1 Definition of the lifting sets used in Theorem 5

In describing the lifting coefficients we need the following definitions.

Definition 1.

- For $i \in N_1^+ \cap N_2^+$ and $b_i < d$ let k_i^1 be the index with the property $\mu_{k_i^1} \le b_i < \mu_{k_i^1+1}$.
- For $i \in N_1^+ \cap N_2^+$ we define k_i^2 as the index with the property $\mu_{k_i^2} \le b_i + v_i < \mu_{k_i^2+1}$ if $b_i + v_i < d$ and $k_i^2 = r$ if $b_i + v_i \ge d$.
- For $i \in N_1^- \cap N_2^+$ let k_i^2 be the index with the property $\mu_{k_i^2} \le b_i < \mu_{k_i^2+1}$ if $b_i < d$ and $k_i^2 = r$ if $b_i > d$.
- For $i \in N_1^- \cap N_2^+$ and $d > b_i - v_i \ge 0$ we define k_i^1 as the index with the property $\mu_{k_i^1} \le b_i - v_i < \mu_{k_i^1+1}$.
- For $i \in N_1^- \cap N_2^+$ and $b_i - v_i < 0$ we define k_i^1 as the index with the property $\mu_{k_i^1} - t_i d \le b_i - v_i < \mu_{k_i^1+1} - t_i d$ for a uniquely defined integer $t_i \ge 1$.

1. $i \in N_1^+ \cap N_2^+$:

$$J_i = \left\{ \left(\frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}, g(\mu_{j+1}) - \frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}(\mu_{j+1} - b_i) \right) : k_i^1 < j < k_i^2 \right\}$$
$$\bigcup \left\{ \left(\frac{g(\mu_{k_i^1+1}) - g(b_i)}{\mu_{k_i^1+1} - b_i}, g(b_i) \right) \right\}$$
$$\bigcup \left\{ \left(\frac{g(b_i + v_i) - g(\mu_{k_i^2})}{b_i + v_i - \mu_{k_i^2}}, g(b_i + v_i) - \frac{g(b_i + v_i) - g(\mu_{k_i^2})}{b_i + v_i - \mu_{k_i^2}} v_i \right) \right\}$$

   if $b_i < d$ and $J_i = \{(0,0)\}$ if $b_i \ge d$.

2. $i \in N_1^- \cap N_2^+$ and $b_i - v_i \ge 0$:

$$J_i = \left\{ \left(-\frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}, g(\mu_{j+1}) - \frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}(\mu_{j+1} + b_i) \right) : k_i^1 < j < k_i^2 \right\}$$
$$\bigcup \left\{ \left(-\frac{g(\mu_{k_i^1+1}) - g(b_i - v_i)}{\mu_{k_i^1+1} - b_i + v_i}, g(b_i - v_i) + \frac{g(\mu_{k_i^1+1}) - g(b_i - v_i)}{\mu_{k_i^1+1} - b_i + v_i}(v_i - 2b_i) \right) \right\}$$
$$\bigcup \left\{ \left(-\frac{g(b_i) - g(\mu_{k_i^2})}{b_i - \mu_{k_i^2}}, g(b_i) - 2\frac{g(b_i) - g(\mu_{k_i^2})}{b_i - \mu_{k_i^2}} b_i \right) \right\}$$

   if $b_i - v_i < d$ and $J_i = \{(0,0)\}$ if $b_i - v_i \ge d$.

3. $i \in N_1^- \cap N_2^+$ and $b_i - v_i < 0$: Let s be such that

$$\frac{g(\mu_s) - g(\mu_{s-1})}{\mu_s - \mu_{s-1}} \le \frac{r}{d} < \frac{g(\mu_{s+1}) - g(\mu_s)}{\mu_{s+1} - \mu_s} .$$

   We have 4 subcases.

   (a) Let first $b_i - v_i \le \mu_s - d$ and $b_i \ge \mu_s$.

$$
\begin{aligned}
J_i = &\left\{\left(-\frac{g(\mu_{k_i^1+1}-t_id)-g(b_i-v_i)}{\mu_{k_i^1+1}-t_id-b_i+v_i}, g(b_i-v_i)+\frac{g(\mu_{k_i^1+1}-t_id)-g(b_i-v_i)}{\mu_{k_i^1+1}-t_id-b_i+v_i}v_i\right)\right\} \\
&\bigcup\left\{\left(-\frac{g(\mu_{j+1}-t_id)-g(\mu_j-t_id)}{\mu_{j+1}-\mu_j}, g(\mu_j-t_id)\right.\right. \\
&\qquad\left.\left.+\frac{g(\mu_{j+1}-t_id)-g(\mu_j-t_id)}{\mu_{j+1}-\mu_j}(b_i-\mu_j+t_id)\right) : k_i^1+1\le j\le s\right\} \\
&\bigcup\left\{\left(-\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}, g(\mu_j)+\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}(b_i-\mu_j)\right) : s<j\le k_i^2\right\} \\
&\bigcup\left\{\left(-\frac{g(b_i)-g(\mu_{k_i^2})}{b_i-\mu_{k_i^2}}, g(b_i)\right)\right\}\bigcup\left\{\left(\frac{r}{d}, g(\mu_s)-\frac{r}{d}\mu_s\right)\right\}
\end{aligned}
$$

(b) Let now $b_i - v_i > \mu_s - d$ and $b_i > \mu_s$. Let also q be such that

$$
\frac{g(\nu_q)-g(b_i-v_i)}{\nu_q-b_i+v_i} \le \frac{g(\nu_j)-g(b_i-v_i)}{\nu_j-b_i+v_i},
$$

where $\nu_j \in \{\mu_{k_i^1+1}-d,\dots,\mu_{r-1}-d\}\cup\{\mu_1,\dots,\mu_s\}$.

$$
\begin{aligned}
J_i = &\left\{\left(-\frac{g(\nu_q)-g(b_i-v_i)}{\nu_q-b_i+v_i}, g(b_i-v_i)+\frac{g(\nu_q)-g(b_i-v_i)}{\nu_q-b_i+v_i}v_i\right)\right\} \\
&\bigcup\left\{\left(-\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}, g(\mu_j)+\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}(b_i-\mu_j)\right) : s<j\le k_i^2\right\} \\
&\bigcup\left\{\left(-\frac{g(b_i)-g(\mu_{k_i^2})}{b_i-\mu_{k_i^2}}, g(b_i)\right)\right\} \\
&\bigcup\left\{\left(-\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}, g(\mu_j)+\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}(b_i-\mu_j) : 0<j<s,\right.\right. \\
&\qquad g(\mu_{k_i^1+1}-d)\ge g(\mu_j)+\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}(\mu_{k_i^1+1}-d-\mu_j), \\
&\qquad\left.\left. g(b_i-v_i)\ge g(\mu_j)+\frac{g(\mu_{j+1})-g(\mu_j)}{\mu_{j+1}-\mu_j}(b_i-v_i-\mu_j)\right)\right\}
\end{aligned}
$$

(c) Let $b_i - v_i \le \mu_s - d$ and $b_i \le \mu_s$. Let also q be such that

$$
\frac{g(b_i)-g(\nu_q)}{b_i-\nu_q} \ge \frac{g(b_i)-g(\nu_j)}{b_i-\nu_j},
$$

where $\nu_j \in \{\mu_s-d,\dots,\mu_{r-1}-d\}\cup\{\mu_1,\dots,\mu_{k_i^2}\}$.

$$
\begin{aligned}
J_i = &\left\{ \left(-\frac{g(\mu_{k_i^1+1} - t_i d) - g(b_i - v_i)}{\mu_{k_i^1+1} - t_i d - b_i + v_i}, g(b_i - v_i) + \frac{g(\mu_{k_i^1+1} - t_i d) - g(b_i - v_i)}{\mu_{k_i^1+1} - t_i d - b_i + v_i} v_i \right) \right\} \\
&\bigcup \left\{ \left(-\frac{g(\mu_{j+1} - t_i d) - g(\mu_j - t_i d)}{\mu_{j+1} - \mu_j}, g(\mu_j - t_i d) \right.\right. \\
&\qquad \left.\left. + \frac{g(\mu_{j+1} - t_i d) - g(\mu_j - t_i d)}{\mu_{j+1} - \mu_j}(b_i - \mu_j + t_i d) \right) : k_i^1 + 1 \leq j \leq s \right\} \\
&\bigcup \left\{ \left(-\frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}, g(\mu_j - d) \right.\right. \\
&\qquad \left. + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(b_i - \mu_j + d) \right) : s < j < r - 1, \\
&\qquad g(\mu_{k_i^2}) \geq g(\mu_j - d) + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(\mu_{k_i^2} - \mu_j + d), \\
&\qquad \left. g(b_i) \geq g(\mu_j - d) + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(b_i - \mu_j + d) \right\} \\
&\bigcup \left\{ \left(-\frac{g(b_i) - g(\nu_q)}{b_i - \nu_q}, g(b_i) \right) \right\}
\end{aligned}
$$

(d) Let $b_i - v_i > \mu_s - d$ and $b_i < \mu_s$. Let also q be such that

$$
\frac{g(\nu_q) - g(b_i - v_i)}{\nu_q - b_i + v_i} \leq \frac{g(\nu_j) - g(b_i - v_i)}{\nu_j - b_i + v_i},
$$

where $\nu_j \in \{\mu_{k_i^1+1} - d, \ldots, \mu_{r-1} - d\} \cup \{\mu_1, \ldots, \mu_{k_i^2}\} \cup \{b_i\}$ and let p be such that

$$
\frac{g(b_i) - g(\nu_p)}{b_i - \nu_p} \geq \frac{g(b_i) - g(\nu_j)}{b_i - \nu_j},
$$

where $\nu_j \in \{b_i - v_i\} \cup \{\mu_{k_i^1+1} - d, \ldots, \mu_{r-1} - d\} \cup \{\mu_1, \ldots, \mu_{k_i^2}\}$.

$$
\begin{aligned}
J_i = &\left\{ \left(-\frac{g(\nu_q) - g(b_i - v_i)}{\nu_q - b_i + v_i}, g(b_i - v_i) + \frac{g(\nu_q) - g(b_i - v_i)}{\nu_q - b_i + v_i} v_i \right) \right\} \\
&\bigcup \left\{ \left(-\frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}, g(\mu_j - d) \right.\right. \\
&\qquad \left. + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(b_i - \mu_j + d) \right) : k_i^1 + 1 < j < r - 1, \\
&\qquad g(\mu_{k_i^2}) > g(\mu_j - d) + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(\mu_{k_i^2} - \mu_j + d), \\
&\qquad \left. g(b_i) > g(\mu_j - d) + \frac{g(\mu_{j+1} - d) - g(\mu_j - d)}{\mu_{j+1} - \mu_j}(b_i - \mu_j + d) \right\} \\
&\bigcup \left\{ \left(-\frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}, g(\mu_j) + \frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}(b_i - \mu_j) : 0 < j < k_i^2, \right.\right. \\
&\qquad g(\mu_{k_i^1+1} - d) \geq g(\mu_j) + \frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}(\mu_{k_i^1+1} - d - \mu_j), \\
&\qquad \left.\left. g(b_i - v_i) \geq g(\mu_j) + \frac{g(\mu_{j+1}) - g(\mu_j)}{\mu_{j+1} - \mu_j}(b_i - v_i - \mu_j) \right) \right\} \\
&\bigcup \left\{ \left(-\frac{g(b_i) - g(\nu_p)}{b_i - \nu_p}, g(b_i) \right) \right\}
\end{aligned}
$$