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Day-Ahead Scheduling of Demand Side Resources with Load-Shifting

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Growing installations of intermittent energy sources are increasing the uncertainty in generation for electric energy. Unless the demand can be controlled, this trend requires more flexible generation capacity, which is costly. Emerging smart grid technology offers an opportunity to control the load and to mitigate cost effects of increasing uncertainty. In this paper, we analyze the potential of using non-event-driven demand side resources (DSRs) that are scheduled one day ahead of generation to shape the demand profile on the following day. By dispatching DSRs, load can be shifted from periods with high loads to earlier and later periods by owners of such resources. Our objective is to determine the day-ahead DSR dispatch schedule that minimizes the sum of DSR dispatch cost and expected operating cost of power generating units. We present a stochastic dynamic programming model and develop a solution algorithm that combines an approximate dynamic programming algorithm with stochastic sample-based progressive hedging. We derive a lower bound on the optimal solution and provide convergence results. Using data from the California Independent System Operator region, we show that our approach can solve real world instances in reasonable time and show in an extensive numerical study that the savings potential from using DSRs to level demand is substantial.

Key words: Demand Response, Stochastic Unit Commitment, Approximate Dynamic Programming, Stochastic Sample-Based Progressive Hedging

1. Introduction

The share of intermittent power sources in electric power generation has considerably increased during the last few years. Power generation from wind and solar sources in California, for instance, has increased by 27% from 2011 to 2012 and accounted for 6% of total generation in 2012 (U.S. Energy Information Administration 2013). By 2020, one third of the generated electricity is expected to be delivered from renewable sources and a large share will be generated by intermittent sources (California State Senate 2011). The actual generation of intermittent sources depends to a large extent on external factors that cannot be controlled, such as wind speed and solar irradiation. Therefore, systems with a large intermittent capacity require costly reserve capacity from conventional sources to maintain system stability and to match demand. Especially during peak load hours, when many units are operating at maximum capacity, the marginal cost of generation is high.

Emerging smart grid technology provides a mean to mitigate the cost effect of increased generation volatility. By integrating demand side resources (DSRs) that can be remotely controlled into the distribution network, the demand profile can be influenced to better match generation.

While previous research on DSRs analyzed the use of DSRs to react to short term deviations of intermittent generation sources from forecasted capacity (i.e., event-driven DSRs, e.g., Papavasiliou and Oren 2013a, Sioshansi 2012, Parvania and Fotuhi-Firuzabad 2010), we focus on the potential of DSRs to shape the demand profile (i.e., non-event-driven DSRs, e.g., California ISO 2013). We analyze how DSRs can be used in a day-ahead market to reshape the demand profile of the next operating day to obtain a more leveled demand profile that can be met at a lower expected operating cost.

Non-event-driven DSRs are mainly offered by load aggregators who cooperate with load operators and offer "negative" generation capacity. If the offer of a load aggregator is accepted by an operator, the aggregator is obligated to provide a reduction in consumption of electric energy (California ISO 2011). Prices for electric energy tend to be high during peak demand hours and aggregators offer demand reductions during these hours. However, scheduling DSRs for dispatch typically leads to load shifting. In the day-ahead DSR scheduling model that we consider, a reduction of load in one period can result in an increase of load in previous or subsequent periods and this effect must be anticipated when scheduling generators and DSRs. The problem that we consider exists, for instance, as part of the Residual Unit Commitment process of Independent System Operators (ISOs) that allow DSRs to offer load reduction in the day-ahead market (Harvey et al. 2005).

Our objective is to compute a day-ahead schedule for the dispatch of DSRs on the next day that results in minimum expected total cost. This schedule is submitted one day ahead of the actual dispatch to the load aggregators that operate the DSRs. The resulting demand profile of the next day, after the dispatch of DSRs, must be met by the power generating units in the network. Their optimal dispatch policy can be determined by solving a unit commitment problem (e.g., Wood and Wollenberg 2006). However, the DSR dispatch schedule and the optimal operating schedule of the generators are not independent, and therefore, we must solve an integrated problem that determines the dispatch schedule of the DSRs and the operating policy of the generators jointly.

Including non-event-driven DSRs into unit commitment results in a complex stochastic problem because load can be shifted to previous as well as to subsequent periods. To solve the problem, we develop a stochastic sample-based extension of the progressive hedging algorithm (Rockafellar and Wets 1991), which allows us to efficiently solve our model by solving a sequence of dynamic programs. Given a DSR dispatch schedule, the underlying unit commitment problem is modeled as a dynamic program and solved by approximate dynamic programming (ADP). Stochastic sample-based progressive hedging is used to optimize DSR bids where each "function evaluation" corresponds to solving a unit commitment problem. We also derive a lower bound on the optimal expected cost. In addition, we provide convergence results of the stochastic sample-based progressive hedging algorithm. We also perform an extensive numerical study to assess the value of using non-event-driven DSRs.

Our contribution is threefold. First, we present a non-event-driven DSR model with load shifting. Unlike previous research, we allow for stochastic load shifts to previous as well as to subsequent time periods, which is important, because the load shifts are uncertain and load can be shifted to previous periods when the DSRs are scheduled for dispatch one day ahead. Second, we develop a new solution approach that combines stochastic sample-based progressive hedging and approximate dynamic programming and we derive convergence results for the algorithm that are of interest in their own right. Third, we present numerical results based on real-world data that indicate that substantial savings in energy generation cost can be achieved if non-event-driven DSRs are scheduled with our algorithm.

This paper is organized as follows. In Section 2, we review the related literature on the unit commitment problem, on DSR models, and on the solution approaches that we rely on. In Section 3, we state the problem formally. In Section 4, we present our model. In Section 5, we show how the problem can be solved and derive convergence results for our algorithm. In Section 6, we present numerical results that are based on actual data from the California ISO region. In Section 7, we conclude. The notation is summarized in the appendix, and similarly all proofs are also deferred to the appendix.

2. Literature Review

We build on and extend the literature on unit commitment models (Subsection 2.1) and on solution approaches for such models (Subsection 2.2).

2.1. Unit Commitment Models

An important element of our model is the unit commitment problem. The early versions of the model consider *deterministic* settings without DSRs and differ in the components and constraints that they incorporate. Muckstadt and Koenig (1977) present a basic model and consider only reserve capacity and demand constraints. Subsequent research extends the basic model and includes transmission constraints (Ma and Shahidehpour 1998), voltage constraints (Ma and Shahidehpour 1999), and storage (Al-Agtash 2001). One of the most general models is studied by Baldick (1995), who includes minimum up and downtime constraints, power-flow constraints, line-flow limits, voltage limits, reserve constraints, ramp limits, and total fuel and energy limits on hydro as well as thermal power generating units. Recently, models include *stochastic* elements and allow for stochastic prices and cost (Takriti et al. 2000), stochastic fuel cost (Valenzula and Mazumdar 2003), and stochastic load (Li et al. 2014, Papavasiliou and Oren 2013a,b, Constantinescu et al. 2011, Sioshansi and Short 2009, Morales et al. 2009). Wood and Wollenberg (2006) provide an extensive overview of unit commitment models. Our model differs from these approaches by modeling the unit commitment problem as a multi-stage decision problem, whereas most previous work considers only two stages.

Some authors have included *DSRs* in their models. Zhao and Zeng (2010) analyze a unit commitment model with DSRs that minimizes the worst-case operating cost. Parvania and Fotuhi-Firuzabad (2010) consider a model where DSRs are employed if load exceeds capacity. Pritchard et al. (2010) present a single-settlement, energy-only market model, in which DSRs may offer load reduction in the day-ahead market and can be redispatched in real-time. All three models consider stochastic and responsive loads but neglect load shifting effects.

Another set of authors takes *load shifting* into account but assumes that loads are deterministic. Su (2007) and Su and Kirschen (2009) analyze a manufacturing process where the manufacturer submits price-responsive bidding curves for electric energy. The bids are incorporated in a unit commitment model, such that the production rate can be optimized based on energy-bid prices. Dietrich et al. (2012) and De Jonghe (2011) develop unit commitment models, where a centralized decision maker has the authority to shift loads between periods. Dietrich et al. (2012) consider a model with local decision making, where load shifts are based on bidding curves of electricity prices.

The only work combining stochastic demand and generation with load shifting is by Papavasiliou and Oren (2013a). They consider a stochastic unit commitment model with different demand response paradigms. In the first paradigm, the system operator can centrally control the demand of individual loads and shift load deterministically in real time. In the second paradigm, the distribution of deferrable demands over the time horizon is determined by bid-curves. Our model differs from their model because we consider day-ahead scheduling of DSRs to level the demand profile before demand is realized.

While all previous research analyzes event-driven DSRs, we consider non-event-driven DSRs that are scheduled for dispatch a day ahead of operation and we allow for stochastic load shifts, which is important in this setting, because there is considerable uncertainty about the load-shift effect over the course of a day when the DSR is scheduled and notified in the day-ahead market.

2.2. Solution Approaches

The solution approaches that are typically used to solve stochastic unit commitment problems rely on *scenario-based* representations of uncertainty (Li et al. 2014, Papavasiliou and Oren 2013a,b, Ruiz et al. 2009, Cerisola et al. 2009, Shiina and Birge 2004, Takriti and Birge 2000, Takriti et al. 2000, Nowak and Römisch 2000). Uncertainty about the next stage is incorporated in the model by considering a number of scenarios that are weighted with probabilities. For our setting, the scenario-based approach has drawbacks because we deal with a two-stage decision problem, where the second stage problem itself has multiple stages with stochastic quantities at each stage. In the first stage, we model stochastic load shifts between periods, which results in a stochastic vector of high dimensionality. In the second stage, we model the unit commitment problem as a multi-stage problem and allow for correlation of demand between periods. A scenario-based approach would require a nested scenario tree with many scenarios for load-shifting of each DSR and would result in a model that is computationally intractable for reasonably sized problems.

Therefore, we use a different approach. We model the unit commitment problem that we solve in the second stage as a dynamic program and embed it into a stochastic proximal point algorithm that solves the first stage problem. Because the resulting dynamic program is very large, we use ADP to approximate the problem. We review the related literature on the proximal point algorithm and ADP next.

The *proximal point algorithm* was introduced by Martinet (1970) and Rockafellar (1975) and was extended to the progressive hedging algorithm, which allows for decomposition of problems with a special structure, by Rockafellar and Wets (1991). Recent work on the proximal point algorithm focuses on convergence rates (Yao and Shahzad 2012) and on extensions to inexact proximal steps (He and Yuan 2012). The progressive hedging algorithm has been used as a decomposition approach for solving various problems in scenario-based stochastic programming (e.g., Li et al. 2014, Haugen et al. 2001, Takriti and Birge 2000). We extend both, the proximal point and the progressive hedging algorithm, to stochastic problems, such that the proximal step is taken based on Monte-Carlo sampling and prove convergence of the algorithms for convex problems. Our line of proof follows Bertsekas (2011), who proves convergence of incremental proximal bundle methods. We use the stochastic sample-based progressive hedging algorithm to decompose the first stage decisions of our problem by time period and we use the stochastic proximal point algorithm to develop lower bounds on the optimal solution.

We approximate the second stage of our problem using ADP. ADP has been widely applied to resource allocation problems, which have many similarities to our problem. Especially the application to transportation problems by Powell and Topaloglu (2003) and the framework provided by Powell et al. (2001) share notation and methodology with our ADP algorithm. For an extensive review of ADP the reader is referred to Powell (2007), Powell and Van Roy (2004), Sutton and Barto (1998), and Bertsekas and Tsitsiklis (1996).

3. Problem Description

We consider the DSR scheduling problem of an ISO in a day-ahead market. By dispatching DSRs, the ISO can change the demand profile on the following day to flatten the demand profile and reshape it, such that it becomes less deep and steep (California ISO 2013). The reshaped demand profile is supplied using two types of energy resources, intermittent renewable generation capacity (wind and solar), and conventional generators (hydro-electric, gas, etc.). The capacity of the intermittent resources is determined by external factors and cannot be controlled by the ISO, but the ISO can control the capacity of the conventional generators.

Dispatching DSRs is costly, but it can help reducing the operating cost of the conventional generators (California ISO 2013). Our objective is to determine the dispatch schedule of the DSRs for the next operating day such that the sum of the DSR dispatch cost and the expected operating cost of the generators is minimized. The optimization is subject to a constraint that demand is met with a certain probability and to physical constraints of the conventional generators, such as minimum up and downtimes and ramp limits.

The output of our optimization is a DSR dispatch schedule for the next day and an operating policy for the conventional generators for the next day. The DSR dispatch schedule for the next day is sent to load aggregators. The load aggregators notify contracted loads who then adjust their load profile on the following day according to the accepted offers, e.g., by reducing air conditioning during dispatch and increasing it before and after dispatch (California ISO 2011). Because DSRs are scheduled a day before their actual dispatch, our model allows for participation of DSRs that do not qualify for emergency-triggered real time programs due to longer notification times.

The operating policy for the conventional generators states which actions should be taken in each period, given the state of the system at the beginning of the period. Note that we do not obtain an operating schedule that can be implemented because the actions taken in a period depend on the unknown realizations of random variables on the following day, such as the realizations of demands and renewable generation capacities.

We use a finite planning horizon that is divided into T periods. In our numerical experiments, we use a time period of one hour to reflect the typical resolution of energy system operations (Shahidehpour et al. 2003) and the total time horizon of one day. We denote the day-ahead forecast of the gross demand for energy in period t before the dispatch of DSRs by L_t^G and the forecast of the capacity of the intermittent resources in period t by P_t. At the time at which the model is solved, the quantities L_t^G and P_t are stochastic because we consider the day-ahead problem where only forecasts are available.

We can control the demand to be filled by conventional generators to a certain extent by scheduling DSRs. The dispatch of a DSR in period t reduces the demand in that period, but it can affect demand in other periods. If a DSR is scheduled for a demand reduction, it is notified as soon as day-ahead schedules are posted, i.e., on the day prior to the scheduled period. DSRs may reallocate demand to periods prior to or after the period in which the demand reduction is scheduled. The

ISO must anticipate this effect when determining the DSR dispatch schedule and the resulting demand profile (Black et al. 2008).

We denote the fraction of the demand that is shifted from period t to period t' by $\beta_{t,t'}$. Load can be shifted from period t to earlier ($t' < t$) and later ($t' > t$) periods. Value $\beta_{t,t'}$ is a random variable because it is not known with certainty how much load is shifted to which period when the DSR dispatch decision is made a day ahead of the actual load reduction. For a given DSR schedule, we denote the change in the demand for electric energy in period t that is due to the dispatch of DSRs by L_t^D. Note that L_t^D can be positive or negative. The residual demand that must be filled by conventional generators is $L_t = L_t^G - L_t^D - P_t$. Figure 1 shows an example for the residual demand without and with DSR dispatches. In the example, the expected peak demand around noon is reduced and some demand is shifted to morning and evening hours.

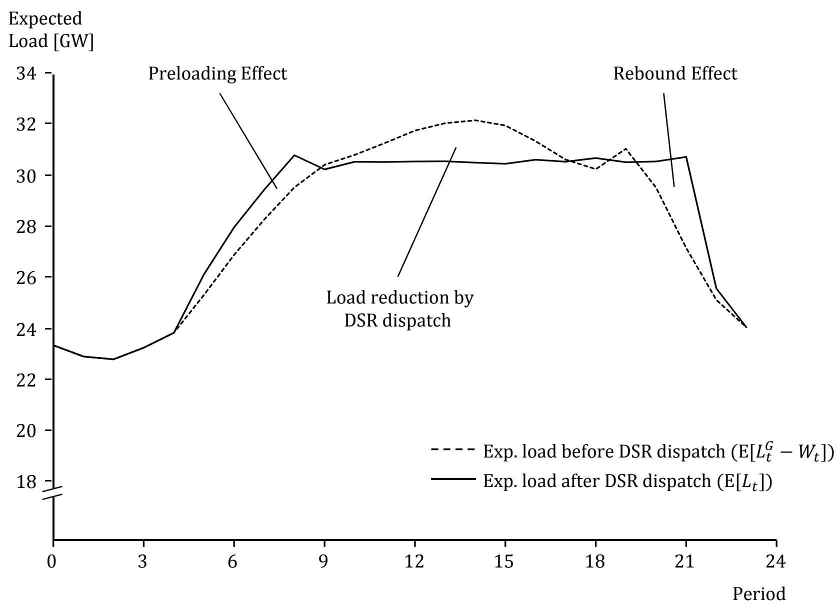
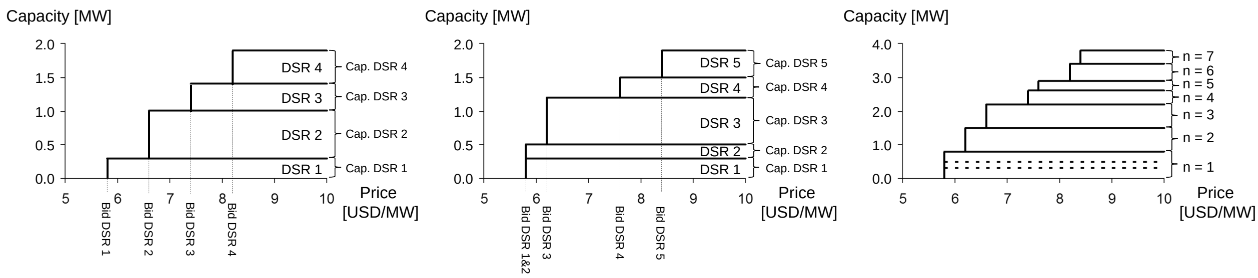


Figure 1 Effect of day-ahead scheduling of DSRs on expected load

To determine the optimal solution, we must trade-off the cost of the conventional generators and the cost of the DSRs. We denote the cost of conventional generators by C_t^C and the dispatch cost of DSRs by C^D. The cost of the conventional generators consists of start-up and actual generation cost. The cost of dispatching DSRs depends on the prices that resources are asking for load reductions. Each resource can offer load reduction capacity at different prices (California ISO 2011). By sorting and aggregating the offers of all resources ascending by price, we obtain an aggregate offer curve that shows the demand reduction as a function of the price. Figure 2



(a) Offer curve Aggregator 1 (b) Offer curve Aggregator 2 (c) Aggregate offer curve

Figure 2 Offer curves for two load aggregators and resulting aggregate offer curve

shows an example, where the aggregate offer curve consists of seven segments. Each segment can be accepted at any fraction, i.e., any load reduction between 0 MW and 4 MW can be chosen.

4. Model

We first develop individual models for stochastic unit commitment and DSRs and then combine them into an integrated model. In Subsection 4.1, we assume that the demand profile is given and develop a model for the stochastic unit commitment problem that determines the minimum expected operating cost of the generators. In Subsection 4.2, we develop a model for the DSRs that determines the cost of DSR dispatches. In Subsection 4.3, we combine both models to obtain an integrated model for determining an optimal DSR schedule, where we take the effect of DSR dispatches on the demand profile and the expected operating cost of the generators into account. This model also determines the optimal operating policy of the generators.

4.1. Stochastic Unit Commitment Model

The objective of the stochastic unit commitment problem is to determine a commitment and dispatch policy for the next-day operation of the generators such that the expected operating cost over the planning horizon is minimized. The stochastic unit commitment problem is a multi-stage stochastic decision problem and we model it as a stochastic dynamic program. For each period t of the next day, the optimal commitment and dispatch policy must be computed given the (stochastic) demand forecast L_t.

The sequence of events on the following day is as follows: At the beginning of time period t, we observe the state of the system, S_t, which includes the demand of period t, l_t[1]. Next, we make commitment and dispatch decisions x_t and determine the cost of the period. The cost depends on the state of the system at the beginning of the period and the decisions that we take in the period. At the end of the period, we update the state of the system.

[1] If the demand is realized after the state is observed, a slight adjustment to the model is needed, but all results and algorithms presented later still hold with minor modifications.

Unlike previous research (e.g., Papavasiliou and Oren 2013a,b, Takriti et al. 1996) we do not classify resources into slow resources which require commitment decisions on the previous day, and fast resources that can be committed in real time. Instead, we model notification times of generators for each generator individually, which allows to commit generators with medium response times (e.g., four hours) more economically.

We next explain the components of the dynamic program in detail and then introduce the optimality equation.

States. The system consists of a set of $\mathcal{I}$ conventional generators with different physical characteristics. The characteristics of a generator that are relevant for us are the current output level, the number of periods the generator has been committed (uptime), and the number of periods it has been uncommitted (downtime). For each generator i, we define vector $R_{t,i}$ with $1 + T_i^u + T_i^d$ coordinates, where T_i^u denotes the minimum uptime and T_i^d denotes the minimum downtime of the generator. The first coordinate is the (continuous) operating level of the generator. The next T_i^u coordinates are binary and are 1 if the generator has an uptime that corresponds to the time period of the underlying coordinate and 0 otherwise. If the uptime is equal to or exceeds the minimum uptime, then the last component of the T_i^u components is 1. The last T_i^d components are defined correspondingly for the downtime. For a generator i with minimum uptime of $T_i^u = 3$ periods and minimum downtime of $T_i^d = 2$ periods, $R_{t,i} = \{0.70, 0, 1, 0, 0, 0\}$ represents the state of the generator with current operating level of 0.70, an uptime of two periods, and a downtime of zero periods.

Our representation of the generator states uses more components of the state vector than alternative representations that use the number of periods that a generator has been online. However, for the algorithms that we use in Section 5 and the proofs in Subsection 5.4, the representation that we use is convenient and we therefore use it for our model.

We allow the demand to be correlated over time. Therefore, we have to keep track of the demands in previous periods and include them in our state. The full representation of the state of the system at the beginning of period t is $S_t = (R_t, l_0, \ldots, l_t)$. We denote the generator state space by $\mathcal{R}_t$, the state space of the demand process from periods 0 to t by $\mathcal{L}_t$, and the resulting state space by $\mathcal{S}_t = \mathcal{R}_t \times \mathcal{L}_t$.

Actions. Two types of actions are relevant, binary commitment actions (turning generators on or off) and continuous dispatching actions (changing the operating levels). We model the binary commitment actions of generator i in period t by binary decision variables $x_{t,i}^{\text{on}}$ and $x_{t,i}^{\text{off}}$. If generator i is committed in period t, then $x_{t,i}^{\text{on}} = 1$ and $x_{t,i}^{\text{on}} = 0$ otherwise. If generator i is decommitted in period t, then $x_{t,i}^{\text{off}} = 1$ and $x_{t,i}^{\text{off}} = 0$ otherwise. We model the dispatch action of generator i in period t by the continuous decision variable $x_{t,i}^d$, $0 \le x_{t,i}^d \le 1$. The decision variable is equal to 0 if the generator operates at the minimum output level and is equal to 1 if the generator operates at the maximum output level, i.e., it measures the percentage of the adjustable generator output. We denote the vector of all decision variables for generator i in period t by $x_{t,i} = (x^{\text{on}}_{t,i}, x^{\text{off}}_{t,i}, x^d_{t,i})$ and the vector of decision variables for all generators by x_t.

We capture physical restrictions of the generators, such as ramping limits, notification times, and minimum up and downtimes, by restricting the feasible set of actions in a period. We denote the set of feasible actions by $\mathcal{X}(S_t)$. Because the actions are restricted by physical constraints that depend on the state of the system, the set of feasible actions depends on the state S_t.

Transition Function. We denote the function that translates the state of the generators at the beginning of time period t and the action of period t to the state of the generators at the beginning of time period $t+1$ by $R^M(R_t, x_t) = AR_t + Bx_t$, where A and B are matrices of appropriate dimension. We provide an example of the function in Appendix EC.1.1. The transition function for the state variables is given by $S_{t+1} = S^M(S_t, x_t, L_{t+1}) = (R^M(R_t, x_t), l_0, \ldots, l_t, L_{t+1})$. Function $S^M(S_t, x_t, L_{t+1})$ translates the state of the system at the beginning of period t, S_t, actions taken at the beginning of period t, x_t, and the demand for period $t+1$, L_{t+1}, into the state at the beginning of time period $t+1$.

Objective Function. The objective function consists of generation cost $c^g_i(R_{t,i}, x^d_{t,i})$ and start-up cost $c^s_i(R_{t,i}, x^{\text{on}}_{t,i})$ (see Appendix EC.1.3 for details). Additional cost components, such as shut-down cost, could easily be incorporated in the model. Our modeling and solution approach does not rely on a specific form of the cost functions and we only require that the generation cost $c^g_i(R_{t,i}, x^d_{t,i})$ is convex in $x^d_{t,i}$ if the generator is committed and that it is zero if the generator is uncommitted. The operating cost of the generators in period t is $\sum_{i\in\mathcal{I}} c^g_i(R_{t,i}, x^d_{t,i}) + c^s_i(R_{t,i}, x^{\text{on}}_{t,i})$.

The power generation in period t is $\sum_{i\in\mathcal{I}} g_i(R_{t,i}, x^d_{t,i})$, where $g_i(R_{t,i}, x^d_{t,i})$ is the energy output of generator i given state $R_{t,i}$ and output level $x^d_{t,i}$ (see Appendix EC.1.3 for details). Our objective is to dispatch the generators, such that they meet the demand. However, demand L_t is stochastic and unless we make strong assumption about the joint distribution of $L_0, \ldots, L_{T-1}$ and the initial state S_0, we cannot guarantee that the system will always be able to match the demand.

To ensure that the problem has always a feasible solution, we introduce imbalance variable b_t, which measures the difference between generation and demand. If $b_t = 0$, then generation is equal to demand. If $b_t > 0$, then generation is greater than demand and we charge penalty cost c^+_t for each unit of excess generation. If $b_t < 0$, then generation is less than demand and we charge penalty cost c^-_t for each unit of excess demand. The penalty cost can be interpreted as the cost of emergency actions, such as load curtailment (Wood and Wollenberg 2006). In our examples, we use sufficiently high values for the penalty cost to make imbalances very rare exceptions.

By adding the relevant cost components, we obtain the following expression for the operating cost of the generators

$$C_t^C(R_t, x_t, b_t) = \sum_{i \in \mathcal{I}} \left(c_i^g(R_{t,i}, x_{t,i}^d) + c_i^s(R_{t,i}, x_{t,i}^{\text{on}}) \right) + c_t^+[b_t]^+ + c_t^-[-b_t]^+, \tag{1}$$

where the system imbalance is computed as $b_t = l_t - \sum_{i \in \mathcal{I}} g_i(R_{t,i}, x_{t,i}^d)$.

Operating Reserve. Operating reserves are used in unit commitment models to hedge against uncertainty in demand. Unit commitment models typically include two types of operating reserve capacity, spinning and non-spinning reserve capacity (Wood and Wollenberg 2006). Spinning reserve capacity is capacity provided by committed generators, as opposed to non-spinning reserve capacity, which is provided by uncommitted generators. Both share the requirement that they must be able to reach a desired output level within a short time interval, e.g., within ten minutes (Ellison et al. 2012). We include operating reserve requirements in the model by formulating a chance constraint that requires that the probability of not having sufficient generation capacity in period $t+1$ is less than ϵ: $\mathbf{P}\left\{ \sum_{i \in \mathcal{I}^{\text{reserve}}(R_t)} g_i^{\max}(R^M(R_{t,i}, x_{t,i})) \leq L_{t+1} \right\} \leq \epsilon$. This constraint captures both, spinning and non-spinning reserve, and can be translated into a deterministic constraint for known distributions of load and intermittent generation.

Function $g_i^{\max}(R_{t,i})$ denotes the maximum output of generator i when in state $R_{t,i}$ (see Appendix EC.1.3 for details). We incorporate random plant failures ($N-1$ condition) into the reserve constraint by summing over all generators excluding the committed generator with maximum currently committed capacity, i.e., by summing over $\mathcal{I}^{\text{reserve}}(R_t)$ that excludes the generator currently running with maximum capacity instead of $\mathcal{I}$ that includes all generators.

We note that most previous research on stochastic unit commitment problems uses scenario-based approaches (e.g., Papavasiliou and Oren 2013a,b, Takriti and Birge 2000, Takriti et al. 1996), where operating reserves are computed in the first stage of the problem and are implicitly defined by the scenario with the maximum load. Because we do not use scenarios, but the entire distribution of the demand, and model notification times exactly we do not rely on a maximum load scenario and use a chance constraint instead. The results are similar, but the approach is different because operating reserves can be adjusted over the day.

Dynamic Programming Model. We denote the optimal value function of the system in state S_t at time t by $V_t(S_t)$, which represents the expected cost-to-go from period t to the end of the planning horizon, if all subsequent actions are taken optimally. The dynamic programming optimality equation for the stochastic unit commitment problem is

$$V_t(S_t) = \min_{x_t \in \mathcal{X}(S_t)} \left\{ C_t^C(R_t, x_t, b_t) + \mathbb{E}_{L_{t+1}}\left[V_{t+1}\left(S^M(S_t, x_t, L_{t+1}) \right) \right] \right\}, \tag{2}$$

subject to

$$b_t = l_t - \sum_{i \in \mathcal{I}} g_i(R_{t,i}, x^d_{t,i}), \tag{3}$$

and

$$\mathbf{P}\left\{ \sum_{i \in \mathcal{I}^{\text{reserve}}(R_t)} g_i^{\max}(R^M(R_{t,i}, x_{t,i})) \leq L_{t+1} \right\} \leq \epsilon. \tag{4}$$

We provide the detailed formulation of the action space $\mathcal{X}_t(S_t)$ in Appendix EC.1.2. Note that the expected value in Equation (2) is defined with respect to the conditional distribution of L_{t+1} given the load history up to period t captured by state S_t.

The dynamic program determines the optimal commitment and dispatch policy for each period. The policy states which commitment and dispatch actions must be taken in each period given the current state of the system S_t for that period. For initial state S_0, the optimal total expected cost over the planning horizon is $V_0(S_0)$. Note that the value functions are non-convex due to the binary commitment decisions.

4.2. Demand Side Resources Model

We model DSRs as offer curves. An offer curve consists of N segments, each characterized by an offer price and an offer quantity. We allow segments to be cleared partially, i.e., an offer can be accepted at any fraction. We consider a single aggregate offer curve in each period and denote the offer price and quantity of segment n in period t by $c^D_{t,n}$ and $g^D_{t,n}$, respectively.

We use a continuous decision variable $z_{t,n} \in [0,1]$ to represent the dispatch decision for segment n in period t. Then, $z = (z_{t,n})_{0 \leq t \leq T-1, 0 \leq n \leq N-1}$ represents a full dispatch schedule for all DSRs. Total dispatch cost of schedule z is given by

$$C^D(z) = \sum_{t=0}^{T-1} \sum_{n=0}^{N-1} z_{t,n} c^D_{t,n}.$$

Scheduling and notifying a DSR in the day-ahead market results in lower expected demand in the period for which it is scheduled for dispatch and may increase expected demand in previous or subsequent periods. We model the fraction of demand of segment n that is shifted from period t to period t' by an arbitrarily distributed random variable $\beta_{t,t',n}$. Reducing expected demand in period t by $z_{t,n} g^D_{t,n}$ leads to an increase of expected demand in period t' of $z_{t,n} g^D_{t,n} \beta_{t,t',n}$. The sum of the shifted demand does not necessarily equal the dispatched capacity. We allow net energy conservation ($\sum_{t' \neq t} \beta_{t,t',n} < 1$) and net increase in expected demand for electric energy ($\sum_{t' \neq t} \beta_{t,t',n} > 1$) induced by dispatch of DSRs in t.

For dispatch schedule z, the expected reduction of demand in period t that is induced by dispatching DSRs is

$$L_t^D(z) = \sum_{n=0}^{N-1} z_{t,n} g_{t,n}^D - \sum_{t'=0}^{T-1} \beta_{t',t,n} \sum_{n=0}^{N-1} z_{t',n} g_{t',n}^D \tag{5}$$

and the residual demand that must be filled by generation by conventional generators is $L_t(z) = L_t^G - P_t - L_t^D(z)$.

4.3. Integrated Model

The stochastic unit commitment model in Subsection 4.1 determines the minimum expected operating cost of the generators for a given stochastic demand profile L_t, $t = 0, \ldots, T-1$. The DSR model in Subsection 4.2 determines the effect of DSR dispatch schedule z on the demand profile and the cost of DSR dispatch. To determine the optimal DSR schedule, we must take the into account effect of DSR dispatch on the DSR dispatching cost and on the expected operating cost of the generators.

Technically, we first replace the variable L_t by function $L_t(z)$ in Function (2) and in Constraints (3) and (4) of the dynamic program to obtain a dynamic program that determines the minimum expected operating cost of the generators for a given dispatch schedule z:

$$V_t(z, S_t) = \min_{x_t \in \mathcal{X}(S_t)} \left\{ C_t^C\left(R_t, x_t, b_t\right) + \mathbb{E}_{L_{t+1}(z)}\left[V_{t+1}\left(z, S^M(S_t, x_t, L_{t+1}(z))\right)\right]\right\}, \tag{6}$$

subject to

$$b_t = l_t(z) - \sum_{i \in \mathcal{I}} g_i(R_{t,i}, x_{t,i}^d), \tag{7}$$

and

$$\mathbf{P}\left\{ \sum_{i \in \mathcal{I}^{\text{reserve}}(R_t)} g_i^{\max}(R^M(R_{t,i}, x_{t,i})) \le L_{t+1}(z) \right\} \le \epsilon. \tag{8}$$

To find the DSR dispatch schedule that minimizes the total expected cost, we add the cost of the DSR schedule and the expected operating cost of the generators and solve the following integrated problem to determine the DSR schedule that minimizes the sum of both costs:

$$\min_z \left\{ C^D(z) + V_0(z, S_0) \right\} \tag{9}$$

subject to Constraints (7) and (8). State S_0 is the state of the system at the beginning of the planning horizon.

The integrated problem is solved a day ahead of the actual generation. The solution consists of an optimal DSR schedule z^* that minimizes expected cost and an optimal operating policy for the generators. The operating policy states the optimal actions for each period as a function of the initial state of the period. Because the demand realization of a period is only revealed at the beginning of the period, we do not obtain a specific schedule but a policy. An exception are slow generators that need to be notified long before their actual dispatch. For such generators, specific schedules might be available at least for the first few hours of operation.

5. Solution Approach

Solving our model is not straightforward because the DSR schedule affects the optimal operating schedule of the generators and vice versa. To solve the problem, we propose a novel solution approach that relies on an extension of the progressive hedging algorithm (Rockafellar and Wets 1991) and we combine it with approximate dynamic programming. Because this approach relies on approximations, we will generally not find the optimal solution. However, we can derive a lower bound on the optimal solution and quantify the maximum gap between the approximated and optimal solution.

In Subsection 5.1, we analyze the problem for a single deterministic realization of the random variables and determine the optimal DSR schedule using an approach that combines progressive hedging with dynamic programming. In Subsection 5.2, we extend the model to incorporate stochastic demand, intermittent generation, and load shifting. In Subsection 5.3, we use approximate dynamic programming to solve the stochastic version of the problem. In Subsection 5.4, we use the results to derive a lower bound on the optimal solution and in Subsection 5.5, we derive convergence results.

5.1. Solution Approach for a Single Demand Realization

Consider a DSR schedule z and a single realization of residual demand for all time periods $l_t(z)$, $t=1,\ldots,T-1$. This setting corresponds to a deterministic unit commitment problem with DSRs that we use later as a building block to solve the stochastic unit commitment problem with DSRs and to derive a lower bound on its optimal solution. To find the optimal DSR schedule z^* that minimizes the sum of the DSR dispatching and operating costs of the generators, we solve

$$z^* = \arg\min_z C^D(z) + V_0(z, S_0) \tag{10}$$

with

$$V_t(z,S_t) = \min_{x_t \in \mathcal{X}(S_t)} C_t^C\left(R_t, x_t, b_t\right) + V_{t+1}\left(z, S^M(S_t, x_t, l_{t+1}(z)\right), \tag{11}$$

where

$$\sum_{i\in\mathcal{I}} g_i(R_{t,i}, x_{t,i}^d) + b_t = l_t(z). \tag{12}$$

We optimize over DSR schedules z that state the fraction $z_{t,n}$ of the offer curve section n is dispatched in period t. One approach to solve Problem (10) is to discretize the decision variables $z_{t,n}$, include them into the state to capture the load shifts between periods, and to apply standard dynamic programming. However, to obtain a reasonably accurate solution, we would require a large state space and solving the resulting dynamic program would become computationally expensive even for the deterministic version of the problem.

Therefore, we take a different approach and use *progressive hedging*. With progressive hedging, we do not use a single DSR schedule z for all periods but introduce a local DSR schedule for each period. We denote the local DSR schedule of period t by $\tilde{z}^t$. The local schedule $\tilde{z}^t$ has the same dimension as the schedule z, i.e., it is a complete schedule for all periods. When we optimize the local DSR schedule of period t, we ignore the local DSR schedules of all other periods. A local schedule is locally optimal for a period, if it minimizes the generation cost for that particular period. For example, consider a problem with a single DSR bid per period and $T = 2$ periods, for which the local DSR schedules $\tilde{z}^1 = (1, 0)$ and $\tilde{z}^2 = (0, 0.8)$ are optimal. With this solution, the cost of period 1 is minimized by dispatching the DSR in period 1 at 100% ($\tilde{z}^1_{1,1} = 1$) and not dispatching it in period 2 ($\tilde{z}^1_{2,1} = 0$). The cost of period 2 is minimized by not dispatching the DSR in period 1 ($\tilde{z}^2_{1,2} = 0$) and dispatching it in period 2 at 80% ($\tilde{z}^2_{2,2} = 0.8$). Obviously, such a solution cannot be implemented because the two local schedules provide differing decisions for both periods. Progressive hedging iteratively synchronizes the local DSR schedules to take a common implementable value.

Using progressive hedging, we do not have to include DSR schedules in the state because we use a local DSR schedule for each period that is independent of the local DSR schedules of other periods. Formally, we solve

$$\left\{\tilde{z}^0, ..., \tilde{z}^{T-1}\right\} = \underset{\left\{\tilde{z}^0,...,\tilde{z}^{T-1}\right\}}{\arg\min} \; C^D\left(\tilde{z}^0\right) + V_0(\tilde{z}^0, \ldots, \tilde{z}^{T-1}, S_0) \tag{13}$$

with

$$V_t(\tilde{z}^t, ..., \tilde{z}^{T-1}, S_t) = \min_{x_t \in \mathcal{X}(S_t)} C^C_t\left(R_t, x_t, b_t\right) + V_{t+1}\left(\tilde{z}^{t+1}, ..., \tilde{z}^{T-1}, S^M(S_t, x_t, l_{t+1}(\tilde{z}^t)\right), \tag{14}$$

where

$$\sum_{i \in \mathcal{I}} g_i(R_{t,i}, x^d_{t,i}) + b_t = l_t(\tilde{z}^t). \tag{15}$$

The result of the optimization is a set of locally optimal DSR schedules $\{\tilde{z}^0, ..., \tilde{z}^{T-1}\}$. Adding the constraint

$$z = \tilde{z}^0 = \tilde{z}^1 = ... = \tilde{z}^{T-1}, \tag{16}$$

to the problem would suffice to obtain a global DSR schedule, but a dynamic program with this constraint would be difficult so solve.

In Appendix EC.2, Proposition EC.1, we show that solving Problem (13) subject to Constraints (14) - (16) is the same as solving Problem (10) subject to Constraints (11) and (12).

As it is unclear how to handle Constraint (16) explicitly in a dynamic program, we dualize it and formulate the augmented Lagrangian of the problem:

$$\left\{z, \tilde{z}^0, ..., \tilde{z}^{T-1}\right\} = \underset{\left\{z, \tilde{z}^0, ..., \tilde{z}^{T-1}\right\}}{\arg\min} \max_{w} C^D\left(\tilde{z}^0\right) + V_0(\tilde{z}^0, ..., \tilde{z}^{T-1}, S_0) + \frac{1}{2\alpha}\sum_t \|z - \tilde{z}^t\|^2 + \sum_t w^t \tilde{z}^t. \quad (17)$$

The terms that were added penalize deviations of the locally optimal schedules from the global DSR schedule. The term $\frac{1}{2\alpha}\sum_t \|z - \tilde{z}^t\|^2$ penalizes deviations of the locally optimal schedules from the global DSR schedule. The term $\sum_t w^t \tilde{z}^t$ contains individual penalty terms w^t for each $\tilde{z}^t$, where the w^t are ordinary Lagrangian multipliers. Unfortunately, we cannot solve Equation (17) directly because the quadratic term links all $\tilde{z}^t$ (as did Constraint (16)) and prevents us from calculating the optimal value functions in Equation (14).

However, we can solve Equation (17) subject to Equations (14) and (15) by progressive hedging. In iteration $k = 0$, we set $z^k_{t,n} = 0$ for all t and n, $w^{t,k} = 0$ for all t and, given z^k_t and $w^{t,k}$, we find the locally optimal solutions $\tilde{z}^{t,k}$ by solving

$$\left\{\tilde{z}^{0,k}, ..., \tilde{z}^{T-1,k}\right\} = \underset{\left\{\tilde{z}^0, ..., \tilde{z}^{T-1}\right\}}{\arg\min} C^D\left(\tilde{z}^0\right) + V_0(\tilde{z}^0, ..., \tilde{z}^{T-1}, S_0) + \frac{1}{2\alpha^k}\sum_t \|z^k - \tilde{z}^t\|^2 + \sum_t w^{t,k}\tilde{z}^t \quad (18)$$

subject to Equations (14) and (15). We can find the solution by first calculating the optimal value functions

$$\bar{V}_t(z^k, w^k, S_t) = \min_{\tilde{z}^t, x_t \in \mathcal{X}(S_t)} C^C_t\left(R_t, x_t, b_t\right) + w^{t,k}\tilde{z}^t + \frac{1}{2\alpha^k}\|z^k - \tilde{z}^t\|^2 + \bar{V}_{t+1}\left(z^k, w^k, S^M(S_t, x_t, l_{t+1}(\tilde{z}^t))\right) \quad (19)$$

subject to Equation (15) by backward dynamic programming and then identifying the local DSR schedules $\tilde{z}^{t,k}$ that are optimal given z^k and w^k, with a forward path. In EC.2, Proposition EC.2, we show that calculating Equation (19) for all t allows us to obtain an optimal solution to Problem (18).

Using the solutions $\tilde{z}^{t,k}$ to Equation (19), we update z^k by $z^{k+1} = \frac{1}{T}\sum_{t=0}^{T-1}\tilde{z}^{t,k}$ and $w^{t,k}$ by $w^{t,k+1} = w^{t,k} + \frac{1}{\alpha^k}(\tilde{z}^{t,k} - \frac{1}{T}\sum_{t=0}^{T-1}\tilde{z}^{t,k})$. Then, we choose a new step size α^k, and repeat the procedure with the updated values. We iterate until all locally optimal solutions $\tilde{z}^t$ converge to a common value, i.e., to the desired global schedule z.

The choice for the sequence of step sizes α^k is crucial for the behavior of the algorithm. In Subsection 5.5, we analyze the properties of the procedure and rules for choosing α^k that guarantee convergence of the algorithm.

Algorithm 1 Solution Algorithm

Initialize: $k=0,\ z^0=0,\ \tilde{z}^{t,0}=0,\ w^{t,0}=0,\ t=1,\dots,t.$

Choose $\alpha^0>0$.

while stopping criterion is not met **do**

  for all $t=T-1,\dots,1$ **do**

    Compute

$$\bar{V}_t(z^k,w^k,S_t)=\min_{\tilde{z}^t,x_t\in\mathcal{X}(S_t)} C_t^C\left(R_t,x_t,b_t\right)+w^{t,k}\tilde{z}^t+\frac{1}{2\alpha^k}\|z^k-\tilde{z}^t\|^2 + \mathbb{E}_{L_{t+1}(\tilde{z}^t)}\left[\bar{V}_{t+1}\left(z^k,w^k,S^M(S_t,x_t,L_{t+1}(\tilde{z}^t))\right)\right].$$

    subject to Equations (15) and (21).

  end for

  Generate samples $l_t^{G,k}$, p_t^k, and β_t^k, $t=1,\dots,t$.

  for all $t=0,\dots,T-1$ **do**

    Compute

$$\tilde{z}^{t,k+1}=\arg\min_{\tilde{z}^t}\min_{x_t\in\mathcal{X}(S_t)} C_t^C\left(R_t,x_t,b_t\right)+w^{t,k}\tilde{z}^t+\frac{1}{2\alpha^k}\|z^k-\tilde{z}^t\|^2+\mathbb{E}_{L_{t+1}(\tilde{z}^t)}\bar{V}_{t+1}\left(z^k,w^k,S^M(S_t,x_t,l_{t+1}^k(\tilde{z}^t))\right)$$

    subject to $\sum_{i\in\mathcal{I}} g_i(R_{t,i},x_{t,i}^d)+b_t=l_t^{G,k}-p_t^k-\sum_{n=0}^{N-1}\tilde{z}_{t,n}^t g_{t,n}^D-\sum_{t'=0}^{T-1}\beta_{t',t,n}^k\sum_{n=0}^{N-1}\tilde{z}_{t',n}^t g_{t',n}^D$ and Eq. (24).

  end for

  Set $z^{k+1}=\frac{1}{T}\sum_t\tilde{z}^{t,k+1}$ and $w^{t,k+1}=w^{t,k}+\frac{1}{\alpha^k}(\tilde{z}^{t,k+1}-z^{k+1})$ for all $t=0,\dots,T-1$.

  Choose α^k and set $k=k+1$.

end while

Return z^k.

5.2. Stochastic Demand and Stochastic Load Shifting

In the previous section, we showed how to solve a deterministic version of the problem. However, demand, intermittent generation, and load shifting are stochastic and we next extend the approach to this setting. Under stochastic demand, generation, and stochastic load shifting, we search for the DSR schedule z that minimizes expected cost. To solve the problem, we again use progressive hedging, which we extend to a stochastic version that incorporates sampling of the random vectors. Algorithm 1 shows the pseudo code of the algorithm.

At the beginning of period t, the demand of period t is given by state S_t but the demands of the following periods are stochastic. We must solve

$$\bar{V}_t(z^k,w^k,S_t)=\min_{\tilde{z}^t,x_t\in\mathcal{X}(S_t)} C_t^C\left(R_t,x_t,b_t\right)+w^{t,k}\tilde{z}^t+\frac{1}{2\alpha^k}\|z^k-\tilde{z}^t\|^2 + \mathbb{E}_{L_{t+1}(\tilde{z}^t)}\left[\bar{V}_{t+1}\left(z^k,w^k,S^M(S_t,x_t,L_{t+1}(\tilde{z}^t))\right)\right] \tag{20}$$

for all S_t subject to Equation (15) and

$$\mathbf{P}\left\{\sum_{i\in\mathcal{I}^{\text{reserve}}(R_t)} g_i^{\max}(R^M(R_{t,i},x_{t,i}))\le L_{t+1}(\tilde{z}^t)\,\middle|\, l_0,\dots,l_t\right\}\le\epsilon. \tag{21}$$

We solve the model by backwards dynamic programming. The solution is a policy that states how $\tilde{z}^t$ must be chosen for given realizations of the gross demand, intermittent generation, and load shift factors. Therefore, the set of optimal local DSR schedules $(\tilde{z}^0, ..., \tilde{z}^{T-1})$ depends on the realizations of these stochastic quantities.

The progressive hedging algorithm needs a single set of local DSR schedules in each iteration that we generate by drawing realizations of the gross demand, $l^{G,k}$, the intermittent capacity, p_t^k, and the load shift factors β_t^k from the corresponding distributions. Then, we find the optimal set of local DSR schedules, given z^k and w^k, with a forward path through

$$\begin{aligned}\tilde{z}^{t,k} = \arg\min_{\tilde{z}^t} \min_{x_t \in \mathcal{X}(S_t)} & C_t^C\left(R_t, x_t, b_t\right) + w^{t,k}\tilde{z}^t + \frac{1}{2\alpha^k}\|z^k - \tilde{z}^t\|^2 \\ & + \mathbb{E}_{L_{t+1}(\tilde{z}^t)}\left[\bar{V}_{t+1}\left(z^k, w^k, S^M(S_t, x_t, L_{t+1}(\tilde{z}^t))\right)\right]\end{aligned} \tag{22}$$

subject to

$$\sum_{i\in\mathcal{I}} g_i(R_{t,i}, x^d_{t,i}) + b_t = l_t^k(\tilde{z}^t). \tag{23}$$

and

$$\mathbf{P}\left\{\sum_{i\in\mathcal{I}^{\text{reserve}}(R_t)} g_i^{\max}(R^M(R_{t,i}, x_{t,i})) \leq L_{t+1}(\tilde{z}^t)\middle| l_0^k, \ldots, l_t^k\right\} \leq \epsilon. \tag{24}$$

The solution approach for the stochastic model is similar to the solution approach for the deterministic model but we solve a stochastic dynamic program as opposed to a deterministic dynamic program.

The outer loop updates z^k and $w^{t,k}$ based on samples of the random variables. It can be interpreted as an extension of the known progressive hedging algorithm to a stochastic version that minimizes the expected value of a function involving random variables. In Subsection 5.5, we formally introduce our extension and formulate conditions under which the algorithms converges to an optimal solution. Next, we present an approximate dynamic programming approach that allows us to find good solutions in a reasonable time.

5.3. Approximation of the Stochastic Dynamic Program

The exact solution of the dynamic program (Equation 20) is computationally intractable for realistic problem sizes and we must rely on alternative solution approaches.

We solve the dynamic program using approximate dynamic programming (Powell 2007) and replace the exact value function by an approximation of sufficiently low dimensionality whose parameters we estimate iteratively. We discretize the continuous operating levels of the generators and the demand history in the state variable. We denote the discretized generator state space by $\hat{\mathcal{R}}_i$ and the discretized space of demand history intervals by $\hat{\mathcal{L}}_t$.

Algorithm 2 ADP Algorithm for the Stochastic Unit Commitment Problem with DSRs

Set $k=0$, $\tilde{z}^{t,k}=0$, $w^{t,k}=0$, and $\bar{c}_t^{g,\max,k}=0$ for all t, and $z^k=0$.
Set $v_{t,i}^{\hat{l},\hat{R},k}=0$, for all $i\in\mathcal{I}$, $\bar{c}_t^{g,\max,k}=0$, $\hat{R}\in\hat{\mathcal{R}}$, $\hat{l}\in\hat{\mathcal{L}}_t$, and t.
Choose arbitrary $\alpha^k>0$.
Set $S_0^k=S_0$, for all $k\geq 0$.
while stopping criterion is not met **do**
  for all $t=0,\dots,T-1$ **do**
    Generate samples β^k, $l_t^{G,k}$, and p_t^k for all t.
    Compute

$$(x_t^k,\tilde{z}^{t,k})=\arg\min_{x_t,\tilde{z}^t} C_t^C(R_t,x_t,b_t)+w^{t,k}\tilde{z}^t+\frac{1}{2\alpha^k}\|z^k-\tilde{z}^t\|^2+\hat{V}_{t+1}\left(S^M(S_t,x_t,l_{t+1}^k(\tilde{z}^t))\right)$$

    subject to $\sum_{i\in\mathcal{I}} g_i(R_{t,i},x_{t,i}^d)+b_t=l_t^{G,k}-p_t^k-\sum_{n=0}^{N-1}\tilde{z}_{t,n}^t g_{t,n}^D+\sum_{t'=0}^{T-1}\beta_{t',t,n}^k\sum_{n=0}^{N-1}\tilde{z}_{t',n}^t g_{t',n}^D$ and Eq. (24).
    Set $S_{t+1}^k=S^M(S_t^k,x_t^k,l_{t+1}^k(\tilde{z}^{t,k}))$.
    Store maximum average generation cost $c_t^{g,\max}$.
  end for
  for all $t=T-1,\dots,0$ **do**
    Set $\bar{c}_t^{g,\max,k+1}=(1-\alpha^k)\bar{c}_t^{g,\max,k}+\alpha^k c_t^{g,\max}$.
    for all $i\in\mathcal{I}$ and $\hat{R}\in\hat{\mathcal{R}}$ **do**
      Compute $\Delta(\hat{R}_{t,i})$.
      Set $v_{t,i}^{l_0^k,\dots,l_t^k,\hat{R}_i,k+1}=(1-\alpha^k)v_{t,i}^{l_0^k,\dots,l_t^k,\hat{R}_i,k}+\alpha^k\Delta(\hat{R}_{t,i})$.
    end for
  end for
  Set $z^{k+1}=\frac{1}{T}\sum_t\tilde{z}^{t,k+1}$, $w^{t,k+1}=w^{t,k}+\frac{1}{\alpha^k}(\tilde{z}^{t,k+1}-z^{k+1})$.
  Choose α^k and set $k=k+1$.
end while
Return z^k.

We choose an additively separable form of the value function that is separable by generator, which is a natural approximation for weakly coupled dynamic programs (Adelman and Mersereau 2008) such as the unit commitment problem. For each generator $i\in\mathcal{I}$, each discretized demand history $(\hat{l}_0,...,\hat{l}_t)\in\hat{\mathcal{L}}_t$, and each operating state $\hat{R}_i\in\hat{\mathcal{R}}_i$, we define a parameter $v_{t,i}^{\hat{l}_0,\dots,\hat{l}_t,\hat{R}_i}$ that represents an estimate of the value that generator i has if its current operating state is $\hat{R}_i$ and if we have observed a demand history that falls in the intervals $(\hat{l}_0,...,\hat{l}_t)$. We replace the exact value function by an additively separable linear approximation

$$\hat{V}_t(z,w,S_t)=\sum_i\hat{V}_{t,i}(z,w,\hat{R}_{t,i},\hat{l}_0,...,\hat{l}_t), \tag{25}$$

where

$$\hat{V}_{t,i}(z,w,\hat{R}_{t,i},\hat{l}_0,...,\hat{l}_t)=\sum_{\hat{R}_i\in\hat{\mathcal{R}}_i}\hat{R}_i v_{t,i}^{\hat{l}_0,\dots,\hat{l}_t,\hat{R}_i}(z,w). \tag{26}$$

In the following we drop the dependency of $\hat{V}$ on z and w, because we store only a single set of $v_{t,i}^{\bar{l}_0,\dots,\bar{l}_t,\hat{R}_i}$ that does not depend on z and w. We update this vector in each iteration with solution information based on new values of w^k and z^k that are computed by the algorithm as explained below.

Algorithm 2 shows the pseudo code of the algorithm. After initialization, we sample the demand, intermittent capacity, and load shift factors in step 7. Then, in the loop consisting of steps 6-11 we step forward in time and solve the single period problems based on the current approximation of the value functions of future time periods. Finally, in steps 12-18 we update the value function approximation and continue the procedure until a stopping criterion is met.

The initialization, demand sampling, and forward optimization steps are standard implementations of approximate dynamic programming (Powell 2007). The update step to estimate parameters v of the value function is specific to the unit commitment problem and we use the following approach. In the forward path through the dynamic program, we compute the maximum average energy generation cost $c_t^{g,\max}$ of each period, i.e., for each committed generator in a period, we calculate the average generation cost by dividing the current output by total generation cost of the generator and select the highest value. We compute

$$c_t^{g,\max,k} = \max_{i \in \mathcal{I}^U} \frac{c_i^g(\hat{R}_{t,i}^k)}{g_i(\hat{R}_{t,i}^k)},$$

where $\mathcal{I}^U$ is the set of all running generators that have reached their minimum uptime. We only consider generators that have reached their minimum uptime because only those generators could in principle be decommitted in this period. We use the value of $c_t^{g,\max}$ to update the estimate of the expected maximum average generation cost in each period by $\bar{c}_t^{g,\max,k+1} = (1-\alpha^k)\bar{c}_t^{g,\max,k} + \alpha^k c_t^{g,\max}$.

Then, going backwards from period $T-1$, we update $v_{t,i}^{\hat{l}_0^k,\dots,\hat{l}_t^k,\hat{R}_i,k}$ for the observed demand history for all $\hat{R}_i \in \hat{\mathcal{R}}_i$, $i \in \mathcal{I}$, and t. For all possible operating states of each generator, we calculate an estimate of the value of being in that operating state. For operating states in which the generator is uncommitted this value is obtained by solving $\min_{x_i \in \mathcal{X}_i(\hat{R}_{t,i})} \Big\{ c_i^s(\hat{R}_{t,i}, x_{t,i}^{\text{on}}) + \hat{V}_{t+1,i}(\hat{R}_{t+1,i}, l_0^k, \dots, l_{t+1}^k) \Big\}$. For operating states in which the generator is running we additionally estimate the savings achieved in this operating state compared to the expected maximum average generation cost in that period. Formally, we compute

$$\Delta(\hat{R}_{t,i}) = \min_{x_i \in \mathcal{X}_i(\hat{R}_{t,i})} \Big\{ c_i^s(\hat{R}_{t,i}, x_{t,i}^{\text{on}}) + \hat{V}_{t+1,i}(\hat{R}_{t+1,i}, l_0^k, \dots, l_{t+1}^k) \Big\} + \begin{cases} g_i(\hat{R}_{t,i}) \left(\frac{c_i^g(\hat{R}_{t,i})}{g_i(\hat{R}_{t,i})} - \bar{c}_t^{g,\max,k+1} \right) & \text{if generator is committed in } \hat{R}_{t,i} \\ 0 & \text{otherwise,} \end{cases}$$

and use $\Delta(\hat{R}_{t,i})$ to update $v_{t,i}^{\hat{l}_0^k,\dots,\hat{l}_t^k,\hat{R}_i,k}$ by the rule $v_{t,i}^{\hat{l}_0^k,\dots,\hat{l}_t^k,\hat{R}_i,k+1} = (1-\alpha^k) v_{t,i}^{\hat{l}_0^k,\dots,\hat{l}_t^k,\hat{R}_i,k} + \alpha^k \Delta(\hat{R}_{t,i})$ and proceed to period $t-1$.

After updating the parameters of the approximation in all periods, we complete iteration k by updating the DSR schedule z^k and multipliers $w^{t,k}$, as introduced in the previous subsection, choosing a new step size α^{k+1}, and incrementing the iteration count.

We continue the procedure until the change in the DSR schedule z^k from one iteration to the next falls below a predefined tolerance and use the last value of z^k as the solution to Problem (9).

5.4. Lower Bound on the Optimal Solution

Our solution approach uses two approximations. We apply the stochastic sample-based progressive hedging algorithm to a non-convex function (Problem (9) is not convex) and we use ADP. Therefore, the solution is not necessarily optimal, but we can obtain an upper bound on the optimal expected cost by simulating the operating policy using Monte-Carlo-Sampling. With existing solution approaches, an optimal solution cannot be computed for problems of relevant size and we cannot evaluate the solution obtained from our algorithm by comparing it to an optimal solution. However, we can compute a lower bound on the optimal solution and use the lower bound as a benchmark. To compute the lower bound, we solve a sequence of relaxed problems based on samples from the distribution of net demand $\mathcal{L}_t$. We show that the sequence of values obtained from these relaxed problems converges to a lower bound on Problem (9).

Proposition 1 provides a convexity result for Problem (9) with relaxed integrality constraints. We use this relaxation as the starting point for the lower bound.

PROPOSITION 1. *The continuous relaxation of the unit commitment problem with DSRs is convex, if $\phi_t^{-1}(\cdot, z)$ is convex in z for $t = 0, \ldots, T-1$, where $\phi_t(\cdot, z)$ denotes the cumulative distribution function of $L_t(z)$.*

Even for the continuous relaxation, it is unclear how the expected value in the dynamic programming recursion can be evaluated. We use the procedure in Theorem 2 that we introduce later in Subsection 5.5 to solve the continuous relaxation of Problem (9) iteratively for z^* by generating samples $(l_0, \ldots, l_{T-1})$ and solving a deterministic unit commitment problem in each iteration. This is equivalent to relaxing the non-anticipativity condition inherent in the dynamic program and yields a lower bound on the optimal cost. We take the expectation over all $L_t(z)$ after minimizing over x to obtain the following lower bound on the unit commitment problem with DSRs.

THEOREM 1. *Problem*

$$\min_z \left\{ C^D(z) + \mathbb{E}_{L_0(z),\ldots,L_{T-1}(z)} \left[\min_{\bar{x} \in \bar{\mathcal{X}}} \sum_{t=0}^{T-1} C_t^C(R_t, \bar{x}_t, b_t) \right] \right\} \tag{27}$$

where

$$\begin{aligned}\bar{\mathcal{X}} = \bar{\mathcal{X}}(z, R_0) = \Bigg\{ &\bar{x} \in [0,1]^{3\times T\times|\mathcal{I}|} : \bar{x} \in \mathcal{X}(S_t), R_{t+1} = R^M(R_t, \bar{x}_t), \\ &\sum_{i\in\mathcal{I}} g_i(R_{t,i}, \bar{x}^d_{t,i}) + b_t = L_t(z), \\ &\mathbf{P}\left\{ \sum_{i\in\mathcal{I}^{reserve}(R_t)} g_i^{max}(R^M(R_{t,i}, \bar{x}_{t,i})) \le L_{t+1}(z) \right\} \le \epsilon, t = 0, \dots, T-1 \Bigg\},\end{aligned} \tag{28}$$

is a lower bound on the stochastic unit commitment problem with DSRs.

We apply the procedure in Theorem 2 to Problem (27) and obtain a sequence of DSR schedules z^k. The following corollary provides convexity of the lower bounding problem and convergence of the obtained sequence z^k to the optimal solution of Problem (27).

Corollary 1. *Problem* (27) *is a lower bound on Problem* (9) *and satisfies all conditions of Theorem 2, if* $\phi_t^{-1}(\cdot, z)$ *is convex in* z *for* $t = 0, \dots, T-1$. *The limit point* z^* *of the sequence* z^k *generated by the procedure in Theorem 2 solves Problem* (27).

While z^* is the optimal solution to Problem (27), we must evaluate the expected value in the objective function to obtain the actual value of the lower bound. We can dualize the constraint $\sum_{i\in\mathcal{I}} g_i(R_{t,i}, x^d_{t,i}) + b_t = L_t(z)$ to obtain a computationally tractable bound on the expected value. Proposition 2 presents the resulting relation.

Proposition 2. *For every* z *and* $\lambda \in \mathbb{R}^T$

$$\begin{aligned}\mathbb{E}_{L_0,\dots,L_T}\left[\min_{\bar{x}\in\bar{\mathcal{X}}} \sum_{t=0}^{T-1} C_t^C(R_t, \bar{x}_t, b_t)\right] \ge \\ \sum_{t=0}^{T-1} \lambda_t \mathbb{E}\left[L_t(z)\right] + \min_{\hat{x}\in\hat{\mathcal{X}}} \left\{ \sum_{t=0}^{T-1} \left(C_t^C(R_t, \hat{x}_t, b_t) - \lambda_t \left(\sum_{i\in\mathcal{I}} g_i(R_{t,i}, \hat{x}^d_{t,i}) + b_t \right)\right)\right\}\end{aligned} \tag{29}$$

holds, where $\hat{\mathcal{X}}$ *is the set obtained when removing the dualized constraint from the definition of* $\bar{\mathcal{X}}$.

Inequality (29) holds for any z and λ. To obtain a tight lower bound, we maximize the right hand side of Inequality (29) for the value z^* obtained from solving Problem (27). This maximization can be performed by subgradient descent and no expected value over a function must be evaluated to obtain a valid lower bound on Problem (9). We report performance results of the lower bound in Section 6.

5.5. Stochastic Proximal Point and Stochastic Sample-Based Progressive Hedging Algorithm

In this section, we formally introduce the stochastic proximal point algorithm and the stochastic sample-based progressive hedging algorithm and prove conditions under which both algorithms converge. We start with the stochastic proximal point algorithm, which is an extension of the proximal point algorithm (Rockafellar 1976, Martinet 1970) to stochastic programming problems. Its objective is to find $(v^*, w^*) \in \text{argminimax}_{v,w} \mathbb{E}[l(v,w,\omega)]$, where $l(v,w,\omega): \mathcal{N} \times \mathcal{M} \times \Omega \to \mathbb{R}, \mathcal{N} \subseteq \mathbb{R}^n, \mathcal{M} \subseteq \mathbb{R}^m$ is a saddle-function, i.e., l is convex in v for all w and concave in w for all v, and $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space. Our extended version finds $(w^*, v^*) = \text{argminimax}_{v,w} \mathbb{E}[l(v,w,\omega)]$ based on drawing samples $\omega \in \Omega$, whereas the existing deterministic version finds $(w^*, v^*) = \text{argminimax}_{v,w} l(v,w,\omega)$ for constant ω.

To show that the algorithm converges to an optimal solution (w^*, v^*) under mild conditions, we need the following assumption:

Assumption 1. *There exists a constant $c \in \mathbb{R}$, such that for all $v, \bar{v} \in \mathcal{N}$, $w, \bar{w} \in \mathcal{M}$, and $\omega \in \Omega$*

$$\max_{g \in \partial_v l(v,w,\omega)} \{\|g\|\} \le c,$$

$$\max_{g \in \partial_w l(v,w,\omega)} \{\|g\|\} \le c$$

and

$$|l(v,w,\omega) - l(\bar{v},w,\omega)| \le c\|v - \bar{v}\|,$$

$$|l(v,w,\omega) - l(v,\bar{w},\omega)| \le c\|w - \bar{w}\|.$$

Also there exist $M(\omega)$ such that $|l(v,w,\omega)| \le M(\omega)$ for every $\omega \in \Omega$, $v \in \mathcal{N}$ and $w \in \mathcal{M}$, and $\mathbb{E}[M(\omega)] < \infty$.

The following known result is the basis for the convergence proofs of many stochastic optimization algorithms and we also rely on it (Bertsekas and Tsitsiklis 1996, Prop. 4.2).

Proposition 3. *Let Y^k, Z^k, and W^k, $k = 0, 1, \ldots,$ be three sequences of random variables and let $\mathcal{F}^k$, $k = 0, 1, \ldots,$ be sets of random variables such that $\mathcal{F}^k \subset \mathcal{F}^{k+1}$ for all k. Suppose that:*

1. *The random variables Y^k, Z^k, and W^k are nonnegative, and are functions of the random variables in $\mathcal{F}^k$.*
2. *For each k, we have*

$$\mathbb{E}\{Y^{k+1} | \mathcal{F}^k\} \le Y^k - Z^k + W^k.$$

3. *There holds $\sum_{k=0}^{\infty} W^k < \infty$.*

Then we have $\sum_{k=0}^{\infty} Z^k < \infty$, and the sequence Y^k converges to a nonnegative random variable Y almost surely.

We now state our main result for the stochastic proximal point algorithm for saddle-functions.

THEOREM 2 **(Stochastic Proximal Point Algorithm)**. *Let $l(v,w,\omega):\mathcal{N}\times\mathcal{M}\times\Omega\to\mathbb{R}$ be convex in v, concave in w for every $\omega\in\Omega$, let $\mathcal{N}\subset\mathbb{R}^n,\mathcal{M}\subset\mathbb{R}^m$ be compact convex sets, and let $(\Omega,\mathcal{F},\mathbb{P})$ be a probability space. Additionally, let l be lower semi-continuous on $\mathcal{N}$ for every $w\in\mathcal{M}$ and let l be upper semi-continuous on $\mathcal{M}$ for every $v\in\mathcal{N}$. Assume there exists a point (v^*,w^*) such that $\mathbb{E}[l(v^*,w,\omega)]\le\mathbb{E}[l(v^*,w^*,\omega)]\le\mathbb{E}[l(v,w^*,\omega)]$ for all $(v,w)\in\mathcal{N}\times\mathcal{M}$.*

Let $k=0$, $v^0\in\mathcal{N}$, $w^0\in\mathcal{M}$ and consider the following procedure:

1. *Sample $\omega^k\in\Omega$.*
2. *Solve*

$$(v^{k+1},w^{k+1})\in \underset{\hat{v}\in\mathcal{N},\hat{w}\in\mathcal{M}}{\mathit{argminimax}}\left\{l(\hat{v},\hat{w},\omega^k)+\frac{1}{2\alpha^k}\|\hat{v}-v^k\|^2-\frac{1}{2\alpha^k}\|\hat{w}-w^k\|^2\right\}.$$

3. *Set $k=k+1$, choose α^k, and go to step 1.*

If the step size α^k satisfies

$$\sum_{k=0}^{\infty}\alpha^k=\infty,\qquad \sum_{k=0}^{\infty}(\alpha^k)^2<\infty,$$

then the sequence $\{v^k,w^k\}$ produced by the procedure converges almost surely to a point $(v^*,w^*)\in\mathcal{N}\times\mathcal{M}$, such that $\mathbb{E}[l(v^*,w,\omega)]\le\mathbb{E}[l(v^*,w^*,\omega)]\le\mathbb{E}[l(v,w^*,\omega)]$ for all $(v,w)\in\mathcal{N}\times\mathcal{M}$ and

$$\liminf_{k\to\infty}\mathbb{E}[l(v^k,w^*,\omega)]-\mathbb{E}[l(v^*,w^k,\omega)]=0.$$

Theorem 2 states that under mild conditions on the saddle-function l and the sequence of step sizes α^k, the stochastic proximal point algorithm finds a saddle-point (v^*,w^*), such that $\mathbb{E}[l(v^*,w,\omega)]\le\mathbb{E}[l(v^*,w^*,\omega)]\le\mathbb{E}[l(v,w^*,\omega)]$ for all $(v,w)\in\mathcal{N}\times\mathcal{M}$. The proof combines the convergence proof of the deterministic algorithm (Rockafellar 1976) and Proposition 3 to show that $Z^k=\alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k])$ is a martingale. This provides convergence of the sequence $\{\|v^k-v^*\|+\|w^k-w^*\|\}_{k\ge0}$, and it can be shown that $\lim_{k\to\infty}\|v^k-\bar{v}\|+\|w^k-\bar{w}\|$ exists for some $(\bar{v},\bar{w})$ with $\mathbb{E}[l(\bar{v},\bar{w},\omega)]=\mathbb{E}[l(v^*,w^*,\omega)]$.

Note that a convex function is a saddle-function and thus all results that hold for saddle-functions include convex functions as a special case. We used Theorem 2 in Subsection 5.4 to calculate the lower bound on our solution, where l corresponds to the objective function of Problem (27), which is convex, v corresponded to the dispatch schedule z, and ω corresponded to the random quantities in the problem, i.e., the load shifting factors β and the residual load L_t^G.

The optimization in Step 2 of Theorem 2 does not require saddle-function l to take a special structure other than the conditions stated in Assumption 1 and Theorem 2. In our application, however, we can exploit the special structure of the problem to decompose the optimization problem in Step 2 into a number of separate optimization problems.

To exploit this structure, we extend the procedure to a decomposable version. Because our extension is an extension of the known progressive hedging algorithm to a sampling-based version, we refer to it as stochastic sample-based progressive hedging.

Let $h : \mathcal{Z} \times \Omega \to \mathbb{R}$ be a function of the form $h(y,\omega) = h(h_0(y,\omega),\ldots,h_{T-1}(y,\omega))$, where h is convex in y for all $\omega \in \Omega$ and components h_t are convex and linked by y only for all t and $\omega \in \Omega$. Our objective is finding $y^* \in \arg\min_y \mathbb{E}[h(y,\omega)]$. Let $\bar{h} : \mathcal{Z}^T \times \Omega \to \mathbb{R}$ be given by $\bar{h}(y_0,\ldots,y_{T-1},\omega) = h(h_0(y_0,\omega),\ldots,h_{T-1}(y_{T-1},\omega))$.

In our problem of Subsection 5.2, the components h_t of function h correspond to the value functions V_t of the dynamic program in Equation (6) that are linked by dispatch schedule z. Components $\bar{h}_t$ correspond to the modified value functions $\bar{V}_t$, where y_t correspond to the local dispatch schedules $\tilde{z}^t$.

Because $\mathbb{E}[\bar{h}(\frac{1}{T}\sum_\tau y_\tau,\ldots,\frac{1}{T}\sum_\tau y_\tau,\omega)] = \mathbb{E}[h(\bar{y},\omega)]$ for $\bar{y} = \frac{1}{T}\sum_\tau y_\tau$, any y_t obtained from solving the problem $\min_{y_0,\ldots,y_{T-1}} \mathbb{E}[\bar{h}(y_0,\ldots,y_{T-1},\omega)]$ subject to $y_t = \frac{1}{T}\sum_\tau y_\tau$ for all t solves $\min_{\bar{y}} \mathbb{E}[h(\bar{y},\omega)]$.

We focus on $\min_{y_0,\ldots,y_{T-1}} \mathbb{E}[\bar{h}(y_0,\ldots,y_{T-1},\omega)]$ subject to $y_t = \frac{1}{T}\sum_\tau y_\tau$ for all t next. The augmented Lagrangian relaxation (e.g., Bertsekas 1982) obtained from dualizing the constraints $y_t = \frac{1}{T}\sum_\tau y_\tau$ for all t reads

$$\bar{l}(y_0,\ldots,y_{T-1},w_0,\ldots,w_{T-1},\omega) = \bar{h}(y_0,\ldots,y_{T-1},\omega) + \sum_{t'}\left(w_{t'}\left(y_{t'} - \frac{1}{T}\sum_\tau y_\tau\right) + \frac{1}{2\alpha}\left\|y_{t'} - \frac{1}{T}\sum_\tau y_\tau\right\|^2\right), \tag{30}$$

where w_t are the Lagrangian multipliers and α is a penalty factor. Function $\bar{l}$ is a saddle-function and satisfies all conditions of Theorem 2. Problem $(y_0^*,\ldots,y_{T-1}^*,w_0^*,\ldots,w_{T-1}^*) \in \text{argminimax}_{y_0^*,\ldots,y_{T-1}^*,w_0^*,\ldots,w_{T-1}^*} \mathbb{E}[\bar{l}(y_0,\ldots,y_{T-1},w_0,\ldots,w_{T-1},\omega)]$ can therefore be solved using the procedure given in Theorem 2.

Note that the derivation of $\bar{l}$ is similar to the derivation of $\bar{V}_t$ in Subsection 5.1. Both terms in the sum over t' in the definition of $\bar{l}$, however, prevent from decomposing the optimization in Step 2 of Theorem 2 into separate optimization problems, each involving only y_t. Following the ideas of the progressive hedging algorithm (Rockafellar and Wets 1991), we modify the augmented Lagrangian representation by introducing an iterative update of the multipliers w_t and by replacing the quadratic penalty term to obtain a decomposable version of the procedure in Theorem 2.

Let k be the iteration count. First, we replace $\left\|y_{t'} - \frac{1}{T}\sum_\tau y_\tau\right\|^2$ in the definition of $\bar{l}$ by $\left\|y_{t'} - \frac{1}{T}\sum_\tau y_\tau^{k-1}\right\|^2$, i.e., we use the solution of iteration $k-1$, $y_0^{k-1},\ldots,y_{T-1}^{k-1}$, in the penalty term. Second, starting from $w_t^0 = y_t^0 - \frac{1}{T}\sum_\tau y_\tau^0$, we update w_t^k by $w_t^{k+1} = w_t^k + \frac{1}{\alpha^k}(y_t^{k+1} - \frac{1}{T}\sum_\tau y_\tau^{k+1})$ instead of maximizing over $w_0,\ldots,w_{T-1}$. Note that now $(y_t - \frac{1}{T}\sum_\tau y_\tau)w_t = y_t w_t$ because $\frac{1}{T}\sum_\tau y_\tau$ and w_t are orthogonal. As a result, the minimization over $y_0,\ldots,y_{T-1}$ is now decomposable. We formally introduce the stochastic sample-based progressive hedging algorithm and provide its convergence in the next theorem.

THEOREM 3 **(Stochastic Sample-Based Progressive Hedging)**. *Let $\mathcal{Z} \subset \mathbb{R}^n$ be a compact convex set, $\mathcal{Y} = \mathcal{Z}^T, T \in \mathbb{N}$ and let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Let $\bar{h}(y,\omega): \mathcal{Y} \times \Omega \to \mathbb{R}$ be convex and lower semi-continuous in y for every $\omega \in \Omega$. Let k be the iteration count and choose an arbitrary $y^0 \in \mathcal{Y}$ and $w_t^0 = y_t^0 - \frac{1}{T}\sum_\tau y_\tau^0$. Consider the following procedure:*

1. *Sample $\omega^k \in \Omega$.*
2. *Solve for all $t = 0, \ldots, T-1$*

$$y_t^{k+1} = \arg\min_{\hat{y}_t \in \mathcal{Z}} \left\{ \bar{h}(\hat{y}_0, \ldots, \hat{y}_t, \ldots, \hat{y}_{T-1}, \omega^k) + \hat{y}_t w_t^k + \frac{1}{2\alpha^k} \left\| \hat{y}_t - \frac{1}{T}\sum_\tau y_\tau^k \right\|^2 \right\}.$$

3. *Set $w_t^{k+1} = w_t^k + \frac{1}{\alpha_k}(y_t^{k+1} - \frac{1}{T}\sum_\tau y_\tau^{k+1})$ for all $t = 0, \ldots, T-1$.*
4. *Set $k = k+1$, choose α^k and go to step 1.*

If the step size α^k satisfies

$$\sum_{k=0}^{\infty} \alpha^k = \infty \qquad \text{and} \qquad \sum_{k=0}^{\infty} (\alpha^k)^2 < \infty,$$

then the sequence $\{y^k\}$ generated by the procedure converges to some $y^* \in \mathcal{Y}$ such that $\mathbb{E}[h(\frac{1}{T}\sum_\tau y_\tau^*, \omega)] \leq \mathbb{E}[h(\frac{1}{T}\sum_\tau y_\tau, \omega)]$ almost surely for all $y \in \mathcal{Y}$.

The proof is based on showing that the sequence (y^k, w^k) generated by the procedure of Theorem 3 is actually the same as the sequence of points generated by the procedure in Theorem 2. Then, the convergence proof of Theorem 2 carries over and provides convergence to the optimal solution for the procedure in Theorem 3.

We apply the procedure of Theorem 3 in Subsection 5.2, where we generate samples of β, P_t, and L_t^G in Step 1 of the procedure. In Step 2, we solve Problem (22) for all t to obtain $\tilde{z}^t$ in a forward path through the dynamic program.

Our extension of progressive hedging to the stochastic case allows us to additionally handle continuous random variables within each of the decomposed subproblems. It is therefore capable of solving problems based on sampling the random variables.

Theorem 3 provides optimality for convex problems. Problem (9) is not convex because of the binary commitment decisions. However, if the algorithm converges, then the following corollary provides local optimality of a solution.

COROLLARY 2. *If the procedure in Theorem 3 is applied to a non-convex problem such that in each iteration the generated solutions y_t^{k+1} are optimal in a δ-neighborhood of y_t^{k+1} for fixed $\delta > 0$ for all* *t and the sequence* $\{y^k, w^k\}_{k\geq 0}$ *does converge to some* (y^*, w^*)*, then* y^* *is a locally optimal solution to the problem*

$$\min_y \mathbb{E}\left[h\left(\frac{1}{T}\sum_t y_t, \omega\right)\right].$$

Corollary 2 provides local optimality of the DSR schedule obtained by applying the procedure in Theorem 3 to Problem (9). It also holds for the approximate model obtained from discretizing the state space of the dynamic program.

6. Computational Results

We apply our approach to the generation system of the California ISO region. This region is of particular interest because it has high installed intermittent generation capacity. We formulate a base case using actual data and perform a sensitivity analysis with respect to the key parameters of the model. We use a planning horizon of one day and a period length of one hour. The algorithms are implemented in C++ and the mixed-integer quadratic problems of Algorithm 2 are solved using CPLEX 12.1 under Linux on a Westmere Hexa-Core Xeon X5650 processor running at 2.66 GHz.

6.1. Base Case

Our model requires data on loads, conventional generation capacity, intermittent generation capacity, and DSR bids. To obtain the data, we rely on a number of sources and we pre-process data when necessary.

Loads. Historical loads of the California ISO region are available from California ISO OASIS (2011). We use the hourly loads of Mondays from April to June of the years 2000 to 2010 to estimate the mean and correlation matrix of the joint normal distribution function of hourly loads on a typical Monday in spring. Average daily load is 654 GWh.

Conventional generation capacity. Data on the types and generation capacities of all grid-connected generators of the California ISO region is available from the California Energy Commission (2011). For our analyses, we use the generators that were connected to the grid of the California ISO region in May 2011. A total of 392 generators with a capacity greater or equal to 20 MW were connected to the grid and had a total capacity of 53,602 MW. Table 1 summarizes the type and characteristics of the generators. Generators with less than 20 MW are not considered because they represent less than 3% of generation capacity and are mostly emergency generators. We use cost factors and physical characteristics of the generators as provided by CAISO for test

Type	Number	Capacity (MW) min	Capacity (MW) max	Min. Uptime (h) avg	Min. Uptime (h) min	Min. Uptime (h) max	Min. Downtime (h) avg	Min. Downtime (h) min	Min. Downtime (h) max
Nuclear	4	1,118	1,124	168.0	168.0	168.0	168.0	168.0	168.0
Coal	7	21	118	96.0	96.0	96.0	48.0	48.0	48.0
Natural Gas	245	20	887	6.3	1.0	96.0	5.1	1.0	24.0
Geothermal	30	20	90	196.0	196.0	196.0	196.0	196.0	196.0
Hydro	88	20	659	196.0	196.0	196.0	196.0	196.0	196.0
Biomass	18	20	81	5.6	1.0	6.0	7.3	1.0	8.0

Table 1 Generator characteristics in the base case

studies in the California and Western Electricity Coordinating Council (WECC) markets (e.g., Price 2013).

Intermittent generation capacity. Intermittent generation capacity consists of wind and solar capacity. We did not find a source of hourly generation of the intermittent sources, and therefore, we simulated the operations of the wind farms and solar panels to estimate the distribution function of intermittent generation for a typical spring day. To simulate the operations of wind farms, we obtained the locations, the capacities, and the turbine types of all major wind farms in the California ISO region from the California Energy Commission (2011), which have a total maximum capacity of 2,709 MW. We obtained the hourly historical wind speeds for April to June of the years 2004 to 2006 at the locations of the wind farms from the National Renewable Energy Laboratory's (NREL) wind integration data set (National Renewable Energy Laboratory 2011b) and used NREL's Solar Advisor Model software[2]to simulate hourly wind power generation for all turbines. We aggregated the hourly generation and estimated the mean and the correlation matrix of the joint normal distribution of total hourly wind power generation for a typical day in spring.

To simulate the operations of the solar panels, we obtained the locations, the capacities, and the panel types of all registered solar panels in the California ISO region from the NREL's OpenPV Project database (National Renewable Energy Laboratory 2011a), which have a total maximum capacity of 511 MW. We obtained the hourly historical solar irradiation intensity for April to June of the years 1998 to 2005 at the locations of the solar panels from the National Climatic Data Center SUNY Gridded Data (National Climatic Data Center 2011) and used NREL's Solar Advisor Model software to simulate hourly solar power generation for all panels. We aggregated the hourly generation and estimated the mean and the correlation matrix of the joint normal distribution of total hourly solar power generation for a typical day in spring.

DSR. Using DSRs to shift load from peak demand periods to off-peak periods is a relatively new approach and economic DSR participation programs have only recently been put into place. Data on actual DSR bid curves and shifting profiles is not publicly available, but we obtained estimates

[2] The Solar Advisor Model was initially designed to simulate the operations of solar panels but now also includes a simulation module for wind turbines.

for the fraction of shiftable load by load type from case studies (Knödler 2012, Engel et al. 2003) and modeled shifting profiles and offer curves based on these sources (Section 6.3). In our base case, we use a bid curve with ten segments and 100 MW bid capacity for each segment, i.e., in each period 1,000 MW of total load reduction by DSR dispatch are possible, with an initial price of 10$/MWh and an increment of 2$/MWh per segment. We assume that load can be shifted to the four previous and four subsequent periods with $\beta_{t,t'}$ uniformly i.i.d. between 0% and 25% with mean 12.5% for all t and t'.

All data used in the computational study can be obtained from the authors' website. *[For the review process, the link has been blinded.]*

6.2. Solution

In this section, we solve the base case and report the performance of the algorithm and lower bound. The average runtime for 500 iterations of our algorithm is 4.2 hours, when starting with a value function approximation initialized to 0. With an initial approximation that carries more information, e.g., from previous optimization runs, and faster CPUs, the run time can be reduced and should be reasonable for real-world applications, where the problem is solved a day ahead of operations.

We first apply our method to a deterministic instance of Problem (9) with and without DSR to assess the quality of the value function approximation and the lower bound. In all instances we use a fixed step size of $\alpha = 0.1$ for Algorithm 2.

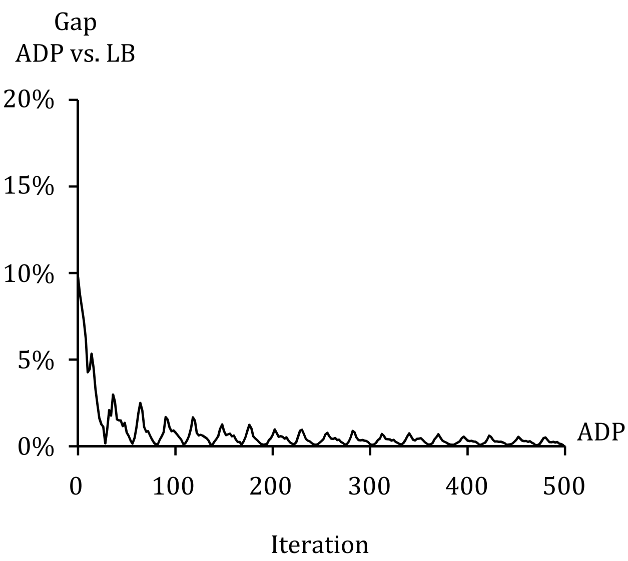


Figure 3 Deterministic, without DSRs

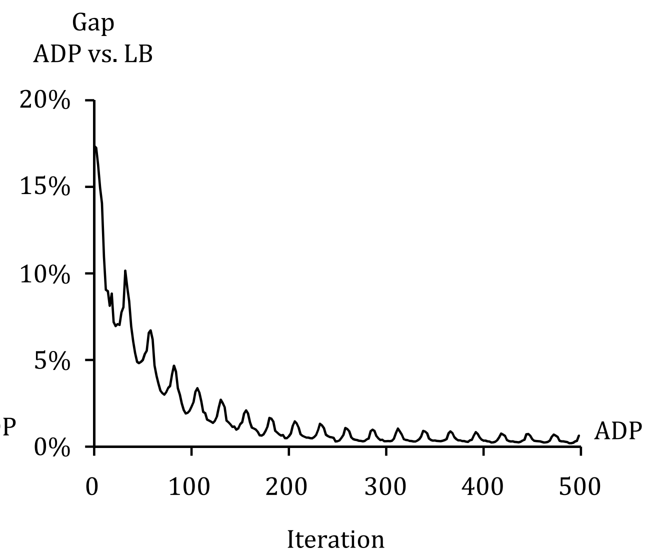


Figure 4 Deterministic, with DSRs

Deterministic instances. Figures 3 and 4 report the relative gap between the solution of Algorithm 2 over 500 iterations and the lower bound (LB) obtained by solving Problem (27) with and without DSRs, respectively. To assess the performance of the lower bound, we additionally solve the lower bounding Problem (27) without relaxed integrality constraints, i.e., the mixed-integer formulation, which provides the optimal value of Problem (9) for deterministic instances.

For the problem without DSRs, the lower bound is 0.017% below the optimal solution. The ADP algorithm found the best solution after 301 iterations. It is 0.031% above the optimal solution and 0.047% above the lower bound. For the problem with DSRs, the lower bound is 0.081% below the optimal solution. The ADP algorithm found the best solution after 432 iterations. It is 0.070% above the optimal solution and 0.151% above the lower bound.

Stochastic instances. For stochastic instances, we cannot solve the model exactly and we report relative gaps between the solution found by our algorithms and the lower bound. We consider the problem with and without DSRs. As the quality of the lower bound depends on the variance of the stochastic quantities, we conducted experiments with varying coefficients of variation (CV) for the demand that must be filled by conventional generators $L_t = L_t^G - L_t^D - P_t$. The CV of the demand that must be filled by conventional generators in the base case is determined by the historic data on load and intermittent generation and we vary the variance in the distribution that we use to generate samples of the random quantities while keeping its mean constant. We perform optimization runs with the CV of the base case, with a CV increased by 50%, and with a CV decreased by 50%. In all three scenarios, we simulated the solution obtained from Algorithm 2 after every 20 iterations for 5,000 draws of the random components. Figures 5 and 6 show how the simulated mean value improves when Algorithm 2 is executed for an increasing number of iterations.

We can observe that the convergence of the algorithm depends on the degree of uncertainty in the optimization problem. For the base case, we obtained optimality gaps of 2.6% and 3.0% for the base cases without DSRs and with DSRs, respectively. Reducing the coefficient of variation by 50% decreased the gaps between the ADP solution and the lower bound to 1.6% and 2.2% for the scenarios without and with DSRs, respectively. Increasing the coefficient of variation by 50% increased the gaps to 3.2% and 3.9%, respectively.

The optimality gaps obtained for the stochastic instances are difficult to compare with the gaps achieved by scenario-based models because our representation of uncertainty follows an alternative approach.

Solution of the base case. Our base case shows potential savings of 1.1 million USD/day (7.54%) when using DSRs as opposed to not using DSRs. If no DSRs are present in the system,

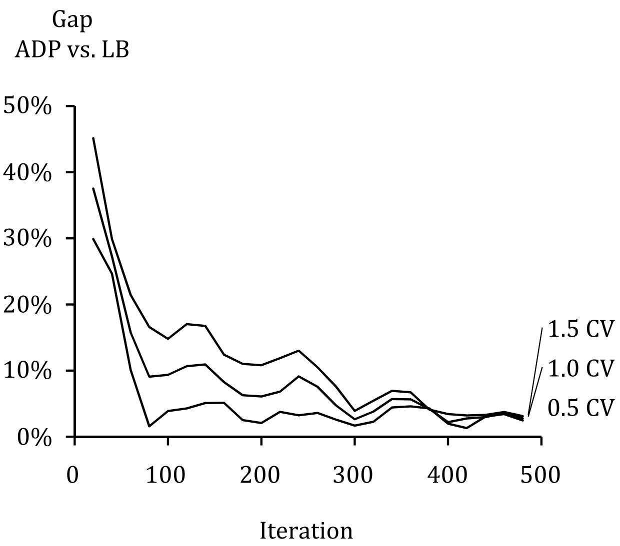


Figure 5 Stochastic, varying CV of L_t, without DSRs

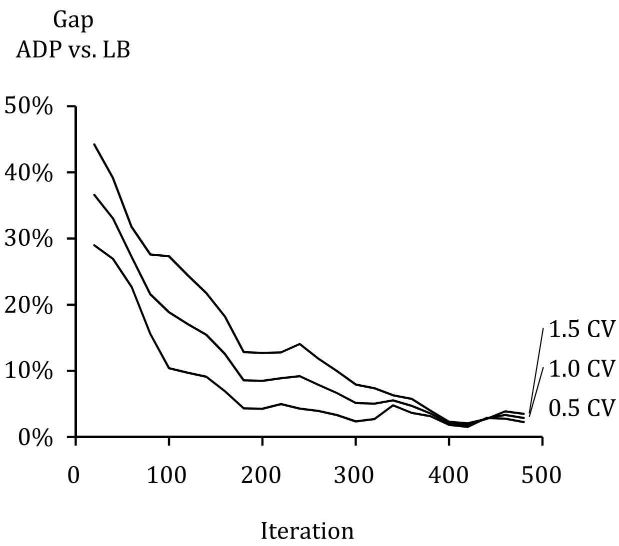


Figure 6 Stochastic, varying CV of L_t, with DSRs

total cost amounts to 15.8 million USD/day compared to 14.7 million USD/day when DSR capacity is dispatched. Especially during peak hours, the dispatch of DSR leads to a net shift of load to off-peak hours. This reduces the number of peak units with high variable cost that are started during peak hours and used only for a small number of periods.

Figure 7 shows the average load in the system with and without DSR dispatch and the shifted load. The peaks are flattened and the dips between periods 0 and 6 and periods 20 and 24 are filled with load from mid-day. Figure 8 shows the average number of generators running to meet current load over the planning horizon without and with dispatch of DSRs. DSRs prevent the start-up of the expensive peak generators during the day at the expense of having more generators running in the early morning and late evening.

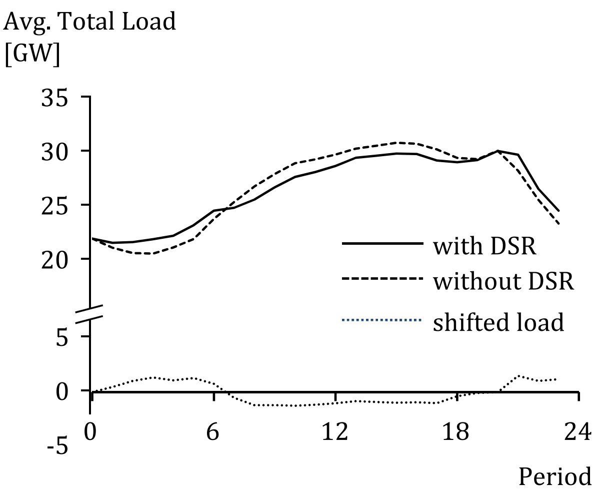


Figure 7 Avg. load and shifted load

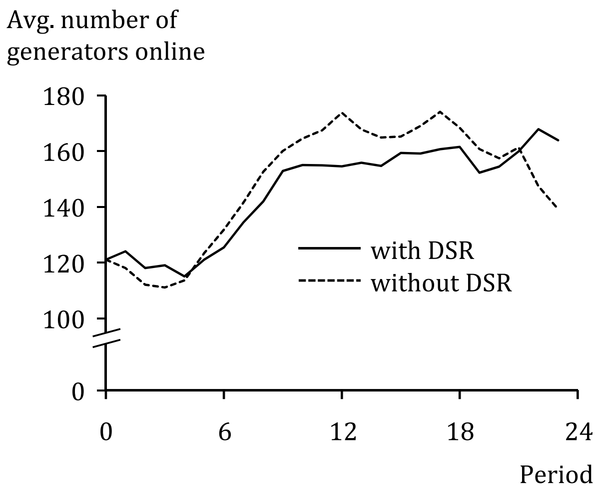


Figure 8 Avg. number of committed generators

6.3. Sensitivity

To analyze how changes in the parameter values affect the solution, we conduct several sensitivity analyses.

Varying uncertainty in intermittent generation forecast. We conducted experiments for varying coefficients of variation (CV) of the quantity $L_t^G - P_t$, which can be interpreted as varying the uncertainty in the forecast of intermittent generation capacity. The quality of the lower bound depends on the variance of the right hand side of Constraint (3). For our analyses, we first calculated the DSR dispatch schedule and value function approximation by executing Algorithm 2 for 500 iterations and then simulated operations of the generators for the obtained DSR dispatch schedule for 5,000 draws of the random variables. We report confidence intervals for our solutions along with the expected total cost.

Figures 9 and 10 show the upper bound(UB) with confidence interval and lower bound (LB) on the optimal expected cost for varying values of CV of the intermittent capacity for cases without and with DSRs. Note that increasing uncertainty causes higher expected costs of operation due to an increase in the right hand side of Constraint (4). The gap between the upper and the lower bound increases in the CV of the intermittent capacity for cases without and with DSRs. This effect can be explained by the fact that Inequality (29) in Proposition 2 becomes less binding as the variance of $L_t^G - P_t$ increases. The lower bound increases only slightly in the CV of the intermittent capacity because Constraint (8) is almost never binding in our problem instance. This is caused by the high share of natural gas powered generators that allow for relatively flexible operations and prevent from committing large reserve capacities.

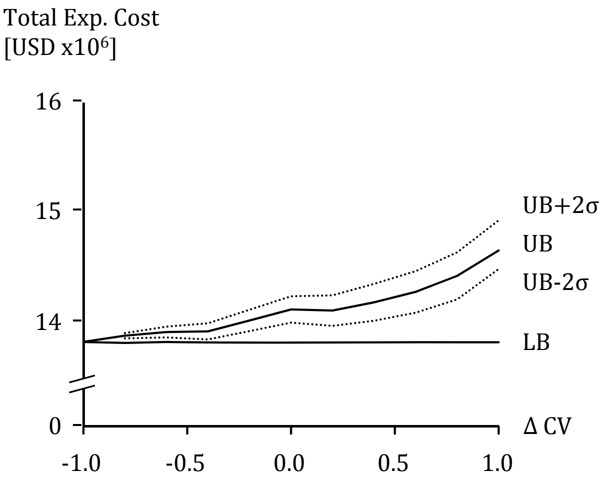


Figure 9 Base Case, without DSR

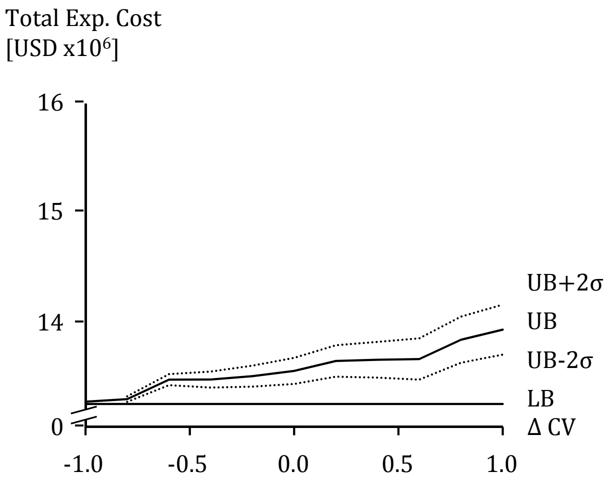


Figure 10 Base Case, with DSR

DSR bid curves. For DSR bid prices, no public data is available and we performed a sensitivity analysis to assess the potential of utilizing DSR for varying offer prices. We used an offer curve with ten segments and 100 MW bid capacity for each segment, i.e., in each period 1,000 MW of total load reduction by DSR dispatch is possible. Each segment of the offer curve is 2$/MWh more expensive than the previous segment. In our experiment, we vary the price for the first segment of the offer curve from 2$/MWh to 58$/MWh. We report total cleared DSR capacity along with total expected cost in Figures 11 and 12.

We can observe that the total cleared DSR capacity decreases monotonically in the bid prices. The total DSR cost, however, first increases and then decreases for minimum offer prices of more than 20$/MWh. Below the peak value, the price increase outweighs the decrease in dispatched capacity. Total cost increases monotonically with increasing offer prices.

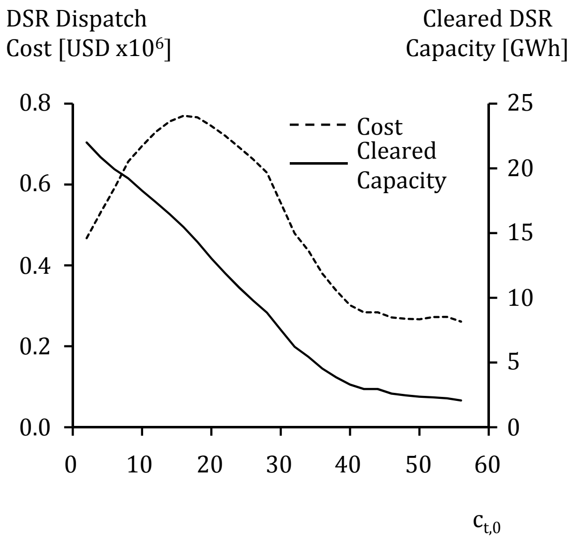


Figure 11 Cleared DSR capacity and cost for increasing offer prices

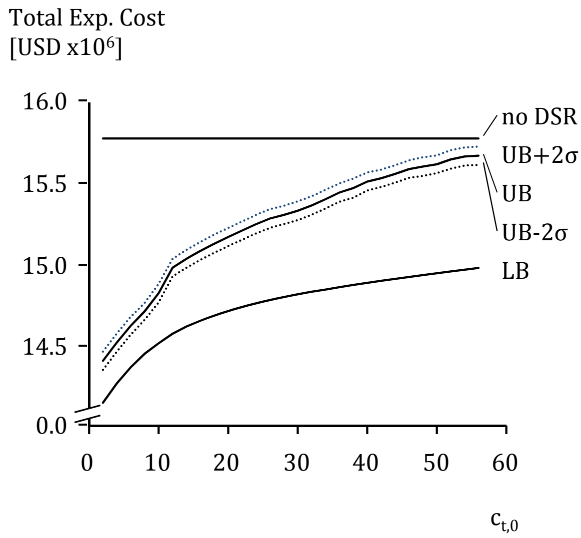


Figure 12 Expected total cost for increasing offer prices

Natural gas prices. 245 of the 392 generators that we consider are powered by natural gas. Prices for natural gas are subject to large fluctuations. We performed a sensitivity analysis to assess how DSRs can mitigate the effect of rising fuel prices on total cost by preventing from start up of expensive peak generators. We varied the price for natural gas, p_g, from -50% to 100% of current prices and report total cleared DSR capacity along with the upper (UB) and lower bound (LB) on the optimal total expected cost in Figures 13 and 14. We do not report the confidence interval on the UB to keep Figure 14 legible.

We can observe that the total cleared DSR capacity increases with increasing fuel prices because dispatching DSRs becomes more profitable. For natural gas price increases of 40% and above all DSR capacity is dispatched and the quantity remains close to its maximum value of 24 GWh.

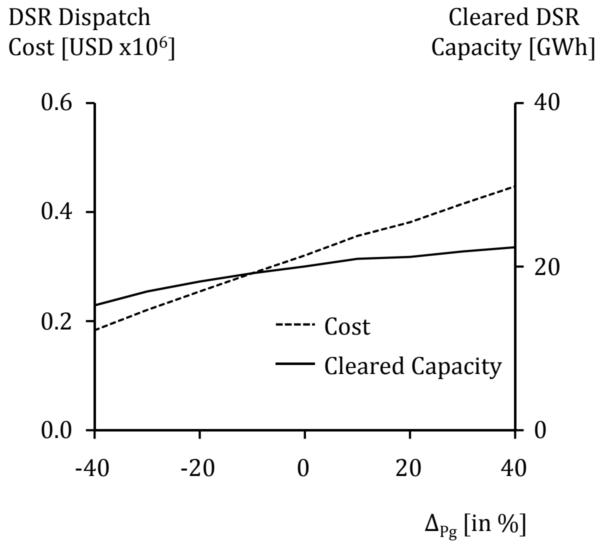


Figure 13 Cleared DSR capacity and cost for increasing natural gas prices

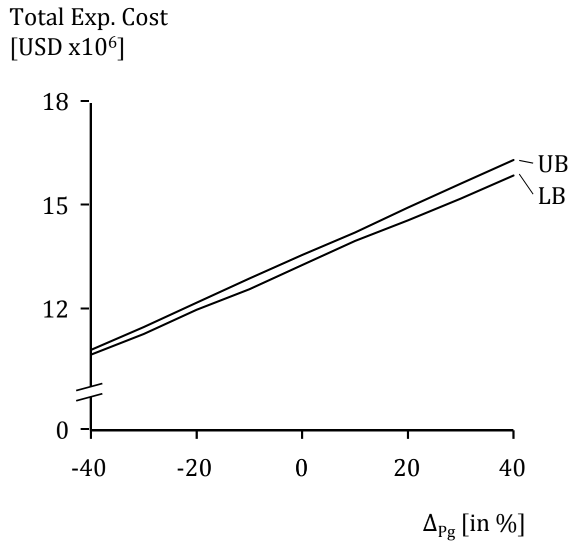


Figure 14 Expected total cost for increasing natural gas prices

7. Conclusion

In this paper, we modeled and analyzed the potential of utilizing DSRs with stochastic load shifting as a tool for leveling demand for electric energy. The goal was to quantify the value of DSRs in an energy generation network, while considering the uncertainty in the load shifting effect induced by dispatch of DSRs in day-ahead markets. Our model allows for participation of "slow" DSRs that do not qualify for emergency-triggered real time programs because DSRs are scheduled a day ahead of their actual dispatch.

To solve our model, we developed a unit commitment model and a solution algorithm based on an extension of the progressive hedging algorithm by Rockafellar and Wets (1991) to stochastic functions. Our algorithm combines elements of ADP and stochastic sample-based progressive hedging and allows for decomposition of decisions within a dynamic program that link time periods.

We provided convergence results for the stochastic proximal point algorithm and the stochastic sample-based progressive hedging algorithm. These algorithms are capable of solving a broad class of convex stochastic programming problems optimally based on Monte-Carlo-Sampling.

To quantify the effect of DSRs on operation and cost of an energy system, we implemented our solution approach and solved problem instances based on real world data. Our implementation is a decision making tool for dispatch of DSRs in energy networks and can be readily applied by system operators. Our results indicate that substantial savings in energy cost can be achieved by utilizing DSRs. These savings increase for growing uncertainty in the forecast of intermittent generation and load, which can be explained by mitigation of demand peaks that are expensive to satisfy.

Our approach is applicable to many other convex stochastic programming problems. For example in problems, where uncertainty is partly modeled as discrete scenarios and partly as continuously distributed random variables can exploit the decomposition property of the algorithm in Theorem 3.

We also hope to spark interest in further investigating the value of DSRs in energy generation. More detailed unit commitment models that include, e.g., transmission constraints or more detailed cost models can help improve the accuracy of the models and improve DSR schedules.

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Appendix. Notation

We use symbols with or without subscript and superscript as required in the corresponding context, e.g, we write z^k to denote $(z^k_{t,n})_{0\le t<T,0\le n<N}$. We do not explicitly list all possible combinations in the following table.

α	algorithm step size
$\beta_{t,t',n}$	fraction of demand shifted from period t to t' by DSR dispatch of offer curve segment n
b_t	imbalance between generation and demand for electric energy
c_i^g	generation cost function of generator i
c_i^s	start-up cost function of generator i
c_t^+, c_t^-	unit penalty cost for positive and negative imbalance energy
$c_{t,n}^D$	cost of DSR supply curve segment n
$c_t^{g,\max}$	maximum average energy generation cost
$\bar{c}_t^{g,\max}$	smoothed maximum average energy generation cost
C_t^C	generation cost of conventional generators with penalties
C^D	dispatch cost for DSRs
δ	indicator function
ϵ	probability of shortage in scheduled capacity
g_i	energy output function of generator i
$g_i^{\max}$	maximum reachable output function of generator i
$g_{t,n}^D$	capacity of DSR supply curve segment n
$G_i^{\max}$	maximum power output of generator i
$G_i^{\min}$	minimum power output of generator i
H	mean value operator
i	subscript for generator index
$\mathcal{I}$	set of all conventional generators
$\mathcal{I}^{\text{reserve}}$	set of all conventional generators minus current maximum capacity committed generator
$\mathcal{I}^U$	set of all conventional generators that have reached their minimum uptime
k	superscript for iteration count
K	duplication operator
λ	penalty factor in Lagrangian relaxation
l_t	realization of L_t in period t
l_t^G	realization of L_t^G in period t
$\hat{l}_t$	discretized realization of L_t in period t
L_t^D	dispatched DSR capacity
L_t^G	forecast of gross demand for energy
L_t	net demand for energy from conventional generators in period t
$\mathcal{L}_t$	joint distribution of $L_0,\ldots,L_t$
$\hat{\mathcal{L}}_t$	discretized joint distribution of $L_0,\ldots,L_t$
$\phi_t(\cdot,z)$	cumulative distribution function of $L_t(z)$
p_g	price of natural gas
P_t	forecast of capacity of intermittent resources
R^M	resource transition function
$R_{t,i}$	state of generator i
$\hat{R}_{t,i}$	discretized state of generator i
RL_i^d	maximum ramp down rate of generator i
RL_i^u	maximum ramp up rate of generator i
$\mathcal{R}_t$	generator state space
$\hat{\mathcal{R}}_t$	discretized generator state space

S^M	state transition function
S_t	state of the system
$\mathcal{S}_t$	state space
t	index of time period
T_i^d	minimum downtime of generator i
T_i^u	minimum uptime of generator i
T	number of planned time periods
$v_{t,i}^{l_0,\ldots,l_t,\hat{R}_i}$	estimate of the value of generator i being in state $\hat{R}_i$ for load history $l_0,\ldots,l_t$
V_t	optimal value function
$\bar{V}_t$	value function for augmented Lagrangian formulation
$\hat{V}_t$	value function approximation
ω	random vector
w^t	Lagrangian multipliers corresponding to $\tilde{z}^t$
$x_{t,i}^{\text{off}}$	binary decision variable for generator i, 'switch generator off'
$x_{t,i}^{\text{on}}$	binary decision variable for generator i, 'switch generator on'
$x_{t,i}^d$	continuous decision variable for generator i, 'output level'
x_t	vector of all decision variables
$\bar{x}_t$	continuous relaxation of x_t
$\mathcal{X}_{t,i}$	set of feasible actions
$\bar{\mathcal{X}}$	continuous relaxation of $\mathcal{X}$
$\hat{\mathcal{X}}$	continuous relaxation of $\mathcal{X}$ with dualized reserve constraints
$z_{t,n}$	dispatch quantity for DSR offer curve segment n in period t
$\tilde{z}^t$	local copy of dispatch schedule z in period t (same dimension as z)

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Appendix

EC.1. Dynamic Programming Model Details

EC.1.1. Transition Function

In this section we give an example for the generator state transition function R^M. We consider a generator with an minimum uptime of $T^u = 2$ periods and a minimum downtime of $T^d = 2$ periods. As a result the generator state vector R_t has $1 + T^u + T^d = 5$ coordinates. The first coordinate is continuous and takes values in the interval $[0, 100]$ representing the operating level. Coordinates two to five are binary, and there is always exactly one binary coordinate taking the value 1, all other binary coordinates take value 0. The binary coordinates indicate for how long the generator has been running, if it is committed, or how long the generator has been turned off, if it is uncommitted. For example, coordinate two takes value 1, if the generator has been running for one hour. All other binary coordinates have value 0. If the uptime exceeds the minimum uptime, the coordinate corresponding to the minimum uptime remains at value 1. If the downtime exceeds the minimum downtime, the coordinate corresponding to the minimum downtime remains at value 1.

The generator state transition equation has the form $R_{t+1} = AR_t + Bx_t$. For our specific example it reads

$$R_{t+1} = \begin{bmatrix} 0&0&0&0&0 \\ 1&1&0&0&0 \\ 0&0&0&0&0 \\ 0&0&1&1&0 \\ 0&0&0&0&0 \end{bmatrix} R_t + \begin{bmatrix} 0&0&1 \\ 0&1&0 \\ -1&0&0 \\ 1&0&0 \\ 0&-1&0 \end{bmatrix} \begin{bmatrix} x_t^{\text{on}} \\ x_t^{\text{off}} \\ x_t^d \end{bmatrix}$$

EC.1.2. Action Space

In this section we provide details on the specific form of the action space $\mathcal{X}(S_t)$ for the dynamic programming formulation of the unit commitment problem. In our model we consider ramping limits and minimum up and downtimes, and provide their specific formulation.

Ramping Limits For each generator, there exist ramping limits, which limit the rate of change in its output level from one period to the next. Let RL_i^u be the maximum rate at which we can increase the output of generator i, and let RL_i^d be the maximum rate at which we can decrease the output of generator i.

$R_{t,i,0}$ denotes the first coordinate of the generator state, i.e., the current operating level of generator i. Then the constraint on ramping up reads $x_{t,i}^d - R_{t,i,0} \leq RL_i^u$ and the constraint on ramping down reads $R_{t,i,0} - x_{t,i}^d \leq RL_i^d$.

Dispatch Restriction A generator can only be dispatched, if it is committed. Let $\mathcal{J}^U$ be the set of indices of those coordinates of generator state $R_{t,i}$ that correspond to states in which the generator is committed. We disallow dispatch of uncommitted generators by the constraint $x^d_{t,i} \leq \sum_{j\in\mathcal{J}^U} R_{t,i,j}$.

Minimum Up- and Downtimes Many generators cannot be committed and decommitted instantaneously, but instead require to remain committed for a minimum amount of time, if they are running, and to remain uncommitted for a minimum amount of time, if they are turned off. We therefore allow to commit a generator on (set $x^{\text{on}}_{t,i} = 1$ in a period) only, when it has reached its minimum downtime and we allow decommitting a generator (set $x^{\text{off}}_{t,i} = 1$ in a period) only, when it has reached its minimum uptime.

Let j^u denote the index of the coordinate of generator state $R_{t,i}$ corresponding to the minimum uptime of generator i and let j^d denote the index of the coordinate of generator state $R_{t,i}$ corresponding to the minimum downtime of generator i. Then the minimum uptime constraint is given by $x^{\text{off}}_{t,i} \leq R_{t,i,j^u}$ and the minimum downtime constraint is given by $x^{\text{on}}_{t,i} \leq R_{t,i,j^d}$.

EC.1.3. Generation and Cost Functions

The energy generation function $g_i(R_{t,i}, x^d_{t,i})$ computes the power output of a generator i given its current state $R_{t,i}$ and dispatch decision $x^d_{t,i}$. Let $G^{\min}_i$ denote the minimum power output of generator i and let $G^{\max}_i$ the maximum power output of generator i. Additionally, let $\mathcal{J}^U$ be the set of indices of those coordinates of generator state $R_{t,i}$ that correspond to states in which the generator is committed. The generation function reads

$$g_i(R_{t,i}, x^d_{t,i}) = G^{\min}_i \sum_{j\in\mathcal{J}^U} R_{t,i,j} + x^d_{t,i}(G^{\max}_i - G^{\min}_i).$$

Maximum output function $g^{\max}_i(R_{t,i})$ computes the maximum achievable output of a generator that is currently in state $R_{t,i}$. For generators currently committed, it is given by the current operating level plus the maximum ramp-up rate, i.e., $R_{t,i,0}(G^{\max}_i - G^{\min}_i) + RL^u_i$. For generators that are uncommitted and cannot be synchronized quickly enough to qualify for non-spinning reserve, it takes value 0, and for generators that are uncommitted but can be synchronized to the grid sufficiently quickly it takes value $G^{\min}_i + RL^u_i$.

Cost function $c^g_i(R_{t,i}, x^d_{t,i})$ represents the energy generating cost of generator i for given generator state $R_{t,i}$ and dispatch decision $x^d_{t,i}$. We model the cost function to take the form

$$c^g_i(R_{t,i}, x^d_{t,i}) = a_i \sum_{j\in\mathcal{J}^U} R_{t,i,j} + b_i x^d_{t,i} + c_i (x^d_{t,i})^2,$$

where a_i, b_i, and c_i are nonnegative scalar values. a_i represents the no-load cost of generator i, i.e., the cost incurred when the generator is committed and running at minimum capacity, and b_i and c_i represent the linear and quadratic coefficients of the incremental generation cost. We model cost parameters to be stationary, but we could also let a, b, and c depend on t.

EC.2. Propositions for Reformulation of the Integrated Problem

This section contains two additional propositions that support the reformulation of the integrated problem (9) in Section 5 into the form necessary to solve the problem using dynamic programming techniques.

PROPOSITION EC.1. *Problem* (9) *subject to Constraints* (7) *and* (8) *is equivalent to problem*

$$\left\{\tilde{z}^0,...,\tilde{z}^{T-1}\right\} = \underset{\left\{\tilde{z}^0,...,\tilde{z}^{T-1}\right\}}{\arg\min}\; C^D\left(\tilde{z}^0\right) + V_0(\tilde{z}^0, S_0) \tag{EC.1}$$

with

$$V_t(\tilde{z}^t,...,\tilde{z}^{T-1}, S_t) = \min_{x_t \in \mathcal{X}(S_t)} C_t^C\left(R_t, x_t, b_t\right) + \mathbb{E}_{L_{t+1}(\tilde{z}^t)}\left[V_{t+1}\left(\tilde{z}^{t+1},...,\tilde{z}^{T-1}, S^M(S_t, x_t, L_{t+1}(\tilde{z}^t))\right)\right], \tag{EC.2}$$

subject to

$$\sum_{i\in\mathcal{I}} g_i(R_{t,i}, x_{t,i}^d) + b_t = l_t(\tilde{z}^t), \tag{EC.3}$$

$$\mathbf{P}\left\{\sum_{i\in\mathcal{I}^{reserve}} g_i^{max}(R^M(R_{t,i}, x_{t,i})) \leq L_{t+1}(\tilde{z}^t) \middle| l_0, \ldots, l_{t+1}\right\} \leq \epsilon \tag{EC.4}$$

and

$$z = \tilde{z}^0 = \tilde{z}^1 = ... = \tilde{z}^{T-1}. \tag{EC.5}$$

We begin by noting that

$$\begin{aligned}&\min_{z_0,\ldots,z_{T-1}} C^D(z_0) + \tilde{V}_0(z_0,\ldots,z_{T-1},S_0) = \min_z C^D(z) + \tilde{V}_0(z,\ldots,z,S_0).\\ &\quad\text{s.t. } z_0 = \cdots = z_{T-1}\end{aligned}$$

Let $(A_k) = V_{T-k}(z, S_{T-k})$ and $(B_k) = \tilde{V}_{T-k}(z,\ldots,z,S_{T-k})$, where $k = 2,\ldots,T$ and (B_k) has parameter z $k-1$ times. We show $(A_k) = (B_k)$ for all k and z.

Base Case. Let $k = 2$. By definition $(A_2) = (B_2)$ holds.

Induction Step. Now consider (A_{k+1}) and (B_{k+1}). The induction assumption is that $(A_k) = (B_k)$. We have

$$\begin{aligned}(B_{k+1}) =&\tilde{V}_{T-k-1}(z,\ldots,z,S_{T-k-1})\\ =&\min_{x_{T-k}\in\mathcal{X}(S_{T-k},z)} C_{T-k}^C(R_{T-k},x_{T-k},b_{T-k}) + \mathbb{E}_{L_{T-k}(z)}\left[\tilde{V}_{T-k}(z,\ldots,z,S_{T-k})\right]\\ =&\min_{x_{T-k}\in\mathcal{X}(S_{T-k},z)} C_{T-k}^C(R_{T-k},x_{T-k},b_{T-k}) + \mathbb{E}_{L_{T-k}(z)}\left[V_{T-k}(z,S_{T-k})\right]\\ =&(A_{k+1}),\end{aligned}$$

where the second and fourth equality are by definition and the third equality follows from the induction assumption. Therefore, $(A_{T+1}) = (B_{T+1})$ for all z. This suffices to show equality of the two problems in the proposition.

Proposition EC.2. *Problem* (18) *is equivalent to problem*

$$\min_{\tilde{z}^0}\left\{C^D(\tilde{z}^0)+\tilde{z}^0w^0+\frac{1}{2\alpha}\|\tilde{z}^0-z\|^2+\min_{x_0}\left\{C_0^C\left(R_0,x_0,b_0\right)+\mathbb{E}_{L_1(\tilde{z}^0)}\left[\bar{V}_1\left(z,w,S_1\right)\right]\right\}\right\} \tag{EC.6}$$

with

$$\bar{V}_t(z,w,S_t)=\min_{\tilde{z}^t}\left\{\tilde{z}^tw^t+\frac{1}{2\alpha}\|\tilde{z}^t-z\|^2+\min_{x_t}\left\{C_t^C\left(R_t,x_t,b_t\right)+\mathbb{E}_{L_{t+1}(\tilde{z}^t)}\left[\bar{V}_{t+1}\left(z,w,S_{t+1}\right)\right]\right\}\right\} \tag{EC.7}$$

and $\bar{V}_T=0$.

We show that for all S_t, w, z, and t equality

$$\min_{\{\tilde{z}^t,\ldots,\tilde{z}^{T-1}\}}V_t(\tilde{z}^t,\ldots,\tilde{z}^{T-1},S_t)+\sum_{\tau=t}^{T-1}\left(\tilde{z}^\tau w^\tau+\frac{1}{2\alpha}\|\tilde{z}^\tau-z\|^2\right)=\bar{V}_t(z,w,S_t) \tag{EC.8}$$

holds. The statement of the proposition then follows from the definition of Problem (18). Throughout the proof we do not explicitly write $x_t\in\mathcal{X}(S_t,z)$, but note that the optimal value of x_t is a function of the state and z, because the distribution of L_t depends on z.

The proof is carried out by induction.

We define problems (A_k) and (B_k) for $k=1,\ldots,T$ as

$$(A_k)\quad \min_{\{\tilde{z}^{T-k},\ldots,\tilde{z}^{T-1}\}}V_{T-k}(\tilde{z}^{T-k},\ldots,\tilde{z}^{T-1},S_{T-k})+\sum_{\tau=T-k}^{T-1}\left(\tilde{z}^\tau w^\tau+\frac{1}{2\alpha}\|\tilde{z}^\tau-z\|^2\right),$$

and

$$\begin{aligned}(B_k)\quad \min_{\tilde{z}^{T-k}}\Big\{\tilde{z}^{T-k}w^{T-k}+\frac{1}{2\alpha}\|\tilde{z}^{T-k}-z\|^2+\min_{x_{T-k}}\Big\{&C_{T-k}^C(R_{T-k},x_{T-k},b_{T-k})\\&+\mathbb{E}_{L_{T-k}(\tilde{z}^{T-k})}\left[\bar{V}_{T-k+1}(z,w,S_{T-k+1})\right]\Big\}\Big\}.\end{aligned}$$

Base case. Let $k=1$. We have

$$(A_1)\quad \min_{\tilde{z}^{T-1}}\tilde{z}^{T-1}w^{T-1}+\frac{1}{2\alpha}\|\tilde{z}^{T-1}-z\|^2+\min_{x_{T-1}}C_{T-1}^C(R_{T-1},x_{T-1},b_{T-1}),$$

and

$$(B_1)\quad \min_{\tilde{z}^{T-1}}\tilde{z}^{T-1}w^{T-1}+\frac{1}{2\alpha}\|\tilde{z}^{T-1}-z\|^2+\min_{x_{T-1}}C_{T-1}^C(R_{T-1},x_{T-1},b_{T-1}).$$

Obviously $(A_1)=(B_1)$ for all w and z.

Induction step. The induction assumption is $(A_k)=(B_k)$ or

$$\begin{aligned}\min_{\{\tilde{z}^{T-k},\ldots,\tilde{z}^{T-1}\}}V_{T-k}(\tilde{z}^{T-k},\ldots,\tilde{z}^{T-1},S_{T-k})+\sum_{\tau=T-k}^{T-1}\left(\tilde{z}^\tau w^\tau+\frac{1}{2\alpha}\|\tilde{z}^\tau-z\|^2\right)\\=\bar{V}_{T-k}(z,w,S_{T-k}).\end{aligned}$$

Consider (A_{k+1})

$$\min_{\{\tilde{z}^{T-k-1},\dots,\tilde{z}^{T-1}\}} V_{T-k-1}(\tilde{z}^{T-k-1},\dots,\tilde{z}^{T-1},S_{T-k-1}) + \sum_{\tau=T-k-1}^{T-1}\left(\tilde{z}^{\tau}w^{\tau} + \frac{1}{2\alpha}\|z_{\tau}-z\|^2\right).$$

We have

$$\begin{aligned}\sum_{\tau=T-k-1}^{T-1}\left(\tilde{z}^{\tau}w^{\tau} + \frac{1}{2\alpha}\|\tilde{z}^{\tau}-z\|^2\right) =& \tilde{z}^{T-k-1}w^{T-k-1} + \frac{1}{2\alpha}\|\tilde{z}^{T-k-1}-z\|^2 \\ &+ \sum_{\tau=T-k}^{T-1}\left(\tilde{z}^{\tau}w^{\tau} + \frac{1}{2\alpha}\|\tilde{z}^{\tau}-z\|^2\right).\end{aligned}$$

Rewriting (A_{k+1}) using this equality gives

$$\begin{aligned}&\min_{\{\tilde{z}^{T-k-1},\dots,\tilde{z}^{T-1}\}}\Bigg\{\tilde{z}^{T-k-1}w^{T-k-1} + \frac{1}{2\alpha}\|\tilde{z}^{T-k-1}-z\|^2 + \sum_{\tau=T-k}^{T-1}\left(\tilde{z}^{\tau}w^{\tau} + \frac{1}{2\alpha}\|\tilde{z}^{\tau}-z\|^2\right) \\ &\qquad + \min_{x_{T-k-1}}\Big\{C^C_{T-k-1}(R_{T-k-1},x_{T-k-1},b_{T-k-1}) \\ &\qquad + \mathbb{E}_{L_{T-k}(\tilde{z}^{T-k})}\left[V_{T-k}(\tilde{z}^{T-k},\dots,\tilde{z}^{T-1},S_{T-k})\right]\Big\}\Bigg\} \\ &= \min_{\tilde{z}^{T-k-1}} \tilde{z}^{T-k-1}w^{T-k-1} + \frac{1}{2\alpha}\|\tilde{z}^{T-k-1}-z\|^2 \\ &+ \min_{\{\tilde{z}^{T-k},\dots,\tilde{z}^{T-1}\}}\Bigg\{\sum_{\tau=T-k}^{T-1}\left(\tilde{z}^{\tau}w^{\tau} + \frac{1}{2\alpha}\|\tilde{z}^{\tau}-z\|^2\right) + \min_{x_{T-k-1}}\Big\{C^C_{T-k-1}(R_{T-k-1},x_{T-k-1},b_{T-k-1}) \\ &\qquad + \mathbb{E}_{L_{T-k}(\tilde{z}^{T-k})}\Big[V_{T-k}(\tilde{z}^{T-k},\dots,\tilde{z}^{T-1},S_{T-k})\Big]\Big\}\Bigg\},\end{aligned}$$

By the induction assumption this is equal to

$$\begin{aligned}&\min_{\tilde{z}^{T-k-1}} \tilde{z}^{T-k-1}w^{T-k-1} + \frac{1}{2\alpha}\|\tilde{z}^{T-k-1}-z\|^2 \\ &\qquad + \min_{x_{T-k-1}}\left\{C^C_{T-k-1}(R_{T-k-1},x_{T-k-1},b_{T-k-1}) + \mathbb{E}_{L_{T-k}}\left[\bar{V}_{T-k}(z,w,S_{T-k})\right]\right\} \\ &= (B_{k+1}).\end{aligned}$$

The statement of the proposition follows from noting that Equation (EC.8) holds for $t=1$ and from the definition of Problem (18) and V_t.

EC.3. Proofs of Theorems

Proof of Theorem 1

We start by noting that relaxing the integrality constraints on x provides a lower bound on the original problem, because $\mathcal{X} \subset \bar{\mathcal{X}}$. Then, because for any function it holds that $\mathbb{E}_{\omega}[\min_x f(x,\omega)] \leq \min_x \mathbb{E}_{\omega}[f(x,\omega)]$, we can move the expectation with respect to L_t in the definition of V_t in front of the minimization over x_t to obtain a lower bound on V_t. We apply this procedure recursively and obtain a deterministic minimization problem inside the expectation over $\{L_0(z),\dots,L_{T-1}(z)\} \in \mathcal{L}_{T-1}$ given in (27).

Proof of Theorem 2

A function is lower (upper) semi-continuous if for every point x_0 in the domain it holds $\liminf_{x\to x_0} \geq f(x_0)$ (which is replaced by $\limsup_{x\to x_0} \leq f(x_0)$ for upper). On several occasions we use the property that if function f is lower semi-continuous, then $\sup_{x\in C} f(x)$ is attained whenever C is a compact set (for upper semi-continuous functions, the sup is replaced by inf). Since $\mathcal{N},\mathcal{M}$ are compact and l is lower (upper) semi-continuous, the argminimax in Step 2 of the procedure in the theorem is non-empty. For each $k=0,1,\dots$ we have that

$$0\in\partial_v l(v^{k+1},w^{k+1},\omega^k)+N_{\mathcal{N}}(v^{k+1})+\frac{1}{\alpha^k}(v^{k+1}-v^k)$$

and

$$0\in\partial_w l(v^{k+1},w^{k+1},\omega^k)-N_{\mathcal{M}}(w^{k+1})-\frac{1}{\alpha^k}(w^{k+1}-w^k). \tag{EC.9}$$

See Du and Pardalos (1995) and Rockafellar 1970, §35, for details on subgradients and optimality conditions of saddle-functions.

Adding $\frac{1}{\alpha^k}(w^{k+1}-w^k)$ on both sides of (EC.9) yields

$$\frac{1}{\alpha^k}(w^{k+1}-w^k)\in\partial_w l(v^{k+1},w^{k+1},\omega^k)-N_{\mathcal{M}}(w^{k+1}),$$

which implies that there exists $d\in\partial_w l(v^{k+1},w^{k+1},\omega^k)$ such that $d-\frac{1}{\alpha^k}(w^{k+1}-w^k)\in N_{\mathcal{M}}(w^{k+1})$ (note that $l(v^{k+1},\cdot,\omega^k)$ is a concave function and the subgradient points in the direction of increase). Then we have that $z-w^{k+1}\in T_{\mathcal{M}}(w^{k+1})$ for each $z\in\mathcal{M}$ and by definition of the normal cone $(d-\frac{1}{\alpha^k}(w^{k+1}-w^k))^T(z-w^{k+1})\leq 0$. We conclude that $\frac{1}{\alpha^k}(w^{k+1}-w^k)^T(z-w^{k+1})\geq d^T(z-w^{k+1})$. Now since $d\in\partial_w l(v^{k+1},w^{k+1},\omega^k)$, we have that for all $z\in\mathcal{M}$

$$l(v^{k+1},w^{k+1},\omega^k)+d^T(z-w^{k+1})\geq l(v^{k+1},z,\omega^k)$$

and as a result

$$l(v^{k+1},w^{k+1},\omega^k)+\frac{1}{\alpha^k}(w^{k+1}-w^k)^T(z-w^{k+1})\geq l(v^{k+1},z,\omega^k). \tag{EC.10}$$

Using similar arguments for convex function $l(\cdot,w^{k+1},\omega^k)$, we obtain

$$l(v^{k+1},w^{k+1},\omega^k)+\frac{1}{\alpha^k}(v^k-v^{k+1})^T(y-v^{k+1})\leq l(y,w^{k+1},\omega^k) \tag{EC.11}$$

for all $y\in\mathcal{N}$.

Expanding the relation $\|v^k-y\|^2+\|w^k-z\|^2=\|v^k-v^{k+1}+v^{k+1}-y\|^2+\|w^k-w^{k+1}+w^{k+1}-z\|^2$ yields

$$\begin{aligned}\|v^{k+1}-y\|^2+\|w^{k+1}-z\|^2=&\|v^k-y\|^2+\|w^k-z\|^2\\&+2(v^k-v^{k+1})^T(y-v^{k+1})\\&+2(w^k-w^{k+1})^T(z-w^{k+1})\\&-\|v^k-v^{k+1}\|^2-\|w^k-w^{k+1}\|^2.\end{aligned}$$

Combining with (EC.10) and (EC.11) yields

$$\begin{aligned}\|v^{k+1}-y\|^2+\|w^{k+1}-z\|^2 \leq &\|v^k-y\|^2+\|w^k-z\|^2\\ &-2\alpha^k(l(v^{k+1},w^{k+1},\omega^k)-l(y,w^{k+1},\omega^k))\\ &-2\alpha^k(l(v^{k+1},z,\omega^k)-l(v^{k+1},w^{k+1},\omega^k))\\ &-\|v^k-v^{k+1}\|^2-\|w^k-w^{k+1}\|^2\\ =&\|v^k-y\|^2+\|w^k-z\|^2\\ &-2\alpha^k(l(v^k,z,\omega^k)-l(y,w^k,\omega^k))\\ &-2\alpha^k(l(y,w^k,\omega^k)-l(y,w^{k+1},\omega^k))\\ &-2\alpha^k(l(v^{k+1},z,\omega^k)-l(v^k,z,\omega^k))\\ &-\|v^k-v^{k+1}\|^2-\|w^k-w^{k+1}\|^2\\ \leq&\|v^k-y\|^2+\|w^k-z\|^2\\ &-2\alpha^k(l(v^k,z,\omega^k)-l(y,w^k,\omega^k))\\ &-2\alpha^k(l(y,w^k,\omega^k)-l(y,w^{k+1},\omega^k))\\ &-2\alpha^k(l(v^{k+1},z,\omega^k)-l(v^k,z,\omega^k)).\end{aligned}$$

Note that both the left- and right-hand side are random variables since we deal with a randomized algorithm and thus we can take conditional expectations on both sides. We now take conditional expectation with respect to $\mathcal{F}^k$ on both sides and set $(y,z)=(y^*,z^*)$, where (y^*,z^*) is a saddle point of function $\mathbb{E}[l(v,w,\omega)]$. The saddle-point exists due to the lower and upper semi-continuity assumptions. We obtain

$$\begin{aligned}\mathbb{E}[\|v^{k+1}-y^*\|^2+\|w^{k+1}-z^*\|^2|\mathcal{F}^k] \leq &\|v^k-y^*\|^2+\|w^k-z^*\|^2\\ &-2\alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k])\\ &-2\alpha^k(\mathbb{E}[l(y^*,w^k,\omega^k)-l(y^*,w^{k+1},\omega^k)|\mathcal{F}^k])\\ &-2\alpha^k(\mathbb{E}[l(v^{k+1},z^*,\omega^k)-l(v^k,z^*,\omega^k)|\mathcal{F}^k]).\end{aligned}$$

From the second condition in Assumption 1 we get $l(y^*,w^k,\omega)-l(y^*,w^{k+1},\omega)\leq c\|w^k-w^{k+1}\|$ and $l(v^{k+1},z^*,\omega)-l(v^k,z^*,\omega)\leq c\|v^{k+1}-v^k\|$ and as a result

$$\mathbb{E}[l(y^*,w^k,\omega)-l(y^*,w^{k+1},\omega)|\mathcal{F}^k]\leq c\mathbb{E}[\|w^k-w^{k+1}\||\mathcal{F}^k]$$

and

$$\mathbb{E}[l(v^{k+1},z^*,\omega)-l(v^k,z^*,\omega)|\mathcal{F}^k]\leq c\mathbb{E}[\|v^{k+1}-v^k\||\mathcal{F}^k]$$

holds. Since $w^k\in\mathcal{M}$ we have $\frac{1}{\alpha^k}(w^k-w^{k+1})\in T_{\mathcal{M}}(w^{k+1})$. On the other hand

$$d+\frac{1}{\alpha^k}(w^{k+1}-w^k)\in N_{\mathcal{M}}(w^{k+1}).$$

From the definition of the normal cone it follows that

$$-\left(d+\frac{1}{\alpha^k}(w^{k+1}-w^k)\right)^T\nu \leq 0 \text{ for every } \nu \in T_{\mathcal{M}}(w^{k+1}).$$

Setting $\nu = \frac{1}{\alpha^k}(w^k - w^{k+1})$ yields

$$\left\|\frac{1}{\alpha^k}(w^k-w^{k+1})\right\|^2 \leq d^T\left(\frac{1}{\alpha^k}(w^k-w^{k+1})\right) \leq \|d\|\left\|\frac{1}{\alpha^k}(w^k-w^{k+1})\right\|.$$

We conclude that $\|\frac{1}{\alpha^k}(w^k-w^{k+1})\| \leq \|d\|$ for all $-d \in \partial_w l(y^*, w^{k+1}, \omega)$.

Now by the first condition in Assumption 1 we have $\|\frac{1}{\alpha^k}(w^k-w^{k+1})\| \leq c$ and as a result

$$\mathbb{E}[l(y^*,w^k,\omega)-l(y^*,w^{k+1},\omega)|\mathcal{F}^k] \leq c\mathbb{E}[\|w^k-w^{k+1}\||\mathcal{F}^k] \leq \alpha^k c^2.$$

Similar arguments show $\mathbb{E}[l(v^{k+1},z^*,\omega)-l(v^k,z^*,\omega)|\mathcal{F}^k] \leq \alpha^k c^2$. We obtain

$$\begin{aligned}\mathbb{E}[\|v^{k+1}-y^*\|^2+\|w^{k+1}-z^*\|^2|\mathcal{F}^k] \leq & \|v^k-y^*\|^2+\|w^k-z^*\|^2 \\ & -2\alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k]) \\ & +4(\alpha^k)^2c^2.\end{aligned}$$

Clearly, we have that $\sum_{k=0}^\infty 4(\alpha^k)^2c^2 < \infty$ since $\sum_{k=0}^\infty (\alpha^k)^2 < \infty$. Note that $\mathbb{E}[l(y^*,w,\omega)] \leq \mathbb{E}[l(y^*,z^*,\omega)] \leq \mathbb{E}[l(v,z^*,\omega)]$ holds for all $(v,w) \in \mathcal{N}\times\mathcal{M}$. Consider $Y^k = \|v^k-y^*\|^2+\|w^k-z^*\|^2$, $W^k = 4(\alpha^k)^2c^2$ and $Z^k = 2\alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k])$. We note that $Z^k \geq 0$. Since

$$Z^k = 2\alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,z^*,\omega^k)+l(y^*,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k]).$$

By Proposition 3 we have that

$$\sum_{k=0}^\infty \alpha^k(\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,z^*,\omega^k)+l(y^*,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k]) < \infty,$$

which implies

$$\liminf_{k\to\infty} \mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,z^*,\omega^k)+l(y^*,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k] = 0$$

and we have that $\|v^k-y^*\|^2+\|w^k-z^*\|^2$ converges to some number $u = u(y^*,z^*)$. Because $\mathbb{E}[l(v^k,z^*,\omega^k)-l(y^*,z^*,\omega^k)|\mathcal{F}^k] \geq 0$ and $\mathbb{E}[l(y^*,z^*,\omega^k)-l(y^*,w^k,\omega^k)|\mathcal{F}^k] \geq 0$ for all k we conclude that there exists a subsequence $S(y^*,z^*)$ such that

$$\lim_{\substack{k\to\infty \\ k\in S(y^*,z^*)}} \mathbb{E}[l(v^k,z^*,\omega)-l(y^*,z^*,\omega)] = 0,$$

$$\lim_{\substack{k\to\infty \\ k\in S(y^*,z^*)}} \mathbb{E}[l(y^*,z^*,\omega)-l(y^*,w^k,\omega)] = 0$$

and

$$\lim_{k\to\infty} \|v^k - y^*\|^2 + \|w^k - z^*\|^2 = u(y^*, z^*). \tag{EC.12}$$

Let $V^* = \arg\min_v\{\max_w \mathbb{E}[l(v,w,\omega)]\}$, $W^* = \arg\max_w\{\min_v \mathbb{E}[l(v,w,\omega)]\}$ and $S(y^*,w^*)$ as defined above with $y^* \in V^*$, $z^* \in W^*$. Sets V^* and W^* are non-empty since there exists a saddle-point. Since $\mathcal{N}\times\mathcal{M}$ is compact, $\{(v^k,w^k)\}_{k\in S(y^*,w^*)}$ has a convergent subsequence C with the limit $(\bar{v},\bar{w})$. Since $\mathbb{E}[l(v,w,\omega)]$ is lower (upper) semi-continuous in v (in w), it follows

$$\lim_{\substack{k\to\infty\\k\in C}} \mathbb{E}[l(v^k,z^*,\omega)] \geq \mathbb{E}[l(\bar{v},z^*,\omega)]$$

and

$$\lim_{\substack{k\to\infty\\k\in C}} \mathbb{E}[l(y^*,w^k,\omega)] \leq \mathbb{E}[l(y^*,\bar{z},\omega)].$$

On the other hand, since $C \subseteq S(y^*,z^*)$, it follows

$$\lim_{\substack{k\to\infty\\k\in C}} \mathbb{E}[l(v^k,z^*,\omega)] = \mathbb{E}[l(y^*,z^*,\omega)]$$

and

$$\lim_{\substack{k\to\infty\\k\in C}} \mathbb{E}[l(y^*,w^k,\omega)] = \mathbb{E}[l(y^*,z^*,\omega)].$$

Since (y^*,z^*) is a saddle point, it follows $\mathbb{E}[l(y^*,z^*,\omega)] = \mathbb{E}[l(\bar{y},z^*,\omega)] = \mathbb{E}[l(y^*,\bar{z},\omega)] = \mathbb{E}[l(\bar{y},\bar{z},\omega)]$, which implies $\bar{y}\in V^*$ and $\bar{z}\in W^*$. From (EC.12) it follows that $\lim_{k\to\infty}\|v^k-\bar{y}\|^2 + \|w^k-\bar{z}\|^2$ exists and we also know that

$$\lim_{\substack{k\to\infty\\k\in C}} \|v^k-\bar{y}\|^2 + \|w^k-\bar{z}\|^2 = 0.$$

This implies that $\lim_{k\to\infty}\|v^k-\bar{y}\|^2 + \|w^k-\bar{z}\|^2 = 0$, i.e., the entire sequence $\{(v^k,w^k)\}$ converges to a point $\{(\bar{v},\bar{w})\}$ such that $\bar{y}\in V^*$ and $\bar{z}\in W^*$.

Proof of Theorem 3

Throughout this proof we write $y_{[t,t']}$ to denote the set of variables $\{y_t, y_{t+1},\ldots,y_{t'}\}$ for $0\leq t\leq t'\leq T$. We define operator $H:\mathcal{Y}^T\to\mathcal{Y}$ as $Hy_{[0,T-1]} = \frac{1}{T}\sum_{t=0}^{T-1} y_t$ and operator $K:\mathcal{Y}\to\mathcal{Y}^T$ as $Ky_t = y_t e'$, where e is a vector of all ones of dimension T. Note that H is the orthogonal projection from $\mathcal{Y}^T$ on $\mathcal{Y}$ and that any $w_{[0,T-1]}$ defined by $w_{[0,T-1]} = y_{[0,T-1]} - KHy_{[0,T-1]}$ is orthogonal to $KHy_{[0,T-1]}$. We introduce subspace $\mathcal{N}\subset\mathcal{Y}$ defined by $\mathcal{N} = \{y\in\mathcal{Y}: KHy = y\}$ and the subspace $\mathcal{M}\subset\mathcal{Y}$ given by $\mathcal{M} = \{y\in\mathcal{Y}: KHy = 0\}$.

Every $y\in\mathcal{Y}$ can be written as $y = u+v$ for $(u,v)\in\mathcal{M}\times\mathcal{N}$, where $u = y - KHy$ and $v = KHy$. This holds since $(KH)^2 = KH$, KH is self-adjoint and $\langle u,v\rangle = 0$. By induction since $(KH)^2 = KH$

it easily follows that $w^k \in \mathcal{M}$ for all k.

For each $\omega^k \in \Omega, k = 0, 1, \ldots,$ we have

$$-w^k - \frac{1}{\alpha^k}\left(y^{k+1} - KHy^k\right) \in \partial\bar{h}(y^{k+1}, \omega^k) + N_{\mathcal{Y}}(y^{k+1}). \tag{EC.13}$$

Noting that $\langle u, v\rangle = 0$ for all $v \in \mathcal{N}$ and $u \in \mathcal{M}$, we can rewrite $\bar{h}(y,\omega) + \langle y, w\rangle$ as $\bar{h}(u+v,\omega) + \langle u, w\rangle$ for every $w \in \mathcal{M}$, $y \in \mathcal{Y}$ for some $(u,v) \in \mathcal{M} \times \mathcal{N}$.

Our claim is that for any fixed k (KHy^{k+1}, w^{k+1}) produced by Steps 2 and 3 of the procedure in the theorem solves the problem

$$(\bar{v}^{k+1}, \bar{w}^{k+1}) = \underset{v\in\mathcal{N}, w\in\mathcal{M}}{\operatorname{argminimax}} \left(l(v, w, \omega^k) + \frac{1}{2\alpha^k}\left\|v - KHy^k\right\|^2 - \frac{1}{2\alpha^k}\left\|w - w^k\right\|^2 \right), \tag{EC.14}$$

where $l(v,w,\omega) = \min_{u\in\mathcal{M}}\{\bar{h}(u+v,\omega) + \delta_{\mathcal{Y}}(u+v) + \langle u, w\rangle\}$ and $\delta_{\mathcal{Y}}(y) = 0$ for $y \in \mathcal{Y}$ and $\delta_{\mathcal{Y}}(y) = \infty$ otherwise. Note that $l(v,w,\omega)$ is convex in v and concave in w for every $\omega \in \Omega$. It is also lower, upper semi-continuous in v, w, respectively, and therefore fulfills the assumptions of Theorem 2. These properties can be shown by using basic mathematics (despite some arguments being technical, we omit these details).

We have that $(x,u) \in \partial l(v,w,\omega^k)$ is the same as $x - w \in \partial(\bar{h}(u+v,\omega^k) + \delta_{\mathcal{Y}}(u+v)) = \partial\bar{h}(u+v,\omega^k) + N_{\mathcal{Y}}(u+v)$ (see §35 and §23 of Rockafellar 1970, and Rockafellar and Wets 1991, Theorem 5.1. with proof).

Let k be fixed and y^k, w^k be the current point of the procedure in the theorem. The solution to Problem (EC.14) at this point satisfies

$$(\frac{1}{\alpha^k}(KHy^k - \bar{v}^{k+1}), \frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k)) \in \partial l(\bar{v}^{k+1}, \bar{w}^{k+1}, \omega^k),$$

where $\partial l = \partial l_v \times \partial l_w$. This gives

$$x - w = \frac{1}{\alpha^k}(KHy^k - \bar{v}^{k+1}) - \bar{w}^{k+1}$$

and

$$u + v = \frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k) + \bar{v}^{k+1}$$

which leads to

$$\frac{1}{\alpha^k}(KHy^k - \bar{v}^{k+1}) - \bar{w}^{k+1} \in \partial\bar{h}(\frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k) + \bar{v}^{k+1}, \omega^k) + N_{\mathcal{Y}}(\frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k) + \bar{v}^{k+1}).$$

Let now $y^{k+1} = \frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k) + \bar{v}^{k+1}$ be the point obtained from Step 2. We then get

$$-\bar{w}^{k+1} - \frac{1}{\alpha^k}(\bar{v}^{k+1} - KHy^k) \in \partial\bar{h}(y^{k+1}, \omega^k) + N_{\mathcal{Y}}(y^{k+1}). \tag{EC.15}$$

We have $y^{k+1} = u + v$ for $v = \bar{v}^{k+1} = KHy^{k+1}$ and $u = \frac{1}{\alpha^k}(\bar{w}^{k+1} - w^k) = y^{k+1} - KHy^{k+1}$. It follows that $\langle u, v\rangle = 0$.

We now consider the subgradient $-w^k - \frac{1}{\alpha^k}(y^{k+1} - KHy^k)$ obtained in (EC.13). We rewrite it as $-w^k - \frac{1}{\alpha^k}(u + KHy^{k+1} - KHy^k)$, which is the same as $-w^k - \frac{u}{\alpha^k} - \frac{1}{\alpha^k}(\bar{v}^{k+1} - KHy^k)$. Substituting u yields

$$-w^k - \frac{1}{\alpha^k}(y^{k+1} - KHy^{k+1}) - \frac{1}{\alpha^k}(\bar{v}^{k+1} - KHy^k) \in \partial\bar{h}(y^{k+1}, \omega^k) + N_{\mathcal{Y}}(y^{k+1}). \tag{EC.16}$$

Comparing (EC.16) to (EC.15), we note that w^{k+1} obtained from the procedure in the theorem equals $\bar{w}^{k+1}$ obtained from (EC.14), if $w^{k+1} = w^k + \frac{1}{\alpha^k}(y^{k+1} - KHy^{k+1})$. But this is precisely the update step taken in Step 3 of the procedure in the theorem. The result now follows from Theorem 2.

EC.4. Proofs of Corollaries

Proof of Corollary 1

Problem (27) is a lower bound on Problem (9) by Theorem 1 and convex by Proposition 1. Additionally, Assumption 1 holds for Problem (9) if maximum system load is assumed to be finite and the slope for the cost functions is bounded (we assume both conditions to be fulfilled in realistic problem instances). Then we can rewrite Problem (27) as

$$\min_z \mathbb{E}_{L_0,\ldots,L_{T-1}}\left[C^D(z) + \min_{\bar{x}\in\bar{\mathcal{X}}} \sum_{t=0}^{T-1} C_t^C(R_t, \bar{x}_t, b_t)\right]$$

and apply the procedure of Theorem 2, which also provides convergence to the optimal z^*.

Proof of Corollary 2

The proof follows the proof of Theorem 6.1. in Rockafellar and Wets (1991), and is only a simple adoption for the stochastic case.

EC.5. Proofs of Propositions

Proof of Proposition 1

We first show convexity of V_t for all t by induction. Throughout the proof, we write $(R_t, l_0, \ldots, l_t)$ instead of S_t.

Cost function $C_t^C(R_t, x_t, b_t)$ is convex, because c_i^g is convex in $R_{t,i}, x_{t,i}^d$, c_i^s is convex in $x_{t,i}^{\mathrm{on}}$ for continuous x_t, and $c_t^+[b_t]^+ + c_t^-[-b_t]^+$ is convex in b_t. We denote the set defined by Constraints (3), (4), the action space, and $x_{t,i} \in [0,1]^3$ for all i by $\bar{\mathcal{X}}_t(z, R_t, l_0, \ldots, l_t)$. $\bar{\mathcal{X}}_t$ is a closed convex set for every $(l_0, \ldots, l_t) \in \mathcal{L}_t$, if $\phi_t^{-1}(\cdot, z)$ is convex in z. As a result $C_t^C(R_t, x_t, b_t) > -\infty$ for $x_t \in \bar{\mathcal{X}}_t(z, R_t, l_0, \ldots, l_t)$.

Base case. Assume $V_T = 0$. For $(R_{T-1}, l_0, \ldots, l_{T-1})$ and z the value function in $T-1$ is defined by $V_{T-1}(z, R_{T-1}, l_0, \ldots, l_{T-1}) = \min_{x_{T-1}} C^C_{T-1}(R_{T-1}, x_{T-1})$, where $x_{T-1} \in \bar{\mathcal{X}}_{T-1}(z, R_{T-1}, l_0, \ldots, l_{T-1})$. Since $\bar{\mathcal{X}}_{T-1}$ is convex for given $l_0, \ldots, l_{T-1}$, $V_{T-1}(z, R_{T-1}, l_0, \ldots, l_{T-1})$ is convex in $S_{T-1} = (R_t, l_0, \ldots, l_t)$ and z.

Induction step. The induction assumption is that $V_{t+1}(z, R_{t+1}, l_0, \ldots, l_{t+1})$ is convex in R_{t+1} and z for all $(l_0, \ldots, l_{t+1})$. We show that $V_t(z, R_t, l_0, \ldots, l_t)$ is convex in R_t and z for every $(l_0, \ldots, l_t)$.

Because R^M is linear in each coordinate, and the proposition assumes $\phi_t^{-1}(\cdot, z)$ to be convex in z, $\mathbb{E}_{L_{t+1}(z)}[V_{t+1}(z, R^M(R_t, x_t), l_0, \ldots, l_t, L_{t+1}(z))]$ is convex in R_t, x_t and z for every $(l_0, \ldots, l_{T-1})$. We next consider

$$\min_{x_t \in \bar{\mathcal{X}}_t} C^C_t(R_t, x_t, b_t) + \mathbb{E}_{L_{t+1}(z)}[V_{t+1}(z, R^M(R_t, x_t), l_0, \ldots, l_t, L_{t+1}(z))].$$

Again, since $\bar{\mathcal{X}}_t(z, R_t, l_0, \ldots, l_t)$ is a convex set for each $(l_0, \ldots, l_t)$ and both summands of the minimization argument are convex in R_t and z for each $(l_0, \ldots, l_t)$, we conclude that $V_t(z, R_t, l_0, \ldots, l_t)$ is convex in R_t and z for each $(l_0, \ldots, l_t)$.

Convexity of the continuous relaxation of the integrated Problem (9) follows from noting that $C^D(z)$, $V_0(z, S_0)$ and the domain of z are convex.

Proof of Proposition 2

The statement follows from noting that the right hand side of inequality (29) is the Lagrangian relaxation of the expected value in (27) obtained by relaxing Constraint (3).