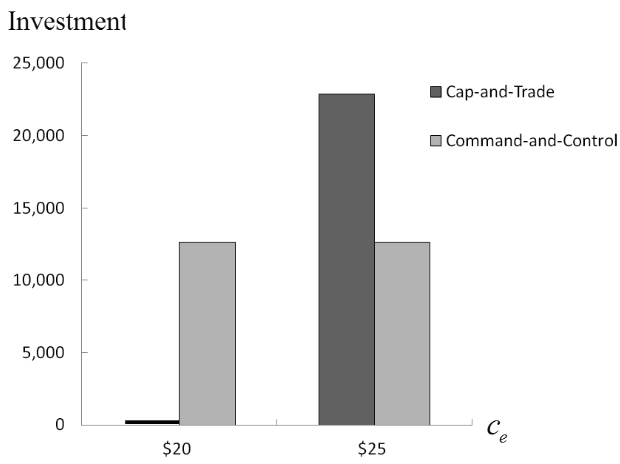
(a) Optimal Solutions



(b) Maximum Profit



(c) Total Emissions



(d) Total Investment

Figure 13: AEP's Comparison with Change of Production Cost

Therefore, for AEP it is not optimal to invest under cap-and-trade. However, based on numerical experiments, it is optimal to invest under command-and-control, with the optimal unit emissions being 0.497 tonne/MWh. Figures 13(b), 13(c), and 13(d) compare AEP's maximum profits, total emissions, and total investments under cap-and-trade and command-and-control. These figures can be understood similarly as those for the case without a change of production cost. □

7 Conclusions

In this paper, we have characterized the optimal investment in process improvement projects and production capacity decisions for an emissions intensive company with several plants under the cap-and-trade and command-and-control regulation. We compare the company's expected profit, total emissions and investment amount and timing under cap-and-trade and command-and-control. Sufficient conditions are given for one regulation to be better than the other for each of the comparison criteria. Our results help the company understand implications of each regulation on its profit, public image in terms of total emissions, and financial resource planning in terms of investment amount and timing.

Economics literature has been touting the benefits of the cap-and-trade over command-and-control regulation. One important managerial insight that can be drawn from our study is that cap-and-trade is not always better. For the company, cap-and-trade may give a lower expected profit for auctioned initial allowances with a large emissions cap. Cap-and-trade may bring a worse public image for the company due to larger total emissions for tight and low emissions caps. In addition, cap-and-trade may not be as effective in motivating investment in emissions reduction projects as command-and-control. If the emissions cap of the company is tight, emissions trading price is small, and the unit investment cost is large comparing to the emissions trading price but not too large comparing to the emissions cap, cap-and-trade may drive a smaller total investment. If in addition, the unit investment cost drops sharply over time, cap-and-trade may also result in a later investment.

Our work can be extended in several ways. In this paper, we frequently assume the investment cost is linear in the emissions level. It would be interesting to study how a nonlinear investment cost impacts our results. Since the main differences of the investment incentives between the cap-and-trade and command-and-control regulations are not influenced by the function forms of the investment cost, we believe similar insights should still hold with a nonlinear investment cost. Nonetheless, it is worthwhile to verify this with a rigorous analysis. We also assume that demand does not change with the company's emissions level. However, demand may change with the company's emissions level due to the fact that consumers may become more and more environmentally conscious. The dependence of demand on the emissions level adds an interesting aspect to our models.

Appendix

In all of the proofs, we assume the technology possible emissions lower bound $\underline{e}$ is equal to zero for simplicity of exposition.

Proof of Proposition 1:

From (1), we get

$$\pi^{ct}(x,e) = (p - c_e e)\mathbb{E}(D) - (p - c_e e)\int_x^{+\infty}(d-x)dF(d) - cx - f(e) + c_e A. \tag{4}$$

Since $f(e) = a(e_0 - e)$, for a given x, we have

$$\frac{\partial \pi^{ct}(x,e)}{\partial e} = -c_e\mathbb{E}(D) + c_e\int_x^{+\infty}(d-x)dF(d) + a. \tag{5}$$

Let $e(x)$ denote the optimal unit emissions for a given production level x. If $a - c_e\mathbb{E}(D) \geq 0$, according to (5), we have $\frac{\partial \pi^{ct}(x,e)}{\partial e} \geq 0$. Therefore, if $a - c_e\mathbb{E}(D) \geq 0$, we have that $\pi^{ct}(x,e)$ increases in e for any given x, and thus $e(x) = e_0$. If $a - c_e\mathbb{E}(D) < 0$, according to (5), we have $\frac{\partial \pi^{ct}(x,e)}{\partial e} = a > 0$ when $x = 0$, while when $x \to +\infty$, according to (5), we have

$$\begin{aligned}
\lim_{x\to+\infty}\frac{\partial \pi^{ct}(x,e)}{\partial e} &= \lim_{x\to+\infty}\left(-c_e\int_0^x \bar{d}dF(\bar{d}) - c_e x\int_x^{+\infty}dF(d) + a\right) \\
&= a - c_e\mathbb{E}(D) - c_e\lim_{x\to+\infty}x(1-F(x)) \\
&= a - c_e\mathbb{E}(D) < 0.
\end{aligned}$$

The last equality is due to the assumption $\lim_{x\to+\infty}x(1-F(x)) = 0$. In addition, we have

$$\frac{d\frac{\partial \pi^{ct}(x,e)}{\partial e}}{dx} = -c_e(1-F(x)) < 0.$$

We conclude that $\frac{\partial \pi^{ct}(x,e)}{\partial e}$ decreases in x, and thus there exists a unique $\bar{x}$ such that $\frac{\partial \pi^{ct}(\bar{x},e)}{\partial e} = 0$, which implies

$$-c_e\mathbb{E}(D) + c_e\int_{\bar{x}}^{+\infty}(d-\bar{x})dF(d) + a = 0. \tag{6}$$

Therefore, if $a - c_e\mathbb{E}(D) < 0$, we have $\frac{\partial \pi^{ct}(x,e)}{\partial e} \geq 0$ and thus $e(x) = e_0$ when $x < \bar{x}$, while we have $\frac{\partial \pi^{ct}(x,e)}{\partial e} < 0$ and thus $e(x) = 0$ when $x \geq \bar{x}$.

It remains to find the optimal production quantity x. We consider three cases.

Case 1 when $a - c_e\mathbb{E}(D) > 0$: We have $e(x) = e_0$, and $\pi^{ct}(x,e_0) = (p - c_e e_0)\mathbb{E}(D) - (p - c_e e_0)\int_x^{+\infty}(d-x)dF(d) - cx + c_e A$, which is concave in x. An optimal x_1^* should satisfy the following first order condition:

$$\frac{\partial \pi^{ct}(x_1^*, e_0)}{\partial x_1^*} = (p - c_e e_0)(1 - F(x_1^*)) - c = 0,$$

which implies that

$$F(x_1^*) = \frac{p - c_e e_0 - c}{p - c_e e_0}.$$

Therefore, we have

$$(x_1^*, e_1^*) = (x^{nv}(p - c_e e_0), e_0).$$

Case 2 when $a - c_e\mathbb{E}(D) \le 0$ and $x \le \bar{x}$: We have $e(x) = e_0$. Similar derivation as in case 1 yields

$$(x_2^*, e_2^*) = \begin{cases} (\bar{x}, e_0) & \bar{x} \le x^{nv}(p - c_e e_0), \\ (x^{nv}(p - c_e e_0), e_0) & \bar{x} > x^{nv}(p - c_e e_0). \end{cases}$$

Case 3 when $a - c_e\mathbb{E}(D) \le 0$ and $x \ge \bar{x}$: We have $e(x) = 0$. Similar derivation as in case 1 implies that if there is no constraint on x, the optimal solution for x is $x^{nv}(p)$. With constraint $x \ge \bar{x}$, because $\pi^{ct}(x, 0)$ is concave in x, the optimal solution is

$$(x_3^*, e_3^*) = \begin{cases} (x^{nv}(p), 0) & \bar{x} \le x^{nv}(p), \\ (\bar{x}, 0) & \bar{x} > x^{nv}(p). \end{cases}$$

Next we need to compare the optimal solutions for the above three cases to get the optimal solution under the cap-and-trade regulation. When $a > c_e\mathbb{E}(D)$, we have $x^{ct} = x^{nv}(p - c_e e_0)$ and $e^{ct} = e_0$. When $a \le c_e\mathbb{E}(D)$, it is clear that, if $\bar{x} \le x^{nv}(p - c_e e_0)$, we have $x^{ct} = x^{nv}(p)$ and $e^{ct} = 0$ since $\pi^{ct}(\bar{x}, e_0) = \pi^{ct}(\bar{x}, 0) \le \pi^{ct}(x^{nv}(p), 0)$. The first equality holds because $\pi^{ct}(\bar{x}, e)$ does not change with e by the definition of $\bar{x}$. If $\bar{x} > x^{nv}(p)$, we have $x^{ct} = x^{nv}(p - c_e e_0)$ and $e^{ct} = e_0$ since $\pi^{ct}(\bar{x}, 0) = \pi^{ct}(\bar{x}, e_0) \le \pi^{ct}(x^{nv}(p - c_e e_0), e_0)$. Further work is needed for the case $x^{nv}(p - c_e e_0) < \bar{x} \le x^{nv}(p)$. We need to compare $\pi^{ct}(x^{nv}(p), 0)$ and $\pi^{ct}(x^{nv}(p - c_e e_0), e_0)$.

Before the comparison, we write the conditions for the optimal values of x^{ct} and e^{ct} in terms of the unit investment cost a. From (6), it follows

$$a = c_e\mathbb{E}(D) - c_e \int_{\bar{x}}^{\infty} (d - \bar{x}) dF(d),$$

which implies that $\bar{x}(a)$ increases as a increases. Let

$$a_1^{ct} = c_e\mathbb{E}(D) - c_e \int_{x^{nv}(p)}^{\infty} (d - x^{nv}(p)) dF(d),$$

and

$$a_2^{ct} = c_e\mathbb{E}(D) - c_e \int_{x^{nv}(p - c_e e_0)}^{\infty} (d - x^{nv}(p - c_e e_0)) dF(d),$$

from which we can observe that $\bar{x}(a_2^{ct}) = x^{nv}(p - c_e e_0)$ and $\bar{x}(a_1^{ct}) = x^{nv}(p)$. Since $\bar{x}(a)$ increases in a, it follows that $\bar{x} \leq x^{nv}(p - c_e e_0)$ if and only if $a \leq a_2^{ct}$, and $\bar{x} \leq x^{nv}(p)$ if and only if $a \leq a_1^{ct}$. In summary, when $a \leq c_e \mathbb{E}(D)$, if $a \leq a_2^{ct}$, then $x^{ct} = x^{nv}(p)$ and $e^{ct} = 0$, if $a > a_1^{ct}$, then $x^{ct} = x^{nv}(p - c_e e_0)$ and $e^{ct} = e_0$.

Next we address the case $a_2^{ct} < a \leq a_1^{ct}$. We compare $\pi^{ct}(x^{nv}(p), 0)$ and $\pi^{ct}(x^{nv}(p - c_e e_0), e_0)$. According to (4), we know

$$\pi^{ct}(x^{nv}(p), 0) = \pi^{nv}(x^{nv}(p), p) - a e_0,$$

and

$$\pi^{ct}(x^{nv}(p - c_e e_0), e_0) = \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0).$$

It follows

$$\pi^{ct}(x^{nv}(p), 0) - \pi^{ct}(x^{nv}(p - c_e e_0), e_0) = \pi^{nv}(x^{nv}(p), p) - \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0) - a e_0,$$

and from this, we can see that $\pi^{ct}(x^{nv}(p), 0) - \pi^{ct}(x^{nv}(p - c_e e_0), e_0)$ is strictly decreasing in a. Let us define

$$a^{ct} = \frac{\pi^{nv}(x^{nv}(p), p) - \pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0)}{e_0}. \tag{7}$$

When $a \geq a^{ct}$, we have $\pi^{ct}(x^{nv}(p), 0) \leq \pi^{ct}(x^{nv}(p - c_e e_0), e_0)$, and otherwise $\pi^{ct}(x^{nv}(p), 0) > \pi^{ct}(x^{nv}(p - c_e e_0), e_0)$.

Therefore, we have when $a \leq c_e \mathbb{E}(D)$, if $a_2^{ct} < a \leq a_1^{ct}$ and $a > a^{ct}$, then $x^{ct} = x^{nv}(p - c_e e_0)$ and $e^{ct} = e_0$, while if $a_2^{ct} < a \leq a_1^{ct}$ and $a \leq a^{ct}$, then $x^{ct} = x^{nv}(p)$ and $e^{ct} = 0$. To simplify these conditions, we show that $a_2^{ct} < a^{ct} < a_1^{ct}$. From (2) and (7), we know that

$$\begin{aligned}
a^{ct} &= \frac{1}{e_0}\Bigg[p\mathbb{E}(D) - p\int_{x^{nv}(p)}^{\infty}(d - x^{nv}(p))dF(d) - c x^{nv}(p) - \\
&\quad \left((p - c_e e_0)\mathbb{E}(D) - (p - c_e e_0)\int_{x^{nv}(p - c_e e_0)}^{\infty}(d - x^{nv}(p - c_e e_0))dF(d) - c x^{nv}(p - c_e e_0)\right)\Bigg] \quad (8) \\
&= a_1^{ct} + \frac{c}{e_0}(x^{nv}(p - c_e e_0) - x^{nv}(p)) - \\
&\quad \frac{(p - c_e e_0)}{e_0}\left[\int_{x^{nv}(p)}^{\infty}(d - x^{nv}(p))dF(d) - \int_{x^{nv}(p - c_e e_0)}^{\infty}(d - x^{nv}(p - c_e e_0))dF(d)\right] \quad (9) \\
&= a_2^{ct} + \frac{c}{e_0}(x^{nv}(p - c_e e_0) - x^{nv}(p)) - \\
&\quad \frac{p}{e_0}\left[\int_{x^{nv}(p)}^{\infty}(d - x^{nv}(p))dF(d) - \int_{x^{nv}(p - c_e e_0)}^{\infty}(d - x^{nv}(p - c_e e_0))dF(d)\right]. \quad (10)
\end{aligned}$$

We also know that

$$
\begin{aligned}
&\int_{x^{nv}(p)}^{\infty}(d-x^{nv}(p))dF(d)-\int_{x^{nv}(p-c_e e_0)}^{\infty}(d-x^{nv}(p-c_e e_0))dF(d)\\
&=-\int_{x^{nv}(p-c_e e_0)}^{x^{nv}(p)}\bar{d}dF(\bar{d})-x^{nv}(p)\frac{c}{p}+x^{nv}(p-c_e e_0)\frac{c}{p-c_e e_0}.
\end{aligned}
\tag{11}
$$

Plugging (11) into (9), we get

$$
\begin{aligned}
a^{ct} &= a_1^{ct}+\frac{c}{e_0}(x^{nv}(p-c_e e_0)-x^{nv}(p))-\\
&\quad \frac{(p-c_e e_0)}{e_0}\left(-\int_{x^{nv}(p-c_e e_0)}^{x^{nv}(p)}\bar{d}dF(\bar{d})-x^{nv}(p)\frac{c}{p}+x^{nv}(p-c_e e_0)\frac{c}{p-c_e e_0}\right)\\
&= a_1^{ct}-\frac{c_e c}{p}x^{nv}(p)+\frac{(p-c_e e_0)}{e_0}\int_{x^{nv}(p-c_e e_0)}^{x^{nv}(p)}\bar{d}dF(\bar{d})\\
&< a_1^{ct}-\frac{c_e c}{p}x^{nv}(p)+\frac{(p-c_e e_0)}{e_0}x^{nv}(p)\left(\frac{p-c}{p}-\frac{p-c_e e_0-c}{p-c_e e_0}\right)\\
&= a_1^{ct}-\frac{c_e c}{p}x^{nv}(p)+\frac{c_e c}{p}x^{nv}(p)\\
&= a_1^{ct}.
\end{aligned}
$$

Substituting (11) in (10), by following the same steps we get $a^{ct} > a_2^{ct}$. Since $a_2^{ct} < a^{ct} < a_1^{ct}$, it follows that when $a \le c_e\mathbb{E}(D)$, we have $x^{ct} = x^{nv}(p)$ and $e^{ct} = 0$ if $a \le a^{ct}$, while $x^{ct} = x^{nv}(p-c_e e_0)$ and $e^{ct} = e_0$ if $a > a^{ct}$. When $a > c_e\mathbb{E}(D)$, we have $x^{ct} = x^{nv}(p-c_e e_0)$ and $e^{ct} = e_0$.

Next we show that $a^{ct} < c_e\mathbb{E}(D)$. According to (8), we have

$$
\begin{aligned}
a^{ct} &= c_e\mathbb{E}(D)+\frac{c}{e_0}(x^{nv}(p-c_e e0)-x^{nv}(p))-\frac{1}{e_0}\times\\
&\quad \left(p\int_{x^{nv}(p)}^{+\infty}(d-x^{nv}(p))dF(d)-(p-c_e e_0)\int_{x^{nv}(p-c_e e_0)}^{+\infty}(d-x^{nv}(p-c_e e_0))dF(d)\right)\\
&= c_e\mathbb{E}(D)-c_e\int_{x^{nv}(p-c_e e_0)}^{+\infty}\bar{d}dF(\bar{d})+\frac{p}{e_0}\left(\int_{x^{nv}(p-c_e e_0)}^{x^{nv}(p)}\bar{d}dF(\bar{d})\right) \qquad (12)\\
&= c_e\mathbb{E}(D)-c_e\int_{x^{nv}(p)}^{+\infty}\bar{d}dF(\bar{d})+(\frac{p}{e_0}-c_e)\left(\int_{x^{nv}(p-c_e e_0)}^{x^{nv}(p)}\bar{d}dF(\bar{d})\right)\\
&< c_e\mathbb{E}(D)-c_e x^{nv}(p)(1-\frac{p-c}{p})+(\frac{p}{e_0}-c_e)x^{nv}(p)\left(\frac{p-c}{p}-\frac{p-c_e e_0-c}{p-c_e e_0}\right)\\
&= c_e\mathbb{E}(D)-c_e x^{nv}(p)\frac{c}{p}+c_e x^{nv}(p)\frac{c}{p}\\
&= c_e\mathbb{E}(D).
\end{aligned}
$$

Since $a^{ct} < c_e\mathbb{E}(D)$, it follows that $x^{ct} = x^{nv}(p)$ and $e^{ct} = 0$ if $a \le a^{ct}$, while $x^{ct} = x^{nv}(p-c_e e_0)$ and $e^{ct} = e_0$ if $a > a^{ct}$.

It remains to show that $a^{ct}(c_e)$ increases in c_e and $a^{ct} = 0$ when $c_e = 0$. From (7), it follows that a^{ct} is a function of c_e, and $a^{ct} = 0$ when $c_e = 0$. Next we show that $a^{ct}(c_e)$ increases in c_e. We know that

$$\pi^{nv}(x^{nv}(p), p) = p\mathbb{E}[\min(x^{nv}(p), D)] - cx^{nv}(p) - ae_0.$$

From the Envelop Theorem it follows that

$$\frac{d\pi^{nv}(x^{nv}(p), p)}{dc_e} = 0. \tag{13}$$

We also know that

$$\pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0) = (p - c_e e_0)\mathbb{E}[\min(x^{nv}(p - c_e e_0), D)] - cx^{nv}(p - c_e e_0) - ae_0.$$

From the Envelop Theorem it follows that

$$\frac{d\pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0)}{dc_e} = -e_0\mathbb{E}[\min(x^{nv}(p - c_e e_0), D)]. \tag{14}$$

From (13) and (14), we conclude

$$\frac{d\pi^{nv}(x^{nv}(p), p)}{dc_e} - \frac{d\pi^{nv}(x^{nv}(p - c_e e_0), p - c_e e_0)}{dc_e} = e_0\mathbb{E}[\min(x^{nv}(p - c_e e_0), D)] > 0,$$

and according to (7), this implies

$$\frac{da^{ct}}{dc_e} > 0.$$

Therefore, a^{ct} increases in c_e. $\square$

Proof of Proposition 2:

For any given e, if there is no emissions bound on total emissions, the optimal $x(e)$ must satisfy

$$\frac{\partial \pi(x(e), e)}{\partial x} = p - c - pF(x) = 0,$$

which implies $F(x(e)) = \frac{p-c}{p}$, and thus the optimal $x(e) = x^{nv}(p)$. Taking into account the emissions bounds, we get that if $x^{nv}(p) \leq B/e$, then $x(e) = x^{nv}(p)$. If $x^{nv}(p) > B/e$, then $x(e) = B/e$. We derive the optimal solution based on two cases of the emissions cap B.

Case 1: The emissions cap is not tight.

If $x^{nv}(p)e_0 \leq B$, then $x^{cc} = x^{nv}(p)$ and $e^{cc} = e_0$. The total emissions under this case are $e_0 x^{nv}(p)$. Thus when the emissions bound is not tight, it has no effect.

Case 2: The emissions cap is tight.

If $x^{nv}(p)e_0 > B$, then the carbon emissions constraint is binding. We have $x(e) = B/e$. By plugging $x(e) = B/e$ into $\pi(x, e)$ to find the optimal emissions level, we have

$$\pi^{cc}(x(e), e) = p\mathbb{E}[D] - p\int_{B/e}^{\infty}(d - B/e)dF(d) - cB/e - a(e_0 - e).$$

It follows that

$$\frac{d\pi^{cc}(x(e), e)}{de} = \frac{B}{e^2}(c - p(1 - F(\frac{B}{e})) + a.$$

If $e \leq B/x^{nv}(p)$, we have $\frac{d\pi^{cc}(x(e),e)}{de} \geq a > 0$ since $F(x^{nv}(p)) = \frac{p-c}{p}$. Because we are maximizing, we conclude that the optimal e must be greater than $B/x^{nv}(p)$. However, at $e = e_0$, we have

$$\frac{d\pi^{cc}(x(e), e)}{de}|_{e=e_0} = \frac{B}{e_0^2}(c - p(1 - F(\frac{B}{e_0})) + a,$$

the sign of which depends on the value of a. Let

$$\underline{a}^{cc} = -\frac{B}{e_0^2}(c - p(1 - F(\frac{B}{e_0})).$$

Since we are in case 2, $e_0 > B/x^{nv}(p)$, which implies $\frac{B}{e_0^2}(c - p(1 - F(\frac{B}{e_0})) < 0$. We conclude that $\underline{a}^{cc} > 0$. Let

$$\bar{a}^{cc} = \max_{B/x^{nv}(p)\leq e\leq e_0} -\frac{B}{e^2}(c - p(1 - F(\frac{B}{e})).$$

Note that $\bar{a}^{cc} \geq \underline{a}^{cc} > 0$. We derive the optimal e for three different intervals of a defined by $\underline{a}^{cc}$ and $\bar{a}^{cc}$ as follows.

(1) When $a \geq \bar{a}^{cc}$, we know that $\pi^{cc}(x(e), e)$ is increasing in the interval $[B/x^{nv}(p), e_0]$. Hence $x^{cc} = B/e_0$ and $e^{cc} = e_0$.

(2) When $\underline{a}^{cc} \leq a < \bar{a}^{cc}$, we know that $\pi^{cc}(x(e), e)$ is increasing in e at the neighborhoods of both $B/x^{nv}(p)$ and e_0. Hence $x^{cc} = B/\tilde{e}$ and $e^{cc} = \tilde{e}$ for some $\tilde{e} \in (B/x^{nv}(p), e_0]$.

(3) When $a < \underline{a}^{cc}$, we have that $\frac{d\pi^{cc}(x(e),e)}{de}$ is greater than 0 at $e = B/x^{nv}(p)$, and smaller than 0 at $e = e_0$. Due to the continuity of $\frac{d\pi^{cc}(x(e),e)}{de}$, there must be some $\bar{e} \in (B/x^{nv}(p), e_0)$ such that $\frac{d\pi^{cc}(x(e),e)}{de}|_{e=\bar{e}} = 0$, and is the global maximizer. Hence $x^{cc} = B/\bar{e}$ and $e^{cc} = \bar{e}$ for some $\bar{e} \in (B/x^{nv}(p), e_0)$.

To summarize case 2, if $x^{nv}(p)e_0 > B$, we have (1) $x^{cc} = B/e_0$ and $e^{cc} = e_0$ if $a \geq \bar{a}^{cc}$, (2) $x^{cc} = B/\tilde{e}$ and $e^{cc} = \tilde{e}$ for some $\tilde{e} \in (B/x^{nv}(p), e_0]$ if $\underline{a}^{cc} \leq a < \bar{a}^{cc}$, and (3) $x^{cc} = B/\bar{e}$ and $e^{cc} = \bar{e}$ for some $\bar{e} \in (B/x^{nv}(p), e_0)$ if $a < \underline{a}^{cc}$. The total emissions under this case are always B.

Note that we also showed

$$\underline{a}^{cc} = -\frac{B}{e_0^2}(c - p(1 - F(\frac{B}{e_0})) \tag{15}$$

and

$$\bar{a}^{cc} = \max_{B/x^{nv}(p)\leq e\leq e_0} -\frac{B}{e^2}(c - p(1 - F(\frac{B}{e})). \tag{16}$$

□

Proof of Theorem 3:
The optimal solution x^{cc} and e^{cc} for command-and-control is feasible under cap-and-trade. Furthermore, by using (4), we have

$$\begin{aligned}\pi^{ct}(x^{cc}, e^{cc}) =& p\mathbb{E}(D) + p\int_{x^{cc}}^{\infty}(x^{cc} - d)dF(d) - cx^{cc} - f(e^{cc}) + c_e(A - e^{cc}\mathbb{E}[\min(x^{cc}, D)]) \\ \geq& p\mathbb{E}(D) + p\int_{x^{cc}}^{\infty}(x^{cc} - d)dF(d) - cx^{cc} - f(e^{cc}) = \pi^{cc}.\end{aligned}$$

This holds due to $e^{cc}\mathbb{E}[\min(x^{cc}, D)] \leq e^{cc}x^{cc} \leq A$. □

Proof of Lemma 4:
As A increases, the optimal value is increasing since we are optimizing over a larger region. Therefore $\pi^{cc}_{\max}(A)$ is nondecreasing in A.

Let $\pi^{ct}(x, e; c_e)$ denote the expected profit under cap-and-trade for production capacity x and unit emissions level e with expected trading price c_e.

Consider $c_{e1} \geq c_{e2}$. For any x and e, we have from (4) that $\pi^{ct}(x, e; c_{e1}) \leq \pi^{ct}(x, e; c_{e2})$. Hence it follows

$$\pi^{ct}(x^{ct}(c_{e1}), e^{ct}(c_{e1}); c_{e1}) \leq \pi^{ct}(x^{ct}(c_{e1}), e^{ct}(c_{e1}); c_{e2}). \tag{17}$$

Here $x^{ct}(c_{e1})$ and $e^{ct}(c_{e1})$ denote the optimal production capacity and unit emissions level under cap-and-trade with unit emissions price c_{e1}, respectively. We know that

$$\pi^{ct}(x^{ct}(c_{e1}), e^{ct}(c_{e1}); c_{e2}) \leq \pi^{ct}(x^{ct}(c_{e2}), e^{ct}(c_{e2}); c_{e2}) \tag{18}$$

since $x^{ct}(c_{e2})$ and $e^{ct}(c_{e2})$ is an optimal solution for unit emissions price c_{e2}. Hence from (17) and (18) we have

$$\pi^{ct}(x^{ct}(c_{e1}), e^{ct}(c_{e1}); c_{e1}) \leq \pi^{ct}(x^{ct}(c_{e2}), e^{ct}(c_{e2}); c_{e2}),$$

which states $\pi^{ct}_{\max}(c_{e1}) \leq \pi^{ct}_{\max}(c_{e2})$. Therefore, $\pi^{ct}_{\max}(c_e)$ is nondecreasing in c_e. □

Proof of Theorem 5:
For any given emissions cost $c_e \geq 0$, we know that

$$\pi^{cc}_{\max}(0) = 0 < \pi^{ct}_{\max}(c_e)$$

and

$$\pi^{cc}_{\max}(x^{nv}(p)e_0) = \pi^{ct}_{\max}(0) \geq \pi^{ct}_{\max}(c_e).$$

Note that $\pi^{cc}_{\max}$ is a function of A. The last inequality is due to $\pi^{ct}_{\max}(c_e)$ being nondecreasing in c_e according to Lemma 4. From Lemma 4, we also know that $\pi^{cc}_{\max}(A)$ is nondecreasing in A. In addition, $\pi_{\max}^{cc}(A)$ is continuous in A according to the Maximum Theorem. Hence there exists an $A^*(c_e)$ such that if $A < A^*(c_e)$, then $\pi_{\max}^{cc}(A) < \pi_{\max}^{ct}(c_e)$, and otherwise $\pi_{\max}^{cc}(A) \geq \pi_{\max}^{ct}(c_e)$.

We have $A^*(0) = x^{nv}(p)e_0$ since the problem under cap-and-trade with 0 emissions price is an uncapped version of command-and-control. The emissions cap under command-and-control does not have an effect as long as it is no less than $x^{nv}(p)e_0$.

The reason that $A^*(c_e)$ is nonincreasing in c_e is as follows. For any $c_{e1} \geq c_{e2}$, let $\pi_{\max}^{cc}(A^*(c_{e1})) = \pi_{\max}^{ct}(c_{e1})$, and $\pi_{\max}^{cc}(A^*(c_{e2})) = \pi_{\max}^{ct}(c_{e2})$. Since $c_{e1} \geq c_{e2}$, we have $\pi_{\max}^{ct}(c_{e1}) \leq \pi_{\max}^{ct}(c_{e2})$ since $\pi_{\max}^{ct}(c_e)$ is nondecreasing in c_e. It follows that $\pi_{\max}^{cc}(A^*(c_{e1})) \leq \pi_{\max}^{cc}(A^*(c_{e2}))$, and thus $A^*(c_{e1}) \leq A^*(c_{e2})$ since $\pi_{\max}^{cc}(A)$ is nondecreasing in A. Therefore, $A^*(c_e)$ is nonincreasing in c_e. □

Proof of Theorem 6:
When $a > a^{ct}$, the total emissions E^{ct} under cap-and-trade are $x^{nv}(p - c_e e_0)e_0$ according to Proposition 1. If $A < x^{nv}(p - c_e e_0)e_0$, then $A < x^{nv}(p)e_0$ since $x^{nv}(p - c_e e_0) \leq x^{nv}(p)$, due to $x^{nv}(\bar{p})$ being nondecreasing in $\bar{p}$. Hence the emissions cap is binding under command-and-control, and therefore, the total emissions E^{cc} under command-and-control are A. We also have $E^{ct} > E^{cc}$ since $A < x^{nv}(p - c_e e_0)e_0$. If $A \geq x^{nv}(p - c_e e_0)e_0$, the total emissions under command-and-control are either A or $x^{nv}(p)e_0$. Therefore, the total emissions under command-and-control are greater than under cap-and-trade.

When $a < a^{ct}$, the total emissions E^{ct} under cap-and-trade are $x^{nv}(p - c_e \underline{e})\underline{e}$ according to Proposition 1. If $A < x^{nv}(p - c_e \underline{e})\underline{e}$, then $A < x^{nv}(p)e_0$ since $x^{nv}(p - c_e \underline{e})\underline{e} < x^{nv}(p)e_0$ due to $x^{nv}(p)$ being nondecreasing in p and $\underline{e} \leq e_0$. Hence the emissions cap is binding under command-and-control. Therefore, the total emissions E^{cc} under command-and-control are A. Therefore, we have $E^{ct} > E^{cc}$ since $A < x^{nv}(p - c_e \underline{e})\underline{e}$. If $A \geq x^{nv}(p - c_e \underline{e})\underline{e}$, the total emissions under command-and-control are either A or $x^{nv}(p)e_0$. We know that $x^{nv}(p)e_0 \geq x^{nv}(p - c_e \underline{e})\underline{e}$ since $x^{nv}(p)$ is nondecreasing in p and $\underline{e} \leq e_0$, and $A \geq x^{nv}(p - c_e \underline{e})\underline{e}$ by assumption. Therefore, the total emissions under command-and-control are greater than under cap-and-trade. □

Proof of Theorem 7:
According to Proposition 2, the total investment I^{cc} under command-and-control is either 0, or $a(e_0 - \bar{e})$ for some $\bar{e} \in (A/x^{nv}(p), e_0)$, or $a(e_0 - \tilde{e})$ for some $\tilde{e} \in (A/x^{nv}(p), e_0]$. According to Proposition 1, the total investment I^{ct} under cap-and-trade is either 0 or ae_0. When $I^{ct} = 0$ and $I^{cc} = a(e_0 - \bar{e})$, it is possible that $I^{cc} > I^{ct}$. By Propositions 1 and 2, the condition for that to happen is $\underline{a}^{cc} > a > a^{ct}$ and $A < x^{nv}(p)e_0$. Recall that $a^{ct} = \dfrac{\pi^{\text{nv}}(x^{nv}(p), p) - \pi^{\text{nv}}(x^{nv}(p - c_e e_0), p - c_e e_0)}{e_0}$, and $\underline{a}^{cc} = -\dfrac{A}{e_0^2}(c - p(1 - F(\dfrac{A}{e_0}))$. When the cost of carbon c_e is 0, we have $\underline{a}^{cc} > a^{ct} = 0$. Due to continuity of a^{ct} as a function of c_e, for every c_e small enough, we have $\underline{a}^{cc} > a^{ct}$.

When $a < a^{ct}$, we have $I^{ct} = ae_0$. Meanwhile, we have $I^{cc} < a(e_0 - A/x^{nv}(p)) < I^{ct}$. □

Proof of Proposition 8

Under the cap-and-trade regulation, the problem in period 2 has exactly the same structure as the single period problem. For a given e_1, there exists an $a_2^{ct}(e_1) = (\pi^{nv}(x^{nv}(p), p) - \pi^{nv}(x^{nv}(p - c_e e_1), p - c_e e_1))/e_1$ such that $x_2^{ct}(e_1) = x^{nv}(p)$ and $e_2^{ct}(e_1) = 0$ if $a_2 \leq a_2^{ct}(e_1)$, and $x_2^{ct}(e_1) = x^{nv}(p - c_e e_1)$ and $e_2^{ct}(e_1) = e_1$ otherwise. According to (12), we have

$$
\begin{aligned}
a_2^{ct}(e_1) =& c_e \mathbb{E}(D) - c_e \int_{x^{nv}(p-c_e e_1)}^{+\infty} d dF(d) + \frac{p}{e_1}\left(\int_{x^{nv}(p-c_e e_1)}^{x^{nv}(p)} d dF(d)\right) \\
\geq& c_e \mathbb{E}(D) - c_e \int_{x^{nv}(p-c_e e_1)}^{+\infty} d dF(d) + \frac{p}{e_1} x^{nv}(p - c_e e_1)\left(\frac{p-c}{p} - \frac{p - c_e e_1 - c}{p - c_e e_1}\right) \\
=& c_e \int_0^{x^{nv}(p-c_e e_1)} d dF(d) + x^{nv}(p - c_e e_1)\frac{c_e c}{p - c_e e_1} \\
\geq& c_e \int_0^{x^{nv}(p-c_e e_1)} d dF(d) + x^{nv}(p - c_e e_1)\frac{c_e c}{p} \\
\geq& c_e \int_0^{x^{nv}(p-c_e e_0)} d dF(d) + x^{nv}(p - c_e e_0)\frac{c_e c}{p}.
\end{aligned}
$$

The last inequality is due to $x^{nv}(p - c_e e_0) \leq x^{nv}(p - c_e e_1)$ since $e_1 \leq e_0$. By defining

$$
a_2^{ct} = c_e \int_0^{x^{nv}(p-c_e e_0)} d dF(d) + x^{nv}(p - c_e e_0)\frac{c_e c}{p},
$$

we have $a_2^{ct} \leq a_2^{ct}(e_1)$ for every $e_1 \leq e_0$. Hence if $a_2 \leq a_2^{ct}$, the firm always invests in period 2 under cap-and-trade. □

Proof of Proposition 9

Under the command-and-control regulation, the problem faced by the firm in period 2 has the same structure as the single period problem:

$$
\begin{aligned}
\pi_2^{cc}(e_1) = \max_{x_2, e_2} \quad & p\mathbb{E}[\min(D_2, x_2)] - cx_2 - f_2(e_2) \\
\text{s.t.} \quad & e_2 x_2 \leq B_2 \\
& 0 \leq e_2 \leq e_1.
\end{aligned}
$$

Hence, for a given e_1, if $e_1 x^{nv}(p) \leq B_2$, then $e_2^{cc}(e_1) = e_1$. If $e_1 x^{nv}(p) > B_2$, there exists $\underline{a}^{cc}(e_1) = -\frac{B_2}{e_1^2}(c - p(1 - F(\frac{B_2}{e_1})))$ such that if $a_2 < \underline{a}_2^{cc}(e_1)$, then $e_2^{cc}(e_1) < e_1$. We want to find a boundary value $\underline{a}_2^{cc}$ such that if $a_2 \leq \underline{a}_2^{cc}$, then the firm invests in period 2 for any possible value of e_1^{cc} if $e_0 x^{nv}(p) > B_2$.

The boundary value $\underline{a}_2^{cc}$ may take different values depending on the value of $e_0 x^{nv}(p)$.

(1) If $B_2 < e_0 x^{nv}(p) \leq B_1$, then $e_1^{cc} = e_0$. We set $\underline{a}_2^{cc} = \underline{a}_2^{cc}(e_0)$. Note that $\underline{a}_2^{cc} > 0$ since $e_0 > B_2/x^{nv}(p)$.

(2) When $e_0 x^{nv}(p) > B_1$, for the 1^{st} period, according to the Envelop Theorem, the optimal e_1^{cc}

must satisfy the following first order conditions:

$$\frac{\partial \pi_1^{cc}(x_1^{cc}(e_1), e_1)}{\partial e_1} = \begin{cases} -\frac{B_1}{e_1^2}(c - p(1 - F(\frac{B_1}{e_1}))) + a_1 - \delta a_2 + \lambda a_2 & e_2^{cc}(e_1) = e_1, \\ -\frac{B_1}{e_1^2}(c - p(1 - F(\frac{B_1}{e_1}))) + a_1 - \delta a_2 & e_2^{cc}(e_1) < e_1. \end{cases}$$

Here $\lambda \geq 0$ is the Lagrangian multiplier for the inequality constraint $e_2 \leq e_1$. Let $\hat{e}$ be the smallest root in the interval $\left[\frac{B_1}{x^{nv}(p)}, e_0\right]$ of $-\frac{B_1}{e^2}(c - p(1 - F(\frac{B_1}{e}))) + a_1 - \delta a_2 = 0$ if there is such a root. Otherwise, we set $\hat{e} = e_0$. We know that e_1^{cc} must be greater than or equal to $\hat{e}$ since $\frac{\partial \pi_1^{cc}(x_1^{cc}(e_1), e_1)}{\partial e_1} > 0$ for any $e < \hat{e}$ under both cases ($e_2^{cc}(e_1) = e_1$ and $e_2^{cc}(e_1) < e_1$). Hence we can add the constraint $e_1 \geq \hat{e}$ to the problem in period 1 without changing the optimal solutions and objective value. Let $\underline{a}_2^{cc} = \min_{\hat{e} \leq e_1 \leq e_0} -\frac{B_2}{e_1^2}(c - p(1 - F(\frac{B_2}{e_1})))$. We know that $\hat{e} > \frac{B}{x^{nv}(p)}$ since $-\frac{B_1}{e^2}(c - p(1 - F(\frac{B_1}{e}))) \geq 0$ for any $e \leq \frac{B}{x^{nv}(p)}$ and $a_1 - \delta a_2 > 0$. Therefore, we have $\underline{a}_2^{cc} > 0$.

If $a_2 \leq \underline{a}_2^{cc}$, then $a_2 < \underline{a}_2^{cc}(e_1)$ for any possible value of e_1^{cc}. Hence when $e_0 x^{nv}(p) > B_2$, the firm always invests in period 2 under command-and-control if $a_2 \leq \underline{a}_2^{cc}$ and $\underline{a}_2^{cc} > 0$. □

Proof of Proposition 10:
Examining the first order condition in period 1, we have

$$\frac{\partial \pi_1^{ct}(x_1, e_1)}{\partial e_1} = -c_e \mathbb{E}(D) + c_e \int_{x_1}^{+\infty} (D - x_1) dF + a_1 + \delta \frac{d\pi_2^{ct}(e_1)}{de_1}.$$

Since $a_2 \leq a_2^{ct}$, we have $\frac{d\pi_2^{ct}(e_1)}{de_1} = -a_2$ according to the Envelop Theorem. Hence we get

$$\frac{\partial \pi_1^{ct}(x_1, e_1)}{\partial e_1} = -c_e \mathbb{E}(D) + c_e \int_{x_1}^{+\infty} (D - x_1) dF + a_1 - \delta a_2. \tag{19}$$

We also know that

$$\frac{\partial \pi_1^{ct}(x_1, e_1)}{\partial x_1} = (p - c_e e_1)(1 - F_{D_1}(x_1)) - c. \tag{20}$$

By examining the first order conditions (19) and (20) in period 1, we find that the problem in period 1 has a similar structure as the single period problem except that now the effective unit investment cost is $a_1 - \delta a_2$. The optimal solution can be obtained similarly. Let $a_1^{ct} = (\pi^{nv}(x^{nv}(p), p) - \pi^{nv}(x^{nv}(p - c_e e_0, p - c_e e_0))/e_0$. If $a_1 - \delta a_2 \leq a_1^{ct}$, then $x_1^{ct} = x^{nv}(p)$ and $e_1^{ct} = 0$. Otherwise, $x_1^{ct} = x^{nv}(p - c_e e_0)$ and $e_1^{ct} = e_0$.

In summary, under the assumption that $\alpha \leq \frac{a_2^{ct}}{a_1}$, if $a_1 \leq a_1^{ct}$, then the firm invests in period 1. If $a_1 > a_1^{ct}$, $\alpha \leq \frac{a_2^{ct}}{a_1}$ and $(1 - \delta\alpha) \leq \frac{a_1^{ct}}{a_1}$, then the firm also invests in period 1. In this case, α exists if $a_1 \leq \delta a_2^{ct} + a_1^{ct}$. If $a_1 > a_1^{ct}$, $\alpha \leq \frac{a_2^{ct}}{a_1}$ and $(1 - \delta\alpha) > \frac{a_1^{ct}}{a_1}$, the firm does not invest in period

1. $\square$

Proof of Proposition 11:

For the problem in period 1, if $e_0 x^{nv}(p) \leq B_1$, then $x_1^{cc} = x^{nv}(p)$ and $e_1^{cc} = e_0$. If $e_0 x^{nv}(p) > B_1$, we can add the constraint $e_1 \geq \hat{e}$ to the problem without impacting optimality according to the proof of Proposition 9. Plugging $x_1^{cc}(e_1) = \dfrac{B_1}{e_1}$ into the objective function $\pi_1^{cc}(x_1, e_1)$, we have $\dfrac{\partial \pi_1^{cc}(x_1^{cc}(e_1), e_1)}{\partial e_1} = \dfrac{B_1}{e_1^2}(c - p(1 - F(\dfrac{B_1}{e_1}))) + a_1 - \delta a_2$ since $a_2 \leq \underline{a}_2^{cc}$. Recall that if $a_2 \leq \underline{a}_2^{cc}$, the firm invests in period 2 and hence $\dfrac{d\pi_2^{cc}(e_1)}{de_1} = -a_2$ according to the Envelop Theorem. Note that the problem in period 1 has the same structure as the single period problem. Therefore, the optimal solution can be obtained similarly. Let

$$\underline{a}_1^{cc} = -\frac{B_1}{e_0^2}(c - p(1 - F(\frac{B_1}{e_0}))$$

and

$$\bar{a}_1^{cc} = \max_{B_1/x^{nv}(p) \leq e_1 \leq e_0} -\frac{B_1}{e_1^2}(c - p(1 - F(\frac{B_1}{e_1})).$$

If $a_1 - \delta a_2 < \underline{a}_1^{cc}$, the firm invests in period 1. If $a_1 - \delta a_2 \geq \bar{a}_1^{cc}$, the firm does not invest in period 1.

In summary, under the assumption that $a_2 \leq \underline{a}_2^{cc}$, i.e. $\alpha \leq \dfrac{\underline{a}_2^{cc}}{a_1}$, when $e_0 x^{nv}(p) \leq B_1$, the firm does not invest in period 1. When $e_0 x^{nv}(p) > B_1$, the firm does not invest if $a_1(1 - \delta\alpha) \geq \bar{a}_1^{cc}$. This implies that $a_1 \geq \bar{a}_1^{cc}$ since $\alpha \geq 0$ and $\delta \geq 0$. When $e_0 x^{nv}(p) > B_1$, the firm invests in period 1 if $a_1 < \underline{a}_1^{cc}$, or if $a_1 \geq \underline{a}_1^{cc}$ and $(1 - \delta\alpha) < \dfrac{\underline{a}_1^{cc}}{a_1}$. Note that if $\alpha \leq \dfrac{\underline{a}_2^{cc}}{a_1}$ and $(1 - \delta\alpha) < \dfrac{\underline{a}_1^{cc}}{a_1}$, value α exists if and only if $a_1 < \underline{a}_1^{cc} + \delta \underline{a}_2^{cc}$. $\square$

Proof of Theorem 12:

We derive sufficient conditions under which the firm invests in period 1 under command-and-control but not cap-and trade under the assumption that $\alpha \leq \dfrac{a_2^{ct}}{a_1}$ and $\alpha \leq \dfrac{\underline{a}_2^{cc}}{a_1}$.

- Condition 1: Under condition (3) in Proposition 10, a firm does not invest under cap-and-trade. Under condition (3) in Proposition 11, a firm invests under command-and-control. When putting these conditions together, for a_1 to exist, we need $a_1^{ct} < \underline{a}_1^{cc}$. We know that $a_1^{ct} < \underline{a}_1^{cc}$ if and only if c_e is small enough as shown in Theorem 7.
- Condition 2: Under condition (3) in Proposition 10, a firm does not invest under cap-and-trade. Under condition (4) in Proposition 11, a firm invests under command-and-control. When putting these conditions together, for α to exist, we need $a_1^{ct} < \underline{a}_1^{cc}$. We know that $a_1^{ct} < \underline{a}_1^{cc}$ if and only if c_e is small enough as shown in Theorem 7. For $\alpha \leq \dfrac{a_2^{ct}}{a_1}$ and $\alpha > \dfrac{1}{\delta}(1 - \dfrac{a_1^{ct}}{a_1})$ to hold at the same time, we also need $a_1 < \underline{a}_1^{cc} + \delta a_2^{ct}$.

□

Proof of Theorem 13:
The proof is similar to the proof of Theorem 12 except that we use conditions (1) and (2) in Proposition 10, and condition (1) in Proposition 11. □

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