Integrated Airline Planning

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1. Introduction

Airline business processes related to tactical planning consist of schedule planning, fleeting, aircraft rotation, and crew pairing (see e.g. Klabjan [10]). In *schedule planning*, a set of flights with specific departure and arrival times is constructed. Next is *fleeting*, which assigns an equipment type to each individual flight. The objective of the *fleet assignment model* (FAM) is to maximize profit subject to the number of available aircraft and other operational constraints. The problems that follow decompose based on the fleeting solution, i.e. there is a problem for every equipment type. The *aircraft rotation* problem is to find a set of generic aircraft routes that satisfy maintenance requirements. The *crew pairing* (CP) optimization follows. In crew pairing a set of crew itineraries or *pairings* is constructed. The goal is to minimize the crew cost and each flight must be cover by exactly one pairing. A few weeks before the day of operations, the actual tail numbers are assigned to each flight and monthly crew rosters or bidlines are assigned to every individual crew member. Throughout tactical planning, revenue management related processes match demand with supply, which is the sit capacity of a given equipment type.

At present, each one of these problems is modeled independently of the remaining problems even though there is a clear interaction between them. Basically, the output of one stage is the input to the next stage. Recently attempts have been made to integrate planning stages. Barnhart et al. [1] present a model that to some extent integrates FAM and crew pairing. They consider duties and not pairings in long-haul operations. Barnhart et al. [2] propose a model for integrating FAM and aircraft rotation. Cordeau et al. [6], Barnhart et al. [3], and Klabjan et al. [9] present models for integrated aircraft rotation and crew pairing. Similar integration efforts have been undertaken in mass transit scheduling by Huisman et al. [8].

We propose a model that considers fleeting, aircraft rotation, and crew pairing simultaneously. Pairings are modeled explicitly and the rotation problem is captured by the plane count constraints from Klabjan et al. [9]. Thus in absence of maintenance requirements, our model completely integrates these three stages. The model is solved by a combination of Lagrangian decomposition and column generation. The latter is used to price out favorable pairings. The main contributions of our work are in the model itself and the novel solution methodology for solving it. The pricing problem is solved by constrained shortest path, where additional labels are required to capture different equipment types. In Section 2 we present the model. Section 3 outlines the solution methodology. Finally, in Section 4 we present preliminary computational experiments.

2. The Model

We start by describing the underlying network required for model description. Consider a station o. An activity or event presents either a landing or a takeoff event. For the departure of flight l, let t_l be the departure time. For the arrival of flight l, let t_l be the arrival time plus the minimum aircraft turn time mt, called also the ready time. The *ready-time-space network* (RTN) has a node

(o,s) for every station o and every activity s at this station. There is a *flight arc* between every departure event and arrival event of the same flight. For every station o we order the activities based on t_l, i.e. $t_1 \leq t_2 \leq \cdots \leq t_n$, where n is the number of activities at the station. The network has a *ground arc* g = $((o,s_i), (o,s_{i+1}))$ for i=1,2,..,n. There is a wraparound arc between the first and the last nodes of the time horizon. The set of all ground arcs is denoted by G. The ground arc time interval is the time in between the two activities that define the ground arc.

The traditional leg-based FAM is to find an optimum fleeting subject to the assignment constraints (each flight assigned to a fleet), flow balance (every aircraft that lands must take off), and the plane count constraints (not to use more aircraft than there are available). The traditional stand-alone CP problem is modeled as the set partitioning problem, where each binary variable corresponds to a pairing p. The idea behind the integrated model is to assign pairings to fleets, which requires expanding the crew pairing variables and to link them with the fleet assignment variables.

Our model has three types of variables. The FAM related variables are the fleet assignment variables x and the ground arc variables z. For each flight arc l and each equipment type f, x_{fl} is 1 if flight l is assigned the equipment type f. The nonnegative variable z_{gf} counts the number of planes on the ground of equipment type f during the ground arc g time interval. Let MD be a fixed time, which corresponds to a time with low activity at any station, e.g. 3 am.

In our integrated model, we use the aforementioned FAM constraints. In order to capture pairings within fleeting, we modify the pairing variables to y_{fp}, where p is a pairing covering any subset of flights among all the flights in the schedule and f is a fleet index. y_{fp} is 1 if pairing p is assigned to fleet f. The fleet-pairing linking constraints must model that a pairing p assigned to fleet f covers flight l only if l is assigned to the same fleet f (determined by the corresponding x variable).

So far we have neglected the aircraft rotation issue. In traditional crew pairing, pairings are generated based on the minimum sit connection time ms, unless the crew follows the aircraft turn (see Barnhart et al. [4] for details on crew scheduling). It has been shown by Klabjan et al. [9] that by reducing ms, additional pairings become feasible, which results into lower crew cost. We use the approach from Klabjan et al. [9] to incorporate routing constraints. In the absence of maintenance constraints, they show that to integrate aircraft rotation and crew pairing it suffices to add the plane count constraints to the set partitioning formulation of crew pairing. Suppose we consider pairings with the minimum sit time equal to mt. Pairings from a solution with a connection shorter than ms imply a plane turn. Such connections are called *forced turns*. A set of forced turns can be extended into a plane count feasible rotation if and only if the number of planes on the ground at any time imposed by the forced turns does not exceed the plane count obtained from the corresponding ground arc value in the FAM solution. The plane count constraints model this relationship. In order to embed this into our integrated model, we have to observe that the ground arc value now corresponds to a decision variable and is not a fixed value. A technical difficulty is the fact that the RTN uses ready times however pairings are based on the actual arrival times.

Let the *actual-time-space network* (ATN) be defined in the same way as RTN except that for each arrival of flight l, t_l is the actual arrival time l_a of leg l. The set of all ground arcs in ATN is denoted by G' and the ground arcs are denoted by g'. *Essential ground arcs* are those ground arcs, which are defined by a departure followed by an arrival. Plane count constraints associated with non-essential ground arcs are redundant (for proof see Klabjan et al. [9]). Let $E(G') \subset G'$ be the set of essential ground arcs.

Let P be the set of all pairings covering any subset of legs in the flight schedule and with the minimum sit connection time equal to mt. Let $P_{g'}$ be the set of pairings, which have a forced turn that includes g'. The plane count constraints from Klabjan et al. [9] read

$$\sum_{p \in P_{g'}} y_p \;\le\; b_{g'} \qquad g' \in E(G'),$$

where, $b_{g'}$ is the number of aircraft on the ground in the time interval defined by ground arc g'.

It remains to be seen how to convert these constraints into a fleet based setting, i.e. we have to link $b_{g'}$ with the ground arc variables z. For each $g' \in E(G')$, there exists a corresponding ground arc $m_{g'}$ in RTN. Note that g' is defined by the arrival of a leg l. This ground arc $m_{g'}$ is defined by the arrival leg l's ready time and an earlier activity, see Figure 1. Let $dep(g')$ be the set of all flights that depart in the time interval $[l_a , l_a + mt)$. We replace the right hand side of the plane count constraints by

$$z_{m_{g'}f} + \sum_{l \in dep(g')} x_{fl}$$

This expression states that the value of the ground arc corresponding to an arrival event of leg l in ATN is equal to the ground arc value of the same flight in RTN plus the number of departures in the set $dep(g')$.

The integrated model reads

$$\max \sum_{f,l} r_{fl} x_{fl} - \sum_{f,p} c_p y_{fp}$$

$$\sum_{f \in F} x_{fl} \;=\; 1 \qquad l \in L \tag{1}$$

$$\sum_{l \in O(v)} x_{fl} + z_{o(v)f} - \sum_{l \in I(v)} x_{fl} - z_{i(v)f} \;=\; 0 \qquad v \in V,\, f \in F \tag{2}$$

$$\sum_{l \in M} x_{fl} + \sum_{g \in W} z_{gf} \;\le\; N_f \qquad f \in F \tag{3}$$

$$\sum_{l \in p} y_{fp} \;=\; x_{fl} \qquad l \in L,\, f \in F \tag{4}$$

$$\sum_{p \in P(g')} y_{fp} \le z_{m_{g'}f} + \sum_{l \in dep(g')} x_{fl} \qquad g' \in E(G'),\, f \in F, \tag{5}$$

where

$I(v)$	: set of flights to node v in RTN	L	: set of all flight arcs
$O(v)$	: set of flights from node v in RTN	F	: set of all fleets
M	: set of flights in the air at MD	V	: set of all nodes in RTN
N_f	: number of aircraft in fleet f	$i(v)$	: ground arc to node v in RTN
W	: set of ground arcs which contain MD	$o(v)$	: ground arc from node v in RTN
c_p	: cost of a pairing p	r_{fl}	: profit of assigning fleet f to leg l.

Constraints (1), (2) and (3) are the standard FAM constraints, (4) assures that a pairing is assigned to a fleet if and only if all the legs in the pairing are assigned to the same fleet and (5) are the plane count constraints.

In our implementation we have also modeled constraints to prevent crew *double overnights* (Clarke et al. [5]). We add constraints to FAM using legal rest arcs and mid-day breakouts such that the fleeting obtained is more "crew friendly".

3. Solution Methodology

Our approach to solve the model is by a combination of Lagrangian decomposition and column generation. First we approximate the model by changing the partitioning requirement (4) into covering constraints. Instead of considering all pairings at once, we generate a subset of pairings. The corresponding restricted master problem is then solved by Lagrangian decomposition after relaxing constraints (4) and (5). The obtained Lagrangian multipliers are then used to find new pairings, which are added to the restricted master problem.

The solution methodology is next detailed step-wise.

1. *Initialization*: Solve FAM, aircraft rotation and CP using the traditional approach. We use the fleeting obtained from FAM and the aircraft routes to solve the CP problem. The pairings obtained from CP are used as the initial set of columns in the restricted master problem.
2. *Computing Lagrangian multipliers* (major iteration): Solve the Lagrangian dual problem with the current set of columns. This gives us a set of Lagrangian multipliers. Note that after relaxing (4) and (5), the resulting model is the traditional FAM model.
3. *Pricing*: Use the Lagrangian multipliers to price out favorable pairings using the constrained shortest path algorithm (see e.g. Desaulniers et al. [7]). Formally, we have to solve

$$\min_{f} \min_{p} \; (c_p - \sum_{l \in p} \lambda_{fl} + \sum_{g' : p \in P_{g'}} \mu_{fg'}),$$

   where λ's are obtained from (4) and μ's from (5). It is easy to see that this can be reduced to the constrained shortest path problem, where for each fleet we have to introduce a new label to capture the dependency of λ's and μ's on fleet type. Thus the number of required labels becomes $k+|F|$, where k is the number of all required labels to capture pairing cost and feasibility rules. In addition, connection arcs in the pairing timeline network (see e.g. Barnhart et al. [4]) get a weight, Figure 2.

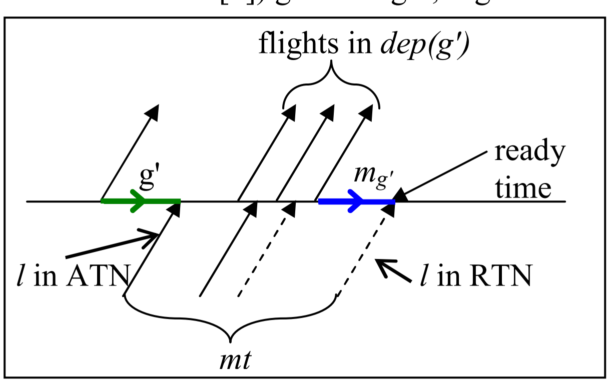

Figure 1: Timeline conversion

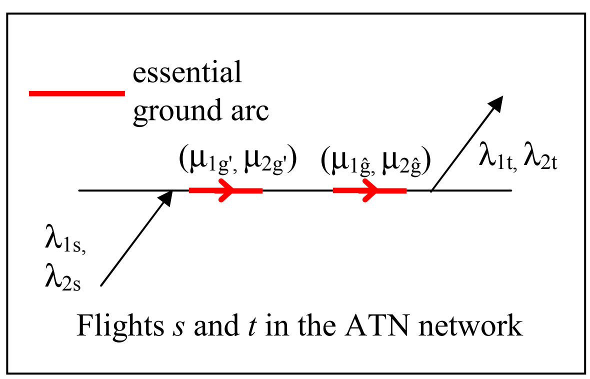

Figure 2: Connection arc weights for $|F| = 2$

4. *Iterate*: Add the obtained pairings and go to step 2. If no additional pairings are generated, go to step 5.

5. *Obtaining the final solution*: The final solution is obtained by using the produced fleeting. Aircraft routes and crew pairings are obtained using the traditional approach.

4. Computational Experiments

The computing environment consists of a cluster of 27 dual 900 Mhz Itanium 2 processors running Linux operating system and g++ development environment. We tested the integrated model on four data sets. Real world data from a major US carrier were used. The carrier has a heavy hub-and-spoke network structure with five crew bases. Crew feasibility rules and cost function comply with airline rules. We consider daily problems and the presented computational results do not include the plane count constraints. For discretionary purposes, the real profit numbers are fudged but the presented numbers show correct proportions and magnitudes.

We control tractability in the following way. First FAM is solved over all fleets and flights. Next we pick a subset of 2 or 3 fleets and the corresponding flights. The integrated model is then solved on this subset of fleets and flights. In other words, instead of considering all fleets at once, we can consider subsets of fleets.

The profits obtained from the sequential versus the integrated approach are shown in Table 1. Profit here is defined as the combined profit resulting from FAM and CP, i.e. for the integrated approach it corresponds to the objective value.

Test Case		Traditional Approach	Integrated Approach	
Fleets	Flights	Profit ($)	Profit ($)	Increase in Profit ($)
3	250	598,940	637,367	38,427
2	215	640,276	681,258	40,982
2	205	551,412	561,932	10,520
2	145	232,814	265,725	32,911

Table 1: Profits

In the last column we show the increase in profit obtained by using the integrated approach instead of the sequential method. It is worth noting that most of the increased profit does not come from diminishing revenue in FAM but it mostly comes from the reduced crew cost. The increased profits shown are for a daily activity of the airline. For an entire year, these additional profits are in the range of several millions.

For the instance with 3 fleets, Figure 3 shows the improvement in the objective value after every major iteration. The computation times for each major iteration were approximately one hour, where about half of that time is spent in subgradient optimization and the rest in pricing.

5. Future Directions

First we need to embed the plane count constraints in our implementation. This would make our computational experiments much more realistic. We see potential improvement in the running time by using the bundle algorithms instead of the vanilla subgradient optimization.

On the algorithmic front, we plan to test two alternative methodologies. The first one is Benders decomposition, where we relax integrality of pairing variables. Based on the incumbent fleeting solutions, for each fleet we solve the crew pairing LP relaxation and add Benders cuts to fleeting. Another algorithm is aligned with the existing one. We propose to directly solve the Lagrangian dual. In every subgradient or bundle iteration, we price out all pairings to adjust the current iterate.

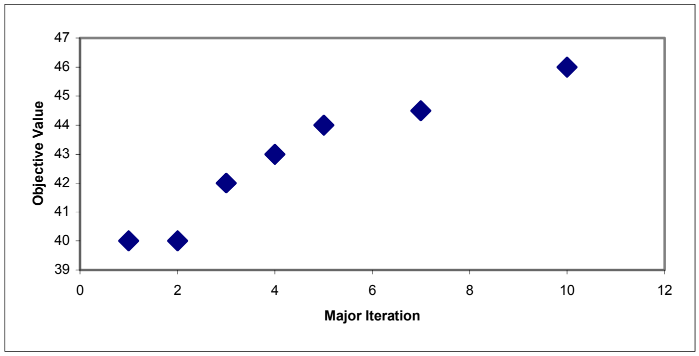

Figure 3: Objective value improvements

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