Modeling Interplanetary Logistics: Evaluation of Fueling Strategies for Space Exploration

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The objective of this paper is to demonstrate a methodology for designing and evaluating the operational planning for interplanetary exploration missions. A primary question for space exploration mission design is how to best design the logistics required to sustain the exploration initiative. Using terrestrial logistics modeling tools that have been extended to encompass the dynamics and requirements of space transportation, an architectural decision method has been created. The model presented in this paper is capable of analyzing a variety of mission scenarios over an extended period of time with the goal of defining interesting architectural scenarios for space logistics. This model can be utilized to evaluate different logistics trades, such as where the push-pull boundary for commodities exists, which can aid in the decision of where to pre-position commodities for later use. In the final paper, the results of this implementation will be presented for a lunar campaign using estimated surface demands for exploration. In addition, the final paper will explore the use of pre-positioning of propellant for later use and incorporate the capability for refueling during the mission to gain a greater understanding of the benefits to exploration of this technology development.

I. Introduction

On January 14th, 2004, President Bush set forth a new exploration initiative to achieve a sustained human presence in space. Included in this directive is the return of humans to the Moon by 2020 and the human exploration of Mars thereafter.[1] The President has tasked NASA with the development of a sustainable space transportation system that will enable continual exploration of the Moon, Mars and 'beyond'.

Inherent to the problem of transporting people to the Moon, Mars, and 'beyond' is sustaining the people and the operations while in transit and at the respective destinations. Especially for long-term missions, the amount of consumables required becomes a significant issue in terms of mass in LEO which translates to mission cost. In order to develop a sustainable space transportation architecture it is critical that interplanetary supply chain logistics be considered. The goal of the supply chain logistics problem is to adequately account for and optimize the transfer of supplies from Earth to locations in space. Although the commodities themselves may be of low value on Earth the consideration of these commodities is of high importance and can directly impact the mission success. As such, it is desirable to find low cost yet reliable methods of transporting these supplies to the destinations.

The space exploration missions will evolve over time which will generate an increased demand at in-space locations. In order to develop a sustainable architecture it is necessary to recognize the interdependencies between missions and how this coupling could effect the logistics planning. By viewing the set of missions together, as a space network, and optimizating the operations of the transportation system that provides the

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logistics for the exploration missions, a reduction in cost can be achieved which promotes a more sustainable system architecture.

The integration of the multiple missions creates an opportunity to analyze the effect of pre-positioning and combining missions. Specifically, the effect of pre-positioning fuel from one mission to the next can effect both the safety and the cost of the missions. The idea of pre-positioning fuel can take on two forms. In one instance, fully fueled propulsion elements can be pre-positioned at a fuel depot or at strategic locations for future use. However, what may prove to be more effective in the future is to establish inteplanetary fueling stations where propulsive elements can be re-fueled for continued use. Therefore, it is necessary to develop a model for space logistics that can evaluate these different scenarios.

There exists a great deal of literature on the design of transportation networks on Earth. For example in Reference 2 the design of the school bus routing problem is presented and solved. In this problem there exists a number of restrictions on feasible solutions, including time window constraints on pick-up and delivery, which add to the complexity of a large-scale problem. In Reference 3 a smaller aircraft network design problem is considered to understand the effects of the network design and vehicle selection on the system cost. By defining three different classes of aircraft, small, medium, and large, the optimal allocation of vehicles to routes can be defined to meet the given demand.

Many of the tools and methods of terrestrial logistics can be extended to space networks. Specifically, time expanded networks represent a method for modeling transportation systems that are operated over time.[4] Using this modeling technique the static network is expanded and time is incorporated directly into the network definition. As shown in Reference 5 time expanded networks were used to plan the routing of trucks for companies that rely on less than truck load carriers for shipping products to customers.

In Section two the general problem is defined. Included in this section is the definition of the commodities or supplies and the elements or physical containment and propulsion units used to transport the commodities. The definition provided is extensive to clarify terminology developed to integrate the aerospace community with the terrestrial logistics community. Furthermore, the network definition is presented as well as the definition and description of the time expanded network which is the terrestrial modeling technique employed for the space logistics model. Section three presents the problem formulation and constraints. Section four presents an overview of the solution methodolgy and Section five provides a quick example, using Apollo to clarify this complex formulation. Section six details the future work that will be conducted for the final paper.

II. Problem Definition

A. Commodities

The goal of the space logistics project is to determine how to meet the demand for the exploration missions. As such, we are investigating how to optimally ship multiple types of commodities. For the purpose of the logistics problem, a commodity will be defined as a high-level aggregate of a type of supply. Thus, we will define a set of $k = 1, \ldots, K$ commodities, each with the following parameters.

- Denote the demand of each commodity as d^k.
- Denote the origin of each commodity as so^k.
- Define the destination of each commodity as sd^k.
- Define the availability interval of each commodity as $to^k = \left[sto^k, eto^k\right]$, where sto^k is the starting time of the interval and eto^k is the ending time of the interval.
- Define the delivery interval of each commodity as $td^k = \left[std^k, etd^k\right]$, where std^k is the starting time of the interval and etd^k is the ending time of the interval.
- Define the maximum time that a commodity can be in transit as t^k_{max}.
- Define the unit mass of each commodity as m^k when it arrives at the destination.
- Define the unit volume of each commodity as v^k when it arrives at the destination.

- Define an absolute mass gain/loss factor for each commodity after being available at so^k for τ periods as fm_τ^k where $fm_\tau^k < 0$ if the commodity gains mass over time, $fm_\tau^k > 0$ if the commodity loses mass over time and $fm_\tau^k = 0$ if the commodity mass remains constant over time. [a]
- Define an absolute volume gain/loss factor for each commodity after being available at so^k for τ periods as fv_τ^k where $fv_\tau^k < 0$ if the commodity gains volume over time, $fv_\tau^k > 0$ if the commodity loses volume over time and $fv_\tau^k = 0$ if the commodity volume remains constant over time.

B. Elements

Elements are physical, indivisible functional units that transport the commodities from origin to destination. An element is classified by the amount of commodity capacity and propulsive capability it possesses. Elements can be divided into two classes: non-propulsive elements $\mathcal{M}_N$ and propulsive elements $\mathcal{M}_P$. The element parameters are (cf. Figure 1):

- The maximum fuel mass of a propulsive element m, $m \in \mathcal{M}_P$ is defined as mf^m.
- The fuel volume of a propulsive element m is defined as vf^m.
- The structural mass of element m is defined as ms^m.
- The mass capacity of element m is defined as CM^m.
- The volume capacity of element m is defined as CV^m.

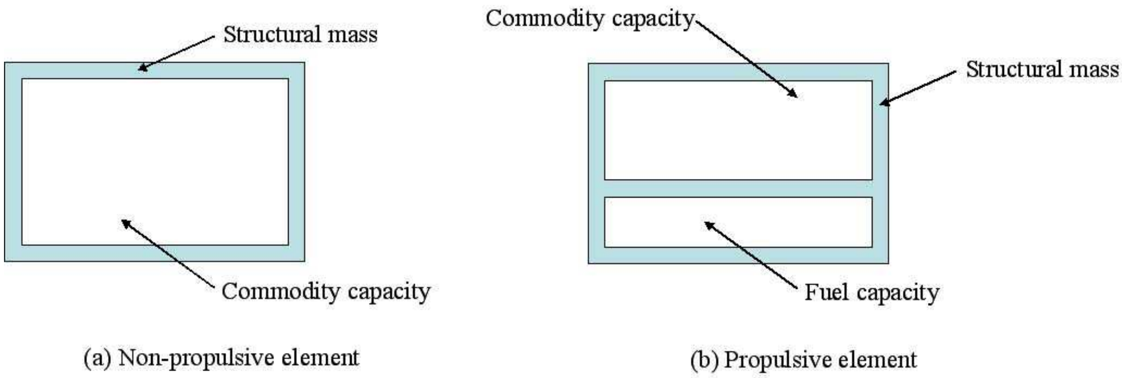

Figure 1. Element Representation

1. *Assumption*

We make the following assumptions regarding elements and stacks.

Consecutive Burns An active element burns only on consecutive burns. Once an element becomes active, it stays active for a certain number of burns. As soon as it becomes passive, it can no longer be active again unless it is refueled. During two consecutive burns, an active element can be idle for an arbitrary length of time. The number of consecutive burns is not constrained.

Fuel Consumption We assume that before every initial burn, the active element is filled to capacity with fuel and after the burns are completed, the remaining fuel is expelled. If an element is later refueled, it is filled to maximum capacity.

For example, consider an element that starts burning. Just before this first burn the element was filled to capacity with fuel. The element then executes four consecutive burns and after the fourth burn it expells any remaining fuel. Then it travels as a passive element for a period of time. If at some point

[a] For example, let commodity k become available at its origin so^k at time $t_o \in to^k$ and arrive at the destination sd^k at time $t_d \in td^k$. For any time $t_c \in [t_o, t_d]$, the unit mass of commodity k at time t_c is $m^k + \sum_{t=t_c}^{t_d} fm_{t-t_o}^k$.

it is refueled, it can remain passive for another period of time before it executes another sequence of burns.

Docking/Undocking We assume that any two elements can be docked and undocked. In addition, if any cost is associated with these operations, it is not explicitly captured. If some elements cannot be docked together, then this must either be captured in stack formation or in a post optimization analysis.

C. Networks

1. Static Network

The physical network, or static network, represents the set of physical locations, or nodes, and the connections, or arcs, between them. The physical nodes, or static nodes, represent the different physical destinations in space, including the origin and destination of all the commodities, as well as the possible locations for transshipment. Three types of nodes have been identified: Body nodes, Orbit nodes, and Lagrange point nodes. The physical arcs, or static arcs, represent the physical connections between two nodes, that is, an element can physically traverse between these two nodes. We define an arc (si, sj) to be a static arc that represents a feasible transfer from static node si to static node sj.

The mathematical description of the static network is given below.

- Define the static network as a graph GS, where $GS = (NS, AS)$.
- Define the set of nodes, $NS = \{s1, \ldots, sn\}$, in the static network.
- Define the set of arcs, $AS \subseteq NS \times NS$ in the static network.

Example 1 Consider 2 commodities shown in the following table.

Commodity	Origin	Destination	Availability Interval	Delivery Interval
Commodity 1	1	3	$[1, 2]$	$[5, 7]$
Commodity 2	2	3	$[1, 3]$	$[5, 6]$

The corresponding static network is shown in Figure 2. As indicated by the arcs in the network, there exists a transfer arc from node 1 to node 3, node 2 to node 3, node 1 to node 4, node 2 to node 4, and node 4 to node 3. Note that node 4 here is neither an origin nor destination of any commodity. It is included in the static network since both commodities can be transhipped at node 4, which may result in a lower cost.

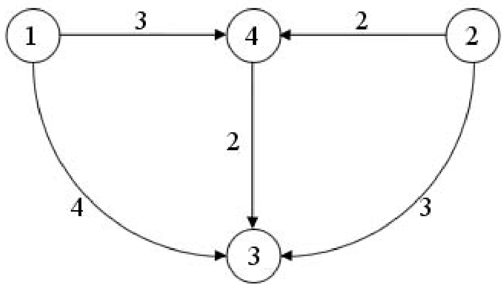

Figure 2. Example of a Static Network

2. Time Expanded Network

The space logistics project is investigating the design of a sequence of missions that evolve over an extended period of time. In addition, certain properties of the space network are time-varying. For these reasons we have chosen to introduce time expanded networks as a modeling tool. In the time expanded network, the absolute time interval under consideration, is discretized into T time periods of length Δt. A copy of each static node is made for each of the time points and the nodes are connected according to the following rules.

- The arc must exist in the static network.

- The arc must create a connection that moves forward in time.
- The arc must represent a feasible transfer, in terms of orbital dynamics.

The mathematical description of the time expanded network is given below.

- Define the time expanded network as a graph $\mathcal{G}$, where $\mathcal{G} = (\mathcal{N}, \mathcal{A})$.
- Define the set of nodes in the time expanded network as $\mathcal{N} = \{i = (si, t) \mid si \in NS,\ t = 1, \ldots, T\}$. To simplify the notation, for a given node $i \in \mathcal{N}$, let $s(i)$ and $t(i)$ denote the physical node and the time period corresponding to node i, i.e., if $i = (si, t)$ then $s(i) = si$ and $t(i) = t$.
- Define node s as the general source that generates the supply of elements. This node is connected to every node in the network where an element can originate (e.g. in the current setting s is connected to every node i with $s(i)$ corresponding to LEO).
- Define the set of arcs in the time expanded network as $\mathcal{A} \subseteq \mathcal{N} \times \mathcal{N}$. An arc $a = (i, j) = ((si, t), (sj, t + T^t_{si,sj}))$ exists if and only if there exists an arc (si, sj) in the static network, and the transit time from static node si to static node sj starting at time t is $T^t_{si,sj}$. Note that if $si = sj$, then $T^t_{si,sj} = 1$ for all t.
- Define path p as a sequence of nodes. In particular, let $f(p)$ and $l(p)$ denote the first node and the last node of path p. If path p originates at node s, $f(p) = s$ for all such p.

Example 2 Consider the static network shown in Example 1. Let the numbers next to each arc (si, sj) indicate the transit time from node si to node sj. The corresponding time expanding network is shown in Figure 3. Note that $T = \max_k etd^k = 7$.

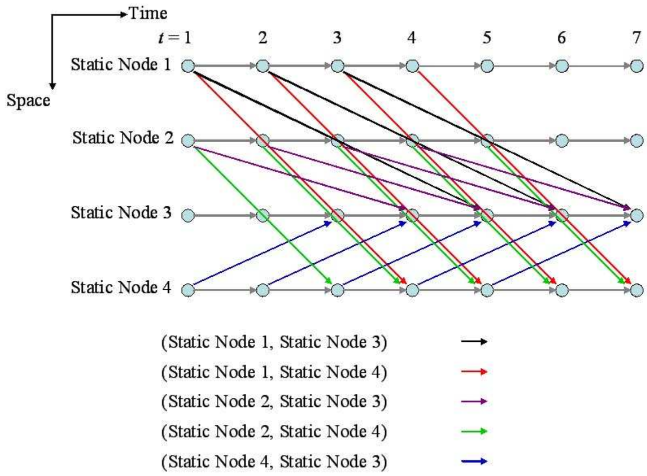

Figure 3. Example of a time expanded network

To account for the fact that on certain transfer arcs two burns occur, we slightly modify the time expanded network. We first introduce a new fictitious static node labeled fic. Note that this node is not related to the static network. On every transfer arc $(i, j), s(i) \neq s(j)$ requiring two burns we add a new auxiliary node $k = (fic, t)$ with two arcs; one connects i to k and the other one k to j. The value of t is irrelevant. In this new network, each arc (i, j) with $s(i) \neq s(j)$ corresponds to a single burn. All such arcs are called *burn arcs* and we denote them by $\mathcal{A}_B$.

The mass fraction for element m to execute the burn corresponding to arc $a \in \mathcal{A}_B$ is defined as

$$\phi^m_a = 1 - \exp\left(\frac{-\Delta V_a}{I^m g_0}\right).$$

III. Formulation

A. Commodity Flows

1. *Commodity Path Feasibility*

In order to understand how each commodity should move through the network it is not sufficient to know which arcs are traversed. Instead, it is necessary to determine the path followed from the origin node to the destination node where the commodity fulfills the specified demand. If we define a path variable p, then for each commodity k it is possible to determine a set of feasible paths $\mathcal{P}^k$.

For a given commodity k, the path p is feasible only if it originates at node $i = (so^k, t)$ with $t \in to^k$ and terminates at node $j = (sd^k, t')$ with $t' \in td^k$. Moreover, we require that the transit time along the path p is no greater than the maximum travel time for commodity k, i.e.,

$$t(l(p)) - t(f(p)) = \sum_{(i,j) \in p} (t(j) - t(i)) \leq t^k_{max} \qquad p \in \mathcal{P}^k.$$

Example 3 Consider commodity 1 in Example 2. If $t^1_{max} = \infty$, all the feasible paths for commodity 1 are shown in Figure 4, and each color corresponds to a different path. However, if we let $t^1_{max} = 5$, the green path is infeasible since the total traveling time along this path is 6.

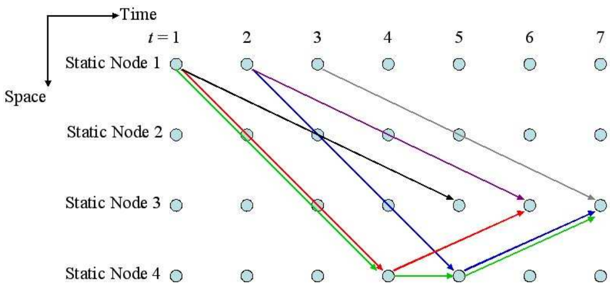

Figure 4. Example of a Feasible Path

2. *Commodity Flow Variables and Constraints*

We need to determine how many units of commodity k are transported on path p, for any k and $p \in \mathcal{P}^k$. Therefore, for every k and $p \in \mathcal{P}^k$ we have a decision variable $x^k_p \geq 0$ such that

$$x^k_p = \text{number of units of commodity } k \text{ traveling on path } p.$$

In order to satisfy the demand d^k of a given commodity x^k_p, we have

$$\sum_{p \in \mathcal{P}^k} x^k_p = d^k \qquad \text{for every commodity } k. \tag{1}$$

B. Element Flows

1. *Element Flow Variables*

For any non-propulsive element $m \in \mathcal{M}_N$, let us define the decision variable y^m_p such that

$$y^m_p = \begin{cases} 1 & \text{if non-propulsive element } m \text{ travels on path } p \\ 0 & \text{otherwise,} \end{cases}$$

for each feasible path p in the time expanded network.

Moreover, for any propulsive element $m \in \mathcal{M}_P$,

$$z^m_{p,q} = \begin{cases} 1 & \text{if element } m \text{ is fueled at the first node of } p \text{ and is active during sub-path } q \text{ of path } p \\ 0 & \text{otherwise,} \end{cases}$$

where p is any feasible path in the time expanded network and q is a sub-path of p. Note that $\sum_q z_{p,q}^m = 1$ if and only if element $m \in \mathcal{M}_P$ travels on path p.

For each path p, the element m can only be refueled at most once at the first node of p, and there is at most one sub-path q such that the element m is active. Note that some arc $a \notin \mathcal{A}_B$ may be included in the active sub-path q. It is possible for an element to enter the network without fuel, and be fueled at a node i. To capture this situation, we allow q to be empty if p is the first path of the element, i.e., the first node of p, $f(p)$ is s. As illustrated in Figure 5, this definition allows the tracking of refueling, and the active sub-path q is empty for the first path p_0.

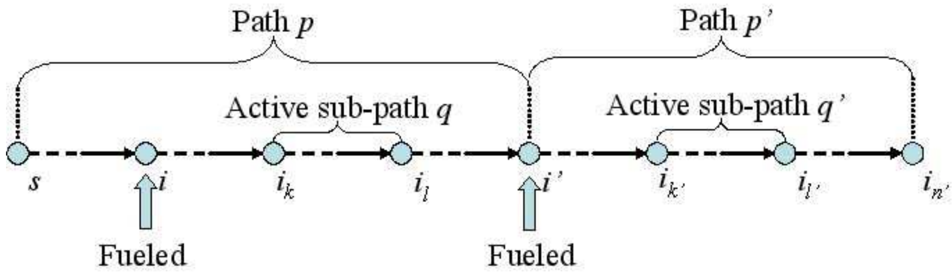

Figure 5. Illustration of the Propulsive Element Flow Variables

2. *Element Flow Constraints*

- A non-propulsive element can only travel on a single path,

$$\sum_p y_p^m \leq 1 \qquad m \in \mathcal{M}_N. \tag{2}$$

- For active elements, we constrain at most one element to be active on any burn arc,

$$\sum_{m \in \mathcal{M}_P} \sum_p \sum_{q:a \in q} z_{p,q}^m \leq 1 \qquad a \in \mathcal{A}_B. \tag{3}$$

- A non-propulsive element $m \in \mathcal{M}_N$ can travel on an arc a only if there is an active element on that arc,

$$\sum_{p:a \in p} y_p^m \leq \sum_{m' \in \mathcal{M}_P} \sum_p \sum_{q:a \in q} z_{p,q}^{m'} \qquad a \in \mathcal{A}_B, m \in \mathcal{M}_N. \tag{4}$$

- A propulsive element $m \in \mathcal{M}_P$ can travel on an arc a only if there is an active element on that arc,

$$\sum_{p:a \in p} \sum_q z_{p,q}^m \leq \sum_{m' \in \mathcal{M}_P} \sum_p \sum_{q:a \in q} z_{p,q}^{m'} \qquad a \in \mathcal{A}_B, m \in \mathcal{M}_P. \tag{5}$$

- To obtain a valid formulation for refueling, we require the flow conservation constraints for each element $m \in \mathcal{M}_P$,

$$\sum_{p:f(p)=s} \sum_q z_{p,q}^m \leq 1 \qquad m \in \mathcal{M}_P \tag{6}$$

$$\sum_{p:l(p)=i} \sum_q z_{p,q}^m = \sum_{p:f(p)=i} \sum_q z_{p,q}^m \qquad m \in \mathcal{M}_P, i \in \mathcal{N}. \tag{7}$$

C. Fuel Flows

For every node i where a propulsive element m will be fueled, there should be enough supply of fuel. From our assumptions it follows that the required amount of fuel be mf^m for element m. Therefore, the amount of fuel required at node i can be regarded as a special commodity, with the demand of $\sum_{m\in\mathcal{M}_P}\sum_{p:f(p)=i}\sum_q mf^m z^m_{p,q}$. For each of the feasible path p, we define the decision variable

$$w_p = \text{amount of fuel traveling on path } p.$$

Obviously, $w_p \geq 0$, and the demand of fuel at node i is satisfied by the fuel transported on path p ending at node i, i.e.,

$$\sum_{p:l(p)=i} w_p = \sum_{m\in\mathcal{M}_P}\sum_{p:f(p)=i}\sum_q mf^m z^m_{p,q} \qquad i\in\mathcal{N}. \tag{8}$$

D. Capacity

For space travel, it is necessary that all commodities be transferred by elements. As such, we must relate the amount of commodities (both mass and volume) available at each time to the total capacity available at that time. Since the mass and volume loss/gain factors for different commodities can be both positive and negative, for each arc $a=(i,j)$, we consider the capacity at both $t(i)$ and $t(j)$.[b]

First, for a given arc $a=(i,j)$, let us consider the mass capacity constraint at time $t(i)$.

- For any commodity k, for some path p such that $a\in p$, we need to consider the mass of the amount x^k_p at time $t(i)$. Commodity k traveling along path p enters the network at the first node of p, i.e., at time $t(f(p))$ and arrives at the destination at the last node of p, i.e., at time $t(l(p))$. According to our definition of the mass loss/gain factor, its mass at time $t(i)$ is $\left(m^k+\sum_{t=t(i)}^{t(l(p))} fm^k_{t-t(f(p))}\right)x^k_p$.
- We need to consider the fuel mass w_p such that $a\in p$.
- The total mass capacity available at arc $a=(i,j)$ is

$$\sum_{m\in\mathcal{M}_P}\sum_{p:a\in p}\sum_q CM^m z^m_{p,q} + \sum_{m\in\mathcal{M}_N}\sum_{p:a\in p} CM^m y^m_p.$$

Therefore, the corresponding capacity constraint is

$$\begin{aligned}\sum_k\sum_{p:a\in p}\left(m^k+\sum_{t=t(i)}^{t(l(p))} fm^k_{t-t(f(p))}\right)x^k_p + \sum_{p:a\in p} w_p \leq \sum_{m\in\mathcal{M}_P}\sum_{p:a\in p}\sum_q CM^m z^m_{p,q} +&\\ \sum_{m\in\mathcal{M}_N}\sum_{p:a\in p} CM^m y^m_p \qquad & a=(i,j)\in\mathcal{A}_B.\end{aligned} \tag{9}$$

Similarly, we can get the mass capacity constraints at time $t(j)$,

$$\begin{aligned}\sum_k\sum_{p:a\in p}\left(m^k+\sum_{t=t(j)}^{t(l(p))} fm^k_{t-t(f(p))}\right)x^k_p + \sum_{p:a\in p} w_p \leq \sum_{m\in\mathcal{M}_P}\sum_{p:a\in p}\sum_q CM^m z^m_{p,q} +&\\ \sum_{m\in\mathcal{M}_N}\sum_{p:a\in p} CM^m y^m_p \qquad & a=(i,j)\in\mathcal{A}_B.\end{aligned} \tag{10}$$

As for volume capacity constraints, we have identical terms except for the fuel carried on arc a. The

[b] If we allow nonlinear loss/gain functions, we need to evaluate the capacity of arc a at any time $t\in[t(i),t(j)]$. However, it is a direct extension of constraints discussed here.

volume of the fuel traveled on path p such that $a \in p$ is $\frac{vf^m}{mf^m} w_p$. Hence, the volume capacity constraints are

$$\sum_k \sum_{p:a \in p} \left(v^k + \sum_{t=t(i)}^{t(l(p))} fv^k_{t-t(f(p))} \right) x^k_p + \sum_{p:a\in p} \frac{vf^m}{mf^m} w_p \leq \sum_{m \in \mathcal{M}_P} \sum_{p:a\in p} \sum_q CV^m z^m_{p,q} + \sum_{m\in\mathcal{M}_N} \sum_{p:a\in p} CV^m y^m_p \qquad a = (i,j) \in \mathcal{A}_B \tag{11}$$

$$\sum_k \sum_{p:a \in p} \left(v^k + \sum_{t=t(j)}^{t(l(p))} fv^k_{t-t(f(p))} \right) x^k_p + \sum_{p:a\in p} \frac{vf^m}{mf^m} w_p \leq \sum_{m \in \mathcal{M}_P} \sum_{p:a\in p} \sum_q CV^m z^m_{p,q} + \sum_{m\in\mathcal{M}_N} \sum_{p:a\in p} CV^m y^m_p \qquad a = (i,j) \in \mathcal{A}_B. \tag{12}$$

E. Capability

The capability constraint determines if enough fuel is available to perform a burn. A single propulsive element can only burn on consecutive burn arcs. All fuel is assumed to be consumed or dropped after the final burn. The propulsive element cannot be reused until after it is refueled.

Here we model that the total fuel of the active element performing the burn on a sub-path q must be enough to carry the total cumulative mass along every arc in q. Let q be an arbitrary sequence of possible consecutive burns and let $a^l = (i^l, j^l)$ be the lth burn arc in q for $l = 1, \dots, |q|$. Here $|q|$ denotes the number of arcs in q. Let $r(p,q)$ denote the sub-path along path p from the first node of p to the first node of q, if q is not empty. For example, $r(p,q)$ is the sub-path from node i to node i_k for the path p shown in Figure 5.

The resulting constraint family reads

$$mf^m \sum_p z^m_{p,q} + M\left(1 - \sum_p z^m_{p,q}\right) \geq \sum_{l=1}^{|q|} \Phi^m_{q,l} \times \Bigg[\sum_{m' \in \mathcal{M}_P} \sum_{p:a^l\in p} \sum_{q'} ms^{m'} z^{m'}_{p,q'} + \sum_{m'\in\mathcal{M}_N} \sum_{p:a^l \in p} ms^{m'} y^{m'}_p + mf^m + \sum_{\substack{m'\in\mathcal{M}_P \\ m' \neq m}} \sum_p \sum_{q':a^l \in r(p,q')} mf^{m'} z^{m'}_{p,q'} + \sum_k \sum_{p:a^l\in p} \left(m^k + \sum_{t=t(i^l)}^{t(l(p))} fm^k_{t-t(f(p))} \right) x^k_p + \sum_{p:a^l\in p} w_p \Bigg] \qquad m \in \mathcal{M}_P,\ \text{path } q, \tag{13}$$

where

$$\Phi^m_{q,l} = \phi^m_{a^l} \prod_{l'=l+1}^{|q|} (1 - \phi^m_{a^{l'}}).$$

F. The Complete Model

Since the cost to route commodities is negligible, we include only the refueling cost and the cost associated with elements. The objective function reads

$$\min \sum_{m\in\mathcal{M}_p} c^m \sum_{\substack{p \\ f(p)=s}} \sum_q z^m_{p,g} + \sum_{m\in\mathcal{M}_n} c^m \sum_p y^m_p + f \sum_p w_p + f \sum_{m\in\mathcal{M}_p} mf^m \sum_{\substack{p \\ f(p)=s}} \sum_{\substack{q \\ q\neq\emptyset}} z^m_{p,q},$$

where c^m is the cost of using element m and f is the per unit fuel cost. Note that c^m should not include the fuel cost. The last term captures the fuel cost of the propulsive elements on their very first path originating at s and assuming they burn (i.e. $q \neq \emptyset$). This is the fuel that is preloaded on the Earth into selected propulsive elements.

The model includes constraints (1) through (13). In addition, all x and w variables are nonnegative and all z and y variables are binary.

IV. Solution Methodology

Having defined the inputs to the problem and the model, it can readily be seen that although this problem is linear, it is quite large, and with discrete variables, will require a complex solution methodology. Using techniques for large-scale optimization, such as column generation and branch-and-price[6] we can sequentially solve a problem of this size efficiently.

Using column generation we select only a sub-set of the variables to be present on each iteration. Using this sub-set, the branch-and-price computation begins by relaxing the integrality constraints on the variables and solving the resulting linear program. This iterative process continues by using a previous solution and resolving the problem after selectively imposing values on certain variables based on the previous solution. Once converged, the solution is evaluated to determine if there is a better solution exists when using another sub-set of variables. The optimization algorithm terminates when a better solution does not exist.

A. Initial Solution Methodolgy

In order to effectively utilize the above solution methodology a good feasible initial solution is required. Due to the size and complexity of the model, it is necessary to utilize optimization techniques prior to the full-scale optimization to create a very good initial solution that provides both a starting point for the optimization and a bound for comparison during the optimization process. Therefore the initial solution is obtained by employing less computationally intensive optimization techniques.

The initial optimization solution has three components: path assignment, commodity to element assignment and propulsive element to burn arc assignment. Figure 6 provides a representation of how these components integrate to form the initial solution.

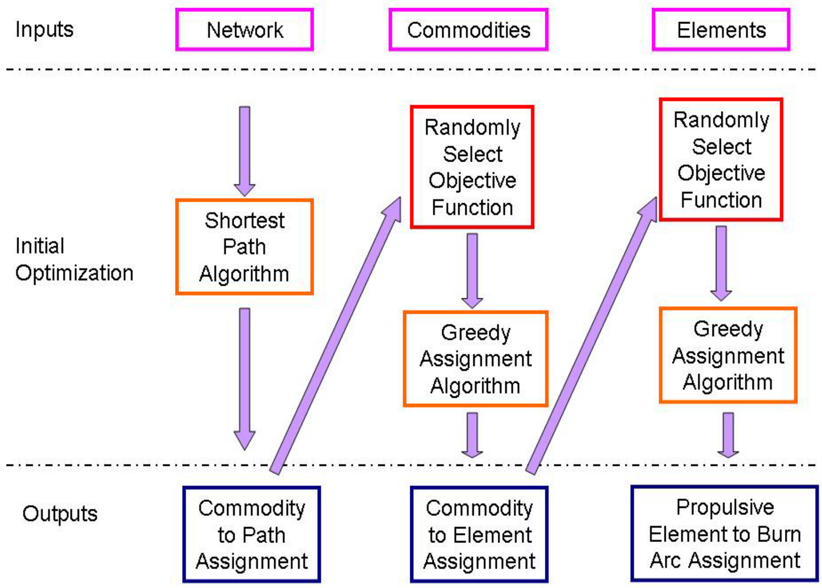

Figure 6. Diagram of Initial Optimization Solution Methodology

The path assignment is performed by employing a shortest path algorithm where the cost of the arcs is proportional to the Δ V of the arcs. The shortest path for each commodity is then determined. In addition, a benefit is placed on using a previously assigned arc to minimize the number of transfers required. The path assigned to a commodity is feasible, given the starting and ending node and time intervals. For commodities with a restricted travel time, it is necessary to utilize a constrained shortest path algorithm where paths can only be selected if they satisfy the addditional time constraints.

Following the path assignment, it is necessary to allocate commodities to elements based on a surrogate cost. Here a greedy algorithm is used in conjunction with random surrogate cost selection to rank the elements. The surrogate cost of an element is based on an approximate measure of the value of the element. Equation 14 lists the surrogate costs employed.

$$\begin{aligned}
\mathcal{J}_1 &= \frac{Cost}{MassCapacity} \\
\mathcal{J}_2 &= \frac{Cost^2}{MassCapacity} \\
\mathcal{J}_2 &= Cost \\
\mathcal{J}_4 &= \frac{\sqrt{Cost}}{MassCapacity} \\
\mathcal{J}_5 &= \frac{Cost}{MassCapacity^2} \\
\mathcal{J}_6 &= \frac{Cost}{\sqrt{MassCapacity}}
\end{aligned} \tag{14}$$

Given these surrogate costs, the greedy algorithm seeks to make the best assignment at each turn by selecting the element with the lowest surrogate cost first. Given a particular element, the initial optimization routine evaluates if the element can be used to carry the given commodities to the destination. This proceedure repeats until all commodities have been assigned.

Finally, the propulsive element to burn arc assignment is performed. Again, a greedy algorithm is utilized and random surrogate cost functions are applied to each element. The surrogate cost functions for the propulsive element assignment are provided in Equation 15.

$$\begin{aligned}
\mathcal{J}_1 &= \frac{Cost}{Fuel} \\
\mathcal{J}_2 &= \frac{Cost^2}{Fuel} \\
\mathcal{J}_3 &= Cost \\
\mathcal{J}_4 &= \frac{\sqrt{Cost}}{Fuel} \\
\mathcal{J}_5 &= \frac{Cost}{Fuel^2} \\
\mathcal{J}_6 &= \frac{Cost}{\sqrt{Fuel}}
\end{aligned} \tag{15}$$

The greedy algorithm tracks from the end of the path to the beginning and assigns propulsive elements along the burn arcs. For an assignment to be feasible, the amount of fuel currently available in a given propulsive element must be greater than the fuel required to perform the burn, which is a function of the mass that needs to be transported. In this manner, an accurate cost of transporting the commodities to their respective destinations can be computed and a feasilbe initial solution to the optimization can be obtained.

V. Apollo Example

For such a complex problem it is helpful for both generating and understanding the model to examine a well defined problem. Using the Apollo 11 elements, a simple example has been defined to determine how the variables above would be set. The example has two commodities that need to be sent to the Apollo 11 landing site. The commodity properties are listed in Table 1.

In order to transport the commodities from low Earth Orbit (LEO) to the lunar surface we must know the properties of the elements that can transport them. A list of these elements is provided in Table 2.

Figure 7 depicts the solution for this example. As we can see, two different types of elements are used to transport the commodities. The commodities travel two seperate paths. The first commodity is shipped

Table 1. List of Commodities and Properties for Apollo 11 Example

Demand	Starting Node	Time Interval	Ending Node	Time Interval	Max Time	Mass	Volume	Mass Loss	Volume Loss
2	LEO	1, 4	Apollo 11	10, 15	20	100 kg	1 m^3	0	0
1	LEO	1, 6	Apollo 11	12, 12	20	100 kg	1 m^3	0	0

Table 2. List of Elements and Properties for Apollo 11 Example

Element Type	Fuel Mass	Isp (sec)	Structural Mass	Mass Capacity	Volume Capacity	Number Available	Cost (mil)
Saturn V 1st Stage	2150999	304	135218	0	0	4	692
Saturn V 2nd Stage	451730	421	39048	0	0	4	307
Saturn V 3rd Stage	106600	421	13300	0	0	4	151
SLA	0	0	1837	0	0	4	0.9
Command Module	0	0	5806	100	1	4	148
Service Module	18413	314	6110	0	0	4	118
LM Descent Stage	8156	311	1984	500	5	4	57
LM Ascent Stage	2358	311	2189	100	1	4	79

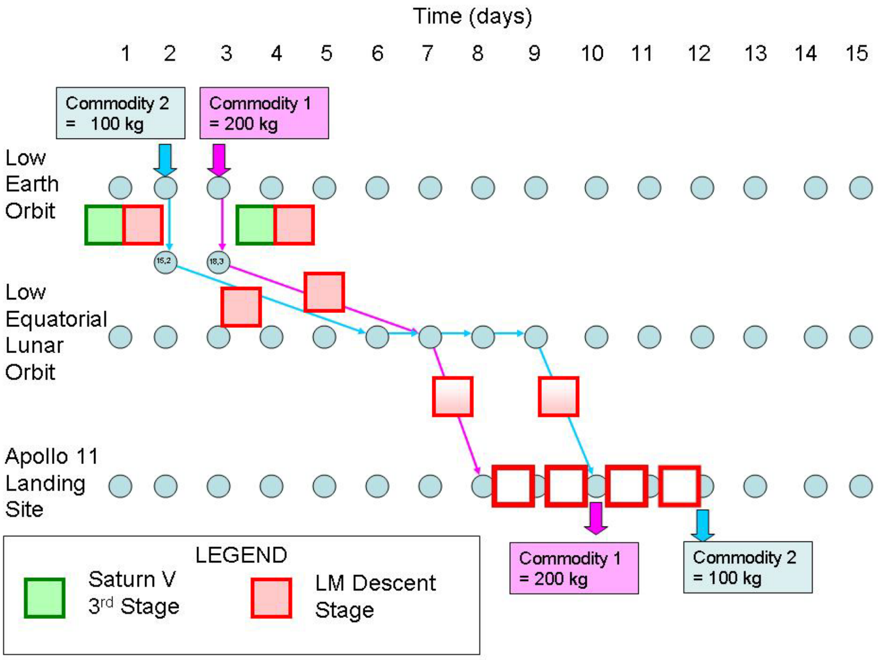

Figure 7. Apollo 11 Example

later but arrives earlier. The second commodity arrives exactly at day 12 because that was part of the input, but since it was shipped earlier, it waits in low Equatorial Lunar orbit for three days. On both paths the Saturn V 3rd Stage is used to perform the lunar insertion burn. The lunar descent module performs both the orbit insertion burn and the descent burn to carry the commodities down to the lunar surface.

VI. Future Work

In the final paper, the focus will shift from the model capability to the solutions obtained from the lunar campaign scenario. Through these solutions an understanding of the architectural decisions necessary to support space exploration will be developed. The complexities of the logistics problem warrant specifically considering logistics in the transportation design and require a formal methodology for solution similar to those considered for terrestrial logistics.

In the final paper, a trade study will be conducted that demonstrates how and when pre-positioning of fuel should be utilized from a system level perspective. In addition, the concept of refueling will be evaluated by examining how the capability effects the solution and what are the system level impacts of this technology on the logistics capabilities of an exploration system.

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