Fleeting with Passenger and Cargo Origin-Destination Booking Control

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Abstract

In tactical planning of an airline, fleeting or capacity planning is the process of assigning an equipment type to each flight. The existing fleeting models take into account only passenger flow while completely ignoring cargo traffic. We propose a fleeting model that captures both passenger and cargo revenue. In addition, the model integrates fleeting and bid price based origin-destination revenue management approach. We use Benders decomposition to solve the model. We give extensive computational experiments on data obtained by a major carrier.

1 Introduction

Airline tactical planning broadly consists of scheduling, fleeting, aircraft routing, and crew scheduling (see e.g. Klabjan (2003) for more details). Our work focuses on *fleeting* or *capacity planning.* The objective of the *fleet assignment model* (FAM) is to assign a particular equipment type to each given leg in a predefined flight schedule while maximizing profit. In a typical FAM, the profit is the revenue earned by transporting passengers minus the operating cost of using a given equipment type. The constraints in the FAM are the assignment constraints (each flight in the schedule is assigned to an equipment type), flow balance (every aircraft that lands must take off), and plane count constraints (we cannot use more than the given number of available aircraft).

The traditional FAM model, also called *leg-based FAM* or *leg-based fleeting*, takes into consideration only the average fare for each leg. In presence of multi-leg passenger itineraries, the fares need to be prorated. This is inconsistent with recent revenue management practices that consider revenue at the itinerary level without any prorating. The leg-based FAM gives solutions that are biased towards larger aircraft, i.e. they produce significant passenger spill, Barnhart et al. (2002b). This leads to better fleeting models and solution methodologies, which are consistent with revenue management practices.

Modeling revenue based on the number of passengers on a given multi-leg passenger itinerary as opposed to on each leg gives a better approximation of the passenger revenue. The *origin-destination fleet assignment model* implements this improved revenue management approach to capture passenger revenue.

Revenue management as defined above only takes into account the passenger fares. But for combination air carriers, revenue management takes into account management of passenger fares and seat capacity along with cargo rates and available cargo space. Airlines have separate cargo and passenger revenue management systems. These problems are solved in a hierarchical fashion. First, the passenger revenue management system determines the number of passengers on each leg. Once this has been decided, the available cargo weight and volume is determined by subtracting the weight of passengers and their bags (as determined by the passenger revenue management) from the payload. The available belly volume for cargo is determined in a similar manner by subtracting the volume of passenger bags from the actual belly volume.

One of the main differences between the two processes is the booking time period. Passengers start booking their itineraries several months in advance and in most cases at least 50% of all bookings are done more than one month in advance. On the other hand, cargo bookings typically start 10 days in advance and the last 50% of bookings are carried out in the last 4 days. While this time difference does not offer great opportunities for integrating the two systems except for data synchronization, there are potential benefits in the fleet assignment phase.

All fleeting models take into account only passenger flow and not the cargo flow. We propose a fleeting model that incorporates both passenger and cargo revenue. The main motivation of this work comes from a changed environment in the airline industry after the events of September 11th. Due to the significant decrease in the number of business class passengers, the passenger revenue is decreasing. To compensate for these losses many airlines started paying more attention to the revenue obtained from cargo. As a consequence, the current trend shows an increase in air cargo volume, which is supposed to double by 2005 (Kasilingam (1996)). It is forecasted that it will grow at the annual rate of 6.4% in revenue tonne-kilometers until 2021, Crabtree et al. (2002). Despite the fact that these numbers are for freight cargo in general, it implies that cargo can be a significant contribution to the revenue of a combination carrier. For example, Singapore Airlines gets 30% of its total revenue from cargo, Lufthansa 20%, and the rising star when it comes to cargo Korean Air up to 33% (see annual reports of these carriers). While all of these three carriers operate pure freighter aircraft, the bellyhold or lower-hold cargo revenue (cargo transported in bellies of passenger aircraft) of these airlines is significant: for Korean Air it is 8% of the total revenue from their flight operations (see the 2004 annual report) and it is 10% for Lufthansa, Froehlich (2004). For United Airlines, who does not have a single pure freighter, the cargo revenue contributes 4% to the total revenue. In addition, based on Crabtree et al. (2002), 56% of the total freight capacity consists of the aircraft belly capacity in passenger aircraft. This number is even larger in Canada, where 85% of the total air cargo is transported as bellyhold cargo on scheduled or charter passenger aircraft, Mathieson (2005). Based on these facts among the combination carriers, there is a need for fleeting models that incorporate cargo and passenger revenue. Such a model will ensure efficient distribution and utilization of aircraft space between passengers and cargo and thereby will provide higher cumulative profit. Fleeting decisions are made several months in advance and traditional models consider forecasted passenger demand. Fleeting models that capture both streams of revenue take into account forecasted passenger and cargo demands.

We present an integrated fleeting model which takes into account both the cargo and passenger revenue. We first propose a cargo routing model called the *cargo mix bid price model*, which has demand, weight and volume constraints. Given a fleeting solution, this model assigns optimal cargo allocations in order to maximize cargo revenue. On the passenger side, we use the *passenger mix bid price model*, which is heavily used in revenue management systems, to allocate passengers to each itinerary while maximizing passenger revenue. The integrated *passenger-cargo fleeting model* is obtained by combining the traditional leg-based FAM model with these two bid price models. The other main distinction between cargo and passengers is the fact that passenger itineraries are given at the leg level, however, cargo demand is given at the origin-destination (O-D) level. Since the number of passenger and even more cargo itineraries is large, the model size is prohibitive. To overcome this, we use Benders decomposition to solve the proposed model.

In our solution methodology, in each iteration, we first obtain a fleeting, then we run the passenger mix bid price model to attain maximum passenger revenue, and then we use the leftover capacity to get the cargo revenue by running the cargo mix bid price model. Based on these two bid price models, Benders cuts are added. Note that the passenger mix bid price model is run before the cargo one. Although the fleeting solution does take into account the revenue contribution of both the entities, the split of capacities is not done in the same proportion. This is aligned with current business practices, where all the priority is given to the passenger revenue.

The main contributions of our work are:

- a comprehensive cargo mix bid price model,
- a fleeting model that considers both passenger and cargo revenue, and
- the solution methodology tailored for our model.

In Section 2 we present a brief review of the traditional FAM and the passenger mix bid price model. We also briefly describe the main differences between the cargo revenue management system and the standard passenger revenue management system. Section 3 describes the cargo mix bid-price model and the integrated fleeting models. Section 4 outlines the solution methodology. In Section 5 we present computational experiments.

2 Traditional Models

2.1 Fleet Assignment Model

Given a set of flights and a set of fleets, the fleeting problem is to find a fleeting that maximizes profit subject to a set of constraints. The assignment constraints ensure that each flight is assigned to an equipment type. Flow balance constraints enforce that each aircraft that lands must take off. In addition, the total number of aircraft used cannot exceed the number available in the fleet. This is taken care by the plane count constraints.

The input consists of a flight schedule, which details the origin/destination stations and the departure/arrival times, the different equipment types and the available number of aircrafts for each equipment type. In strategic planning often it is assumed that the flight schedule is daily, i.e. every flight is repeated every day of the week. Throughout this manuscript, we assume that the flight schedule is daily. The objective function in FAM captures the operating cost and revenue. The average fares per leg, which are calculated based on a prorating scheme, are used to calculate the revenue. The operating costs consist of fixed and variable costs associated with using a specific equipment type for a flight, e.g. fuel cost, fixed cost of using a specific equipment type at a given station, depreciation cost, etc. The model captures revenue on a per leg basis.

We now briefly describe the network that is built to capture all the flights in the schedule. This network is modeled as a timeline at every station. The timeline captures all the activities at the station where an activity is defined as a departure or an arrival. For each activity, let t_l be the time of the activity. For a departure flight, t_l is the actual departure time of the flight. For an arrival flight, t_l is the arrival time plus the minimum aircraft turn time t_{mt}, also called the ready time. The turn time can be dependent on the fleet type. For each activity a at station o, there is a node (o,a) in the timeline for station o. For each flight, we define a *flight arc* between the departure and arrival nodes of the same flight. For every station o, the activities are ordered based on the time t_l, i.e. $t_1 \leq t_2 \leq \cdots \leq t_n$, where n is the number of activities at the station. We also define *ground arcs* $g = ((o,a_i), (o,a_{i+1}))$ for i=1,2,..,n. There is a wraparound ground arc between the first and the last node of the day.

The FAM has two families of variables, the fleet assignment variables x and the ground arc variables z. For each flight in the schedule l and each fleet f, we have a binary variable x_{fl} which is 1 if flight l is assigned the equipment type f. Similarly, for each equipment type f and ground arc g, we define a nonnegative variable z_{gf}, which counts the number of planes of fleet f on the ground during ground arc g time interval. Let MD be a fixed time, which corresponds to a time with low activity at any station, e.g. 3 am. The leg-based FAM model reads

$$\max \sum_{f,l} r_{fl} x_{fl}$$

$$\sum_{f \in F} x_{fl} = 1 \qquad l \in L \tag{1}$$

$$\sum_{l \in O(v)} x_{fl} + z_{o(v)f} - \sum_{l \in I(v)} x_{fl} - z_{i(v)f} = 0 \qquad v \in V,\, f \in F \tag{2}$$

$$\sum_{l \in M} x_{fl} + \sum_{g \in W} z_{gf} \leq N_f \qquad f \in F \tag{3}$$

$$x \text{ binary}, z \geq 0,$$

where

$I(v)$	: set of flights to node v in RTN	L	: set of all flight arcs
$O(v)$	: set of flights from node v in RTN	F	: set of all fleets
M	: set of flights in the air at MD	V	: set of all nodes in RTN
N_f	: number of aircraft in fleet f	$i(v)$	: ground arc to node v in RTN
W	: set of ground arcs which contain MD	$o(v)$	: ground arc from node v in RTN
r_{fl}	: profit of assigning fleet f to leg l.		

Constraints (1) are the assignment constraint, (2) are the flow balance constraints, and (3) are the plane count constraints. This FAM model is described in detail in Hane et al. (1995). Maintenance requirements and some crew constraints can be incorporated into this FAM model as shown in Clarke et al. (1994). Details and enhancements to the FAM model and an extensive literature survey can be found in the survey work by Klabjan (2003).

2.2 Passenger Mix Bid Price Model

Due to multi-leg itineraries, the average fare per leg does not correspond to the correct revenue value. A spilled passenger on one leg effects revenue on all the flights in her itinerary. The leg-based fleeting model does not capture these network effects. Many revenue management systems rely on the marginal values for selling a seat. These are also known as bid prices; see e.g. Williamson (1992), McGill and van Ryzin (1999), or van Ryzin and Talluri (2002). The passenger mix bid price model yields passenger revenue based on a given seat capacity and forecasted demand.

Given a fixed capacity for each leg in the schedule, the passenger mix model decides upon the number of passengers on each given itinerary. Let I be the set of all possible passenger itineraries. Fare classes for passenger itineraries are denoted by k. Let f_i^k be the fare and u_i^k (the decision variable) the number of passengers for a given itinerary i and fare class k. Each fleet has a specific cabin configuration. We index the cabins by j. Let C_{lj} denote the seat capacity of a given leg l in cabin j. For cabin j let F_j be the set of fare classes that use j.

There are two types of constraints in the passenger mix bid price model. The demand constraints for each itinerary ensure that we do not exceed the forecasted demand of passengers on a given itinerary. The demand in this case is the average forecasted demand. Let d_i^k be the forecasted average demand for itinerary i and fare class k. The other family of constraints is the capacity constraints. These impose seat capacity constraints for each leg and cabin. The passenger mix model reads

$$\max \sum_i \sum_k f_i^k u_i^k$$

$$\sum_{i:l \in i} \sum_{k \in F_j} u_i^k \leq C_{lj} \qquad l \in L, \text{cabin } j \tag{4}$$

$$u_i^k \leq d_i^k \qquad i \in I, \text{class } k \tag{5}$$

$$u \geq 0.$$

Constraints (4) are the capacity constraints and (5) are the demand constraints.

A big drawback of this model is that it does not incorporate passenger recaptures. If a passenger spills, then we assume that the revenue is lost, which is not always the case in practice since the passenger might opt for an alternative itinerary with the same carrier. More details on this issue can be found in Kniker (1998).

2.3 Main Differences between Cargo Revenue Management and Passenger Revenue Management

The cargo revenue management system differs from the passenger one in many ways. Unlike passengers, for cargo we have shippers, forwarders, and different shapes and sizes of cargo that need to be taken into account. Next we list the key differences between the two revenue management systems.

Cargo is two-dimensional, i.e. weight and volume. The third dimension corresponding to containers is addressed later. In passenger revenue management since we only need to decide the number of passengers on a given flight, we have restrictions only with respect to the seat capacity on flights. For all practical purposes, weight and volume constraints are redundant. On the other hand, in cargo revenue management, we need to capture weight capacity as well as volume restrictions.

Unlike passengers who follow pre-defined itineraries, cargo can be shipped through any route as long as it reaches the destination before the required delivery time. Another aspect that needs to be taken into consideration is that any given shipment of cargo could be possibly split and shipped to the destination through different routes. Thus cargo demand is given at the O-D level and not at the itinerary level. While passenger demand realizes in a span of approximately six months, most of the cargo bookings are done in the last two weeks before the departure.

Another characteristic specific to cargo is allotments. Big shippers and forwarders reserve some space on particular flights for certain days. This space cannot be used for general cargo. To complicate the matter even further, a shipper or forwarder might deliver less or more cargo than its allotment. Allotments need to be forecasted and taken into account.

In case of a wide body aircraft, the cargo and passenger bags are stored in containers. Containers can be viewed as positions in the belly of a certain shape, size, and volume. In cargo revenue management we need to take into account container availability and space restrictions on the cargo based on the shape and size of the containers.

Along with these basic differences between cargo and passenger traffic, the available cargo capacity is not fixed. In passenger revenue management the number of seats on a given flight is fixed. But in case of cargo, the capacity is determined by payload, available space in the belly and number of passengers and their bags. Payload is not known accurately until the final flight plan is constructed, which is only a few minutes before the departure since it depends on the runway structure, fuel (depends on the flight route), etc. All these differences show that cargo revenue management is more complex than passenger revenue management.

2.4 Literature Review

In this section we present a literature review on O-D fleeting models and work related to cargo revenue management and fleeting.

We first review literature on O-D fleeting. Kniker (1998) gives a comprehensive treatment of fleeting. He proposes enhancements and generalizations to the passenger mix model, which capture spill and recapture. He illustrates pitfalls of leg-based approaches and proposes approaches to enhance them. Barnhart et al. (2002b) present an O-D fleeting model with spill and recapture. Barnhart et al. (2002a) propose an alternative model to O-D fleeting. In their model they use variables that assign a subset of legs to a fleet as opposed to assigning individual legs to fleets. Jacobs et al. (1999) and Smith (2002) propose a different approach to O-D fleeting. They embed the passenger mix bid price model to fleeting and then solve it by Benders decomposition. In this work we build on their model and solution methodology.

On the cargo revenue management side, Hendricks and Kasilingam (1992) present a detailed discussion of the cargo revenue management problem. Kasilingam and Hendricks (1993) discuss the development and implementation of the cargo revenue management system at American Airlines. Some of the major differences between cargo revenue management and passenger revenue management are detailed in Kasilingam (1996). A decision support system to evaluate and optimize a flight schedule of a major cargo airline is presented by Antes et al. (1997). They propose three models to generate schedules; two of them are multi-commodity network flow models with O-D's being modeled as commodities. No time restrictions on the cargo shipments are modeled in these two models. In the third approach they use the path formulation and they take into consideration the time constraints to generate feasible cargo routes. They do not consider containers and the capacity of a leg is one-dimensional.

Chen et al. (2003) propose a model for routing considerations within a cargo revenue management system. Capacity is one-dimensional and they do not incorporate containers. They give a deterministic and a stochastic model. A model for online cargo routing, i.e. developing cargo routes in real time, is proposed by Günther (1998). They first decompose the network by imposing hierarchy among stations with respect to their activities. A Dijkstra type shortest path algorithm is then used to generate cargo routes.

Huang et al. (1999) present a model for simultaneous fleet assignment and cargo routing for a combination carrier. They estimate the passenger revenue on a per leg basis as is modeled in the traditional FAM. The model is solved by Benders decomposition. Our model is more general since we capture passenger revenue at the O-D level, our cargo capacity is 2 dimensional, we consider containers, and we use cargo classes, which are used in current revenue management systems. On the other hand, their model is more general than ours since they consider pure freighter aircraft. As a result of O-D passenger revenue, our Benders decomposition is more involved.

3 The Model

One of the key differences between cargo and passengers as described earlier is the predefined versus flexible routes. Unlike passengers who have fixed itineraries, cargo can be shipped through any route while ensuring that it reaches the destination prior to the specified delivery time. For each given O-D pair, we can have multiple routings (or itineraries or paths). In this section, we first introduce the cargo mix bid price model. Our model is an extension to all existing models since we incorporate many considerations present in practice. For example, we capture two dimensions of the capacity, we consider standard and express shipments, and we consider containers. The three dimensionality of cargo capacity, i.e. volume, weight, and containers, is stressed by several authors, see e.g. Kasilingam (1996), LaDue (2004). We also use cargo rate classes, which are commonly used in modern cargo revenue management systems. We then introduce the integrated passenger cargo fleeting model that is a combination of O-D FAM and the cargo mix bid price model.

3.1 Cargo Mix Bid Price Model

The objective of the cargo mix bid price model is to maximize revenue obtained from cargo without exceeding the available capacity on any given leg and ensuring that the cargo is delivered on time. The input to this model is the forecasted cargo demand and the available weight and volume on each flight. Before introducing the model, we detail notation and terms that are used in the rest of the paper.

Similar to the passenger mix model, we have fare classes in the cargo mix model. These fare classes are based on density of the cargo and are termed *density fare classes* or simply *density classes*. The density classes are denoted by j. Each of these classes corresponds to a range of density values (in lb/m^3). For each density class we also have the corresponding revenue factors (in \$/lb/mile). Note that such density classes correspond to pricing schemes used by the airlines. In our model, we cater to *standard* as well as *express* shipments. Express shipments have to be delivered within a certain time window, e.g. 24 hours. This simply means that the elapsed time of the underlying routes cannot exceed this time. All other shipments are classified as *standard*. For each density class, we therefore have revenue factors for express and standard shipments separately.

We first describe the underlying network to generate cargo itineraries or routes. The network has nodes representing departures and arrivals of flights. A flight arc connects the departure and the arrival node corresponding to the same flight. Connection arcs between a pair of flights are added if the arrival station of the first flight is the same as the departure station of the second flight and the time between the two activities is more than the minimum turn time t_{mt}. Here the connection arcs wrap in time, e.g. a flight arriving at 11 pm can be connected to a flight departing at 6 am from the same station. These network components are common to each O-D pair. For each O-D pair, denoted by od, we make the following changes to the network. Here o corresponds to the origin station and d to the destination station. First, we add a source node s and a sink node t. We remove the flight arcs corresponding to the flights arriving at o. This prevents the generation of cargo itineraries, which visit the origin station more than once in a route. We connect source node s to all flights that depart from o. Similarly we connect all the flights that arrive at d with sink node t and we remove all flight arcs departing from d. A small sample network is depicted in Figure 1. Note that additional nodes and arcs can be removed from the BOS-ORD network.

The presented network is very generic and unfortunately it has cycles. In our implementation, we use a slightly different acyclic network. For ease of exposition, let us assume that each itinerary cannot be longer than two days. In this case, we replicate all the nodes and flight arcs twice. This construction automatically forbids all itineraries that are longer than two days. For each od we do not replicate the flight arcs departing from o on the second day. This ensures that we do not generate duplicate cargo itineraries. It is obvious that every od itinerary corresponds to a path in such a network.

For every given O-D pair od there is a set of feasible cargo itineraries, which is denoted by S_{od}. The set of all O-D pairs is denoted by OD. We also distinguish between the set of itineraries, which cater to express shipments, and those that cater to standard shipments. Let S^e_{od} , S^s_{od} be the set of feasible itineraries for a given od that correspond to express and standard shipments, respectively. Typically $S^e_{od} \subseteq S^s_{od}$. Cargo itineraries are indexed by p.

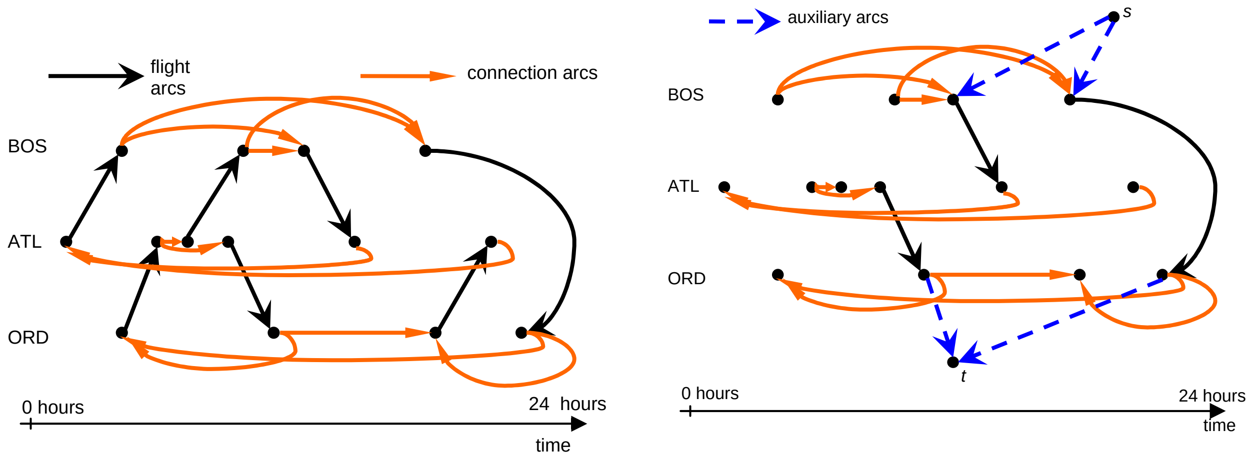


Figure 1: *Generic network on the left, network corresponding to BOS-ORD on the right*

For any given *od* and density class *j*, we assume there are forecasts for the weight of bulk cargo and the number of containers. Note that we use containers in a generic sense since they can correspond to any large shipment. These forecasts are done separately for express and standard shipments. The forecasts are denoted as follows:

$FW^{E}_{od,j}$: forecasted weight for express cargo shipment for a given $od \in OD$ and density class j

$FW^{S}_{od,j}$: forecasted weight for standard cargo shipment for a given $od \in OD$ and density class j

$FC^{E}_{od,j}$: forecasted number of containers for express shipment for a given $od \in OD$ and density class j

$FC^{S}_{od,j}$: forecasted number of containers for standard shipment for a given $od \in OD$ and density class j.

Although in the real world cargo demand is stochastic, here we use only expected values and therefore the resulting model is deterministic. We detail in Section 6 future research using stochastic demand so that the results are more inline with the real world.

Revenue depends on *od*, weight of the shipment, which is captured by decision variables, and the density class. The notation for revenue is as follows:

re^{j}_{od} : revenue obtained from express bulk cargo shipment for density class j for a given $od \in OD$

rs^{j}_{od} : revenue obtained from standard bulk cargo shipment for density class j for a given $od \in OD$

ce^{j}_{od} : revenue obtained from express container shipment for density class j for a given $od \in OD$

cs^{j}_{od} : revenue obtained from standard container shipment for density class j for a given $od \in OD$.

Typically the dependency on *od* is in terms of the geographical location of stations *o* and *d* and the fixed distance factor between these two stations.

For a given leg $l \in L$, let $\overline{W_l}$ be the available weight capacity and let $\overline{V_l}$ be the available volume. Note that these are given since in the cargo mix model we assume that first a fleeting is obtained. The weight capacity is also offset by the weight of passengers and their bags and the available volume by the volume of passengers' bags. For our current model, we assume that all the containers are of the same volume, which is denoted by v. It is easy to extend the model to the more general case of multiple volumes. In addition, let d_j be the representative density of density class j. Since a density class corresponds to a range of densities, typically this is the average density in this range.

We have two types of variables: the cargo weight and the number of containers. There is a variable for each cargo itinerary p and each given density class j. Variables to capture volume of the cargo are not required because volume is implicitly calculated from the weight variables and the corresponding density class. We further decompose the variables to capture the amount shipped as standard and express shipments. The variables are:

we_p^j : amount (weight) of express cargo shipped on itinerary p for a given density class j

ws_p^j : amount (weight) of standard cargo shipped on itinerary p for a given density class j

ne_p^j : number of containers for express shipment on itinerary p for a given density class j

ns_p^j : number of containers for standard shipment on itinerary p for a given density class j.

The cargo mix bid price model reads

$$\max \sum_{od}\sum_{j} re_{od}^j \cdot \sum_{p \in S_{od}^e} we_p^j + \sum_{od}\sum_{j} rs_{od}^j \cdot \sum_{p \in S_{od}^s} ws_p^j + \sum_{od}\sum_{j} ce_{od}^j \cdot \sum_{p \in S_{od}^e} ne_p^j + \sum_{od}\sum_{j} cs_{od}^j \cdot \sum_{p \in S_{od}^s} ns_p^j$$

$$\sum_{l \in p}\sum_{j} \left\{ we_p^j + ws_p^j + v d_j \left(ne_p^j + ns_p^j \right) \right\} \le \overline{W_l} \qquad l \in L \tag{6}$$

$$\sum_{l \in p}\sum_{j} \left\{ \left(1/d_j \right)\left(we_p^j + ws_p^j \right) + v\left(ne_p^j + ns_p^j \right) \right\} \le \overline{V_l} \qquad l \in L \tag{7}$$

$$\sum_{p \in S_{od}^e} we_p^j \le FW_{od,j}^E \qquad od \in OD, \text{class } j \tag{8}$$

$$\sum_{p \in S_{od}^s} ws_p^j \le FW_{od,j}^S \qquad od \in OD, \text{class } j \tag{9}$$

$$\sum_{p \in S_{od}^e} ne_p^j \le FC_{od,j}^E \qquad od \in OD, \text{class } j \tag{10}$$

$$\sum_{p \in S_{od}^s} ns_p^j \le FC_{od,j}^S \qquad od \in OD, \text{class } j \tag{11}$$

$$we \ge 0, ws \ge 0, ne \ge 0, ns \ge 0,$$

$$ne \text{ integer, } ns \text{ integer.}$$

The objective function maximizes the revenue obtained from express and standard cargo shipments for bulk cargo as well as containers. The capacity constraints (6) ensure that the total weight of cargo (bulk cargo and containers) for a given leg does not exceed the available capacity for that leg. Similarly, the volume constraints (7) ensure that the total volume of cargo on a given leg does not violate the available volume on the leg. Constraints (8) and (9) ensure that for any given od and density class j, the weight of cargo shipped does not exceed the forecasted weight of bulk cargo. Constraint (8) is for express shipments and constraint (9) for standard shipments. Constraints (10) and (11) impose similar restrictions on the forecasted number of containers for express and standard shipments for any given *od* and density class *j*.

While our purpose of using the presented cargo mix model is in obtaining a fleeting that captures passenger and cargo revenue, this model can be used in a bid price based cargo revenue management system. In such a setting, the relaxation of the model, where integrality requirements are dropped, is periodically solved. The dual values or bid prices from (6) and (7) are then used to set up the acceptance criteria of a shipment. Clearly the system needs to be run in parallel with the passenger revenue management system since the weight and volume capacity needs to be adjusted accordingly. Additional information on revenue management in the airline industry can be obtained in Williamson (1992), van Ryzin and Talluri (2002) and McGill and van Ryzin (1999).

3.2 Integrated Passenger-Cargo Fleeting Model

The O-D fleet assignment model, Kniker (1998), Barnhart et al. (2002b), Jacobs et al. (1999), Smith (2002), Smith (2004) captures only the passenger revenue at the O-D level. Thus the cargo flow through the network is completely ignored. The fleeting solution determines the available weight and volume on each leg and it therefore affects the cargo routing and revenue. Solving the fleeting and cargo mix models separately can thus generate suboptimal solutions. With the cargo revenue on a rise, suboptimal solutions can imply larger and larger untapped profits. To overcome this, we propose an integrated passenger-cargo fleeting model, which takes into account passenger and cargo revenue.

Example: Typical standard cargo rates per pound are less than 50¢. Assuming that a typical passenger carries two bags each one weighing approximately 20 lbs, it is clear that it is not beneficial to displace a passenger for cargo. The situation is different if expedited handling is considered. A 40 lbs shipment from Midwest to the east coast in the US can easily cost in excess of \$350 with a 2nd day delivery. In this case it might be more profitable to displace passengers.

Even in the presence of only standard shipping, an integrated passenger-cargo approach is beneficial as shown by the following instance. Consider a single flight and two equipment types given in Table 1. We assume a single fare class and a single cabin in both equipment types. For simplicity we neglect the volume constraints and we do not consider containers. The cargo rate of 50¢ per pound implies that passenger displacements are not beneficial. Table 2 shows the seat capacity, the mass payload, and the operating cost of these two fleets. Let us assume that on average each passenger and the corresponding baggage weigh 220 lb.

	Flight
Passenger demand	55
Cargo demand	8,000 lbs
Average fare	\$200
Cargo rate per lb	50¢

Table 1: *Flight data*

	Seat capacity	Payload (lb)	Operating cost
Fleet A	50	15,000	\$5,000
Fleet B	100	25,000	\$7,000

Table 2: *Fleet information*

We have 2 possible assignments, which are analyzed in Table 3.

	Passenger revenue	Cargo revenue	Operating cost	Passenger profit	Total profit
Fleet A	$10,000	$2,000	$5,000	$5,000	$7,000
Fleet B	$11,000	$4,000	$7,000	$4,000	$8,000

Table 3: *Fleet assignments*

Considering solely the passenger revenue, the assignment of Fleet A is optimal (see the next to the last column). When cargo revenue is taken into account (the last column), the assignment of Fleet B becomes the most profitable. Therefore this example shows that even though passenger displacements are not beneficial, it is still beneficial to consider both passenger and cargo revenue. This additional benefit comes from the fact that different equipment types have different payloads and therefore can transport different cargo quantities. □

The integrated fleeting model is obtained by combining leg-based FAM with the passenger and cargo mix bid price models. The linking variables are the fleeting variables x_{fl}, which link the capacities. We replace the right hand sides of (6) and (7) in the cargo mix bid price model by $\sum_f x_{fl} W_{fl}$ and $\sum_f x_{fl} V_{fl}$, respectively. W_{fl} is the payload and V_{fl} is the belly volume for equipment type f on leg l. We treat (4) in a similar fashion. The integrated passenger-cargo fleeting model reads

$$\max \sum_i \sum_k f_i^k z_i^k + \sum_{OD} \sum_j re_{od}^j \cdot \sum_{p \in S_{od}^e} we_p^j + \sum_{OD} \sum_j rs_{od}^j \cdot \sum_{p \in S_{od}^s} ws_p^j + \sum_{OD} \sum_j ce_{od}^j \cdot \sum_{p \in S_{od}^e} ne_p^j$$
$$+ \sum_{OD} \sum_j cs_{od}^j \cdot \sum_{p \in S_{od}^s} ns_p^j - \sum_f \sum_l c_{fl} x_{fl}$$

$$\sum_f x_{fl} = 1 \qquad l \in L \qquad (12)$$

$$\sum_{l \in O(v)} x_{fl} + z_{o(v)f} - \sum_{l \in I(v)} x_{fl} - z_{i(v)f} = 0 \qquad v \in V, f \in F \qquad (13)$$

$$\sum_{l \in M} x_{fl} + \sum_{g \in W} z_{gf} \le N_f \qquad f \in F \qquad (14)$$

$$\sum_{i:l \in i} \sum_{k \in F_j} u_i^k \le \sum_f Cap_{fj} x_{fl} \qquad l \in L, \text{cabin } j \qquad (15)$$

$$u_i^k \le d_i^k \qquad i \in I, \text{class } k \qquad (16)$$

$$\sum_{i:l \in i} \sum_k (wp + wb) u_i^k + \sum_{p:l \in p} \sum_j \left\{ we_p^j + ws_p^j + v\, d_j \left(ne_p^j + ns_p^j \right) \right\} \le \sum_f W_{fl} x_{fl} \qquad l \in L \qquad (17)$$

$$\sum_{i:l \in i} \sum_k vb \cdot u_i^k + \sum_{p:l \in p} \sum_j \left\{ \left(1/d_j \right) \left(we_p^j + ws_p^j \right) + v \left(ne_p^j + ns_p^j \right) \right\} \le \sum_f V_{fl} x_{fl} \qquad l \in L \qquad (18)$$

$$\sum_{p \in S_{od}^e} we_p^j \le FW_{od,j}^E \qquad od \in OD, \text{class } j \qquad (19)$$

$$\sum_{p \in S_{od}^s} ws_p^j \le FW_{od,j}^S \qquad od \in OD, \text{class } j \qquad (20)$$

$$\sum_{p \in S_{od}^{e}} ne_p^j \quad \leq \quad FC_{od,j}^{E} \qquad od \in OD, \text{class } j \quad (21)$$

$$\sum_{p \in S_{od}^{s}} ns_p^j \quad \leq \quad FC_{od,j}^{S} \qquad od \in OD, \text{class } j \quad (22)$$

$$z \geq 0, we \geq 0, ws \geq 0, ne \geq 0, ns \geq 0, u \geq 0$$

$$x \text{ binary}, ne \text{ integer}, ns \text{ integer},$$

where *wp* is the average weight per passenger, *wb* is the average weight of baggage per passenger, and *vb* is the average volume of baggage per passenger. Cap_{fj} is the seat capacity of fleet *f* in cabin *j*.

Constraints (12)-(14) are the standard leg-based fleeting constraints. (15) and (16) are the passenger mix constraints, where the former is linked with the fleeting decision variables. Constraints (17) and (18) are the modified constraints (6) and (7) of the cargo mix bid price model, where we take into account additional weight from passengers and their bags and volume from passenger bags. Constraints (19)-(22) are the demand constraints from the cargo mix bid price model. Constraints (17) and (18) link together the passenger and the cargo mix models.

The solution methodology using Benders decomposition is detailed in Section 4.

4 Solution Methodology

The proposed integrated model is too large to be solved by standard optimization software packages. We can have as many as 100,000 variables and constraints in the integrated models. However, the proposed models can be decomposed into two or three subproblems, which are relatively easy to solve. We use Benders decomposition (Benders (1962)) to solve the problems.

In Benders decomposition, in each iteration the *restricted master problem* (RMP) is solved. The initial RMP consists of a subset of constraints and variables. It includes all integer variables. Given a solution to the RMP, the subproblem is solved. The subproblem consists of the remaining constraints and variables. The dual information from the subproblem is then passed to the RMP as a new constraint called the Benders cut.

In our solution methodology the initial RMP consists of the traditional FAM model. Once the fleeting variables *x* are fixed, the remaining problem is a mixed integer linear program, which can be further decomposed into subproblems. In order for the subproblems to be linear programs (which is needed to obtain the dual values), we solve the relaxed problem, where we relax the integrality of the container variables *n*. Next we detail the approach.

4.1 Benders Reformulation for Passenger-Cargo Fleeting Model

The initial restricted master problem consists of constraints (12)-(14). Once the fleeting variables *x* are obtained, the subproblem consisting of (15)-(22) is a linear program with fixed right hand sides. This subproblem contains both passenger itineraries and cargo itineraries and is therefore a large-scale LP. We further decompose the subproblem by first solving the subproblem consisting of (15)-(16). Note that this linear program is identical to the passenger mix bid price model (4)-(5) with

$$C_{lj} = \sum_{f} Cap_{fj} x_{fl} \qquad (23)$$

for every $l \in L$ and cabin *j*. Once the passenger itinerary variables *u* are obtained, the cargo mix bid price model (6)-(11) (the corresponding constraints in the integrated model are (17)-(22)) is solved next with

$$\overline{W_l} = \sum_f W_{fl} x_{fl} - \sum_{i:l\in i} \sum_k (wp + wb) u_i^k \tag{24}$$

$$\overline{V_l} = \sum_f V_{fl} x_{fl} - \sum_{i:l\in i} \sum_k vb \cdot u_i^k \tag{25}$$

for every $l \in L$. We then add two Benders cuts; one from the passenger mix model and the other one from the cargo mix model. Note that the order of first solving the passenger mix model and then the cargo one reflects the fact that most of the revenue can be attributed to passenger operations. This decomposition of the subproblem is only an approximate methodology for solving the subproblem, which reflects the current practice.

We denote the dual values of (4) and (5) by θ and Π respectively. The duals for (6)-(11) are denoted by γ, δ, ζ, λ, μ and ν, respectively. After solving the two LPs, we use these dual values to generate two Benders cuts. η represents the passenger revenue in the RMP while σ represents the revenue obtained from cargo. We index the Benders cuts by t, where t is the iteration count. The RMP containing FAM constraints and the Benders cuts reads

$$\max \ \eta + \sigma - \sum_{fl} c_{fl} x_{fl}$$

subject to FAM constraints (12), (13) and (14) from the integrated model

$$\eta \le \sum_l \sum_f \sum_j \left(\theta_{lj}^t \ Cap_{fj}\right) x_{fl} + \sum_i \sum_k \pi_{i,k}^t \, d_i^k \qquad t \in T \tag{26}$$

$$\begin{aligned} \sigma \le & \sum_l \sum_f \left(\gamma_l^t \ \overline{W_l} + \delta_l^t \ \overline{V_l}\right) x_{fl} \\ & + \sum_{od} \sum_j \left(\tau_{od,j}^t \ FW_{od,j}^e + \lambda_{od,j}^t \ FW_{od,j}^s + \mu_{od,j}^t \ FC_{od,j}^e + \nu_{od,j}^t \ FC_{od,j}^s\right) \quad t \in T \end{aligned} \tag{27}$$

$$\eta \ge 0, \sigma \ge 0, z \ge 0$$

x binary.

Constraints (26) and (27) are the Benders cuts from the passenger and cargo models respectively that are added to the RMP.

To summarize, in each iteration, first the mixed integer program defined by the RMP is solved. Next the passenger mix bid price model is solved with the seat capacities defined by (23). By using the fleet assignment from RMP and the passenger allocations u, we solve the cargo mix bid price model with weight and volume capacities defined by (24) and (25). Based on the obtained dual prices from the two bid price models, we add two Benders cuts to the RMP, which are given by (26) and (27).

Due to the approximation in subproblem solving, the added Benders cuts overestimate the optimal revenue, i.e. the value of the RMP is a lower bound on the optimal profit. In order to obtain the optimal profit, the resulting fleeting must be plugged into the subproblem consisting of (15)-(22).

5 Computational Experiments

We tested the integrated model on one large data set consisting of five fleets and 1493 flights. The problem that we solve is a daily problem. We have thousands of passenger and cargo itineraries. Real world data from a major US carrier were used. The carrier has a heavy hub-and-spoke network structure with 5 major hubs. The computing environment consists of a PC with a 27 dual 900 Mhz Itanium 2 processor running Red Hat 7.3 operating system and the gcc-3.2 development environment. Due to lack of data for containers, the presented computational results do not take into account containers. For discretionary purposes, the real profit numbers are fudged but the presented numbers show correct proportions and magnitudes. After obtaining the final fleeting by the integrated approach, we do not compute the optimal profit (see the discussion at the end of Section 4.1). Instead the profit is computed based on the decomposition principle presented in Section 4.1, i.e. first the passenger allocation is computed and then the remaining capacity is assigned to cargo. Note that this profit can clearly be materialized.

Since we have used only one data set, we modified cargo rates and made additional test cases to capture the effect of cargo rates on revenue. In what follows, CR denotes the actual cargo rates that we have obtained from the airline. Unless otherwise stated, any increase or decrease is shown with respect to CR and the actual values are not quoted.

The cargo itineraries are generated using an implementation of the k^{th}*-shortest path algorithm*. We generate a pre-defined number of cargo routes for each given *od* pair. The network used is the one described in Section 3.1.

We first report the increase in savings by using our integrated model as opposed to the sequential approach. The traditional sequential approach first solves the O-D based fleeting model with respect solely to the passenger revenue. After this step, the remaining volume and weight are allocated optimality to cargo. The increase in profit is shown in Table 4. We calculate profit by subtracting the operating cost from the revenue obtained from both cargo and passenger contributions. In the last column we show the range (for confidentiality reasons) of the increase in profit obtained by solving the integrated model vs. the traditional sequential approach. For example, in Case 2 the actual additional profit is between \$6,000 and \$7,000. When these numbers are multiplied by the number of days in a year, we obtain yearly profits reaching several million dollars.

	Cargo rate	**Increase in profit (\$)**
Case 1	CR	13,000-14,000
Case 2	CR/3	6,000-7,000
Case 3	2·CR	37,000-39,000

Table 4: *Profits*

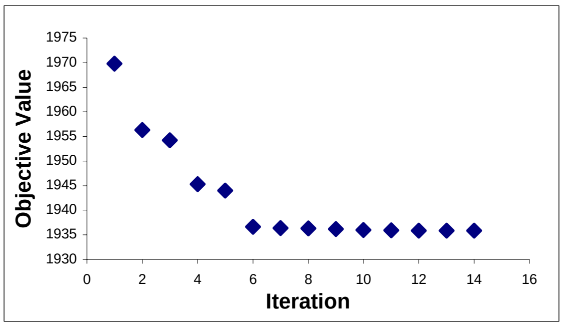
Next in Table 5 we break up the obtained profit into cargo revenue, passenger revenue, and operating cost. The values are reported as percentage increase or decrease values with respect to those obtained by using the sequential model. To ensure confidentiality, we present the range of percentages instead of the actual percentage increase or decrease. For example, the cargo revenue value shown for Case 1 is between 0.9% and 1.0%. This implies that the cargo revenue obtained from the integrated approach is approximately 0.95% higher than the cargo revenue obtained in the traditional case. As expected, the passenger revenue decreases while the cargo revenue increases. The operating cost does not change substantially.

	Cargo revenue (%)	**Passenger revenue (%)**	**Operating cost (%)**	**Savings (%)**
Case 1	[0.9, 1.0]	[-0.15, -0.1]	[0.02,0.03]	[0.07,0.08]
Case 2	[0.9, 1.0]	[-0.02, -0.01]	[0.03,0.05]	[0.04,0.05]
Case 3	[0.6,0.8]	[-0.02, -0.01]	[0.1,0.2]	[0.1,0.2]

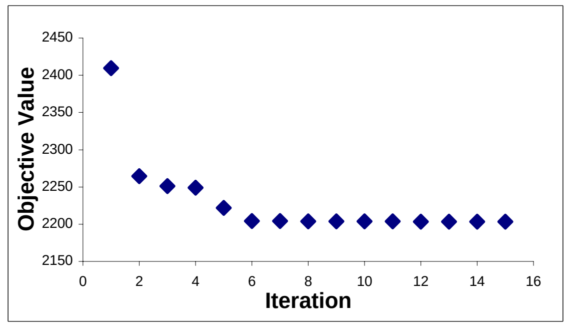
Table 5: *Breakup of profit*

In Case 1 the integrated solution has approximately 97% of tight weight capacity constraints (17). A constraint at equality in the underlying solution is tight. The sequential model gives the same order of tight weight capacity constraints. These numbers are surprising for volume. Approximately 80% of the volume capacity constraints (18) are tight in the integrated solution while this number increases to 87% for the sequential solution. Further analysis of the integrated solution shows that this solution puts aircraft with larger payloads than those of the sequential solution on some of the flights. As a result more cargo demand can be carried on these flights but the new fleet assignment leaves some unused volume capacity.

The objective value of the RMP tends to decrease across iterations. The trends for Case 1 and Case 2 are shown in Figure 2. These plots exhibit the typical tailing effect known for many iterative optimization algorithms. In Benders decomposition it results from degeneracy of the cargo and passenger mix models.



Case 1



Case 2

Figure 2: *Objective value improvements*

The computation times are shown in Table 6. The "average" column for the RMP and LP steps shows the absolute value of the average over all iterations. For these two steps, the "%" column shows the percentage of the total time that is spent in solving this step over all iterations. The last column gives the total time taken to solve the integrated model using Benders decomposition. On an average it takes approximately 6-10 hours to solve the integrated model.

	Time per iteration (min)	RMP		LP subproblems		Total (hrs)
		Average (min)	%	Average (min)	%	
Case 1	25	15	60	10	40	6
Case 2	24	14	58	10	42	6
Case 3	40	30	75	11	25	10

Table 6: *Computation times*

6 Future Directions

The observed convergence rate using Benders decomposition is rather slow for most practical problems. If the primal subproblem is degenerate (either the passenger or the cargo mix bid price model), we may obtain multiple optimal dual solutions, which yield more than one Benders cut. The approach by Magnanti and Wong (1981) produces pareto-optimal cuts, i.e. nondominated cuts. Their procedure requires generating a core point, which is a nontrivial task in our case. Nevertheless, pareto-optimal cuts might produce better solutions. A slightly different methodology for solving the model (12)-(22) is by branch-and-bound, where at each node of the branch-and-bound tree the LP relaxation is solved by Benders decomposition. In this algorithm the RMP is solved as an LP and identical cuts to (26) and (27) are added. After an LP stopping criterion applies, we branch by fixing an assignment variable.

In our current implementation for each O-D pair we generate a predefined number of cargo routes offline and input these as cargo itineraries. Another approach is to use column generation (see e.g. Barnhart et al. (1998)) when solving the cargo mix bid price model. In this approach cargo itineraries are generated dynamically as needed. A constrained shortest path algorithm is required for pricing. This approach could potentially yield more profitable solutions.

We believe the most significant improvement can be obtained by using stochastic models. The passenger mix model used in our integrated model is a deterministic linear program. For each itinerary i and for each fare class k we use the deterministic average demand, which is forecasted in advance. We ignore the stochastic nature of demand. A better approach would be to take into account the complete probability distribution of demand. Luckily the presented model is extendible to this more general case. Next we elaborate more on this extension.

Consider the stochastic passenger mix bid price model, see e.g. van Ryzin and Talluri (2002), defined by

$$\max \sum_i \sum_k f_i^k \, E(\min(u_i^k, D_i^k))$$

$$\sum_{i:l\in i} \sum_{k\in F_j} u_i^k \le C_{lj} \qquad l \in L, \text{cabin } j \tag{28}$$

$$u \ge 0.$$

Here D_i^k is the random variable corresponding to the demand for itinerary i and fare class k. The expression $\min(u_i^k, D_i^k)$ captures the fact that we do not allocate more than the demand. Note that $d_i^k = E(D_i^k)$. As a function of capacities C this is a concave function and therefore Benders decomposition is applicable. In addition, the objective function is separable and concave in u. Given fixed capacities implied by the solution to the RMP, we need to solve this nonlinear program, e.g. by subgradient optimization or randomized linear programming. In the end we obtain bid prices θ_{lj}^t corresponding to (28), where t is the iteration index. Similarly to (25) the Benders cut $\eta \le \sum_l \sum_f \sum_j (\theta_{lj}^t Cap_{fj}) x_{fl}$ is added to the RMP.

Similar treatment holds for the cargo mix bid price model. The stochastic linear program in this case reads

$$\max \sum_{od} \sum_j re_{od}^j \cdot E(\min(\sum_{p\in S_{od}^e} we_p^j, DW_{od,j}^E)) + \sum_{od} \sum_j rs_{od}^j \cdot E(\min(\sum_{p\in S_{od}^s} ws_p^j, DW_{od,j}^S))$$

$$+ \sum_{od} \sum_j ce_{od}^j \cdot E(\min(\sum_{p\in S_{od}^s} ne_p^j, DC_{od,j}^E)) + \sum_{od} \sum_j cs_{od}^j \cdot E(\min(\sum_{p\in S_{od}^s} ns_p^j, DC_{od,j}^S))$$

$$\sum_{l\in p} \sum_j \left\{ we_p^j + ws_p^j + v\, d_j \left(ne_p^j + ns_p^j \right) \right\} \le \overline{W}_l \qquad l \in L \tag{29}$$

$$\sum_{l\in p} \sum_j \left\{ \left(1/d_j \right) \left(we_p^j + ws_p^j \right) + v \left(ne_p^j + ns_p^j \right) \right\} \le \overline{V}_l \qquad l \in L \tag{30}$$

$$we \ge 0, ws \ge 0, ne \ge 0, ns \ge 0,$$

where $DW_{od,j}^E, DW_{od,j}^S, DC_{od,j}^E, DC_{od,j}^S$ are the random variables corresponding to the respective demands. This is again a concave function of weight and volume capacities and therefore bid prices γ, δ for (29) and (30) respectively can be computed by any algorithm for convex optimization. The Benders cut $\sigma \leq \sum_l \sum_f \left(\gamma_l^t \, \overline{W}_l + \delta_l^t \, \overline{V}_l \right) x_{f\,l}$ is then added to the RMP.

It is also possible to model these stochastic linear programs as deterministic ones, Williamson (1992). We show this on the passenger mix model. Let M be the maximum available seat capacity among all the flights. For m = 0,1,…,M let $P_{i,k}$ *(m)* be the probability of selling the m^{th} seat on itinerary i and fare class k. Then the stochastic passenger mix model reads

$$
\begin{aligned}
\max \sum_i \sum_k \sum_{m=1}^{M} f_i^k P_{ik}(m) \, . \, z_{mi}^k & \\
\sum_i \sum_{k \in F_j} \sum_{m=1}^{M_l} z_{mi}^k \ \leq \ C_{lj} & \qquad l \in L, \text{cabin } j \\
0 \leq z_{mi}^k \leq 1, &
\end{aligned}
\tag{31}
$$

where z_{mi}^k is 1 if the m^{th} seat on itinerary i and fare class k is sold, and 0 otherwise. The drawback of this model is a large number of variables. It is straightforward now to change constraints (15)-(18) in our integrated model to use this passenger mix model. The cargo mix model can be changed in the similar fashion.

We did not experiment with these stochastic models. We believe it is a very important line of research since it addresses the concern of a deterministic model in a stochastic environment. Solving the stochastic problem is most likely very challenging due to the poor convergence of the Benders algorithm for such problems, Smith (2005). On the positive note, Smith (2005) presents several techniques for speeding up the algorithm when only the passenger revenue is considered.

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