Optimization of Battery Charging and Purchasing at Electric Vehicle Battery Swap Stations

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***Abstract*—A promising model for providing charging services to owners of electric vehicles is a network of battery swap stations. A swap station operator will need to decide how many batteries to purchase initially, and when, based on dynamic electricity rates, to charge the batteries. We propose a dynamic programming model to assist in making optimal charging decisions, and a master problem embedding the dynamic program for making the purchasing decision.**

I. Problem Definition

The principal challenge that must be surmounted in order to replace petroleum powered passenger vehicles with electric vehicles (EV's) is in providing rapid and flexible charging of batteries. The EV's currently coming onto the market have a range of about 100 miles, which is sufficient for the commute and other daily driving of the vast majority of the world's drivers, but does not allow for longer planned trips. More significantly, the 100 mile limit is close enough to the daily travel distance of many drivers that they would understandably feel constrained and unable to either respond to emergency driving needs or to be spontaneous in their driving habits. This phenomenon is known in the industry as range anxiety.

Unfortunately, current battery technology does not allow for charging in less than half an hour (at best), so charging stations analogous to gas stations, where energy can be restocked to the vehicle in a few minutes, do not currently seem possible. However, if the batteries themselves can be swapped in and out of the EV's, a discharged battery can be replaced with a fully charged one very rapidly. Such swapping stations would require that batteries be easily accessible on the vehicle, and replaceable by an automated process. More problematically, such stations would require a standardization of batteries and interfaces that does not currently exist - a swap station would not be feasible with more than a few different kinds of batteries, just as gasoline stations typically only carry 3 or 4 types of fuel. But despite the difficulties, a company, Better Place, is in the process of deploying a network of swap stations.

In order to serve Better Place's needs, and the needs of the imagined swap station manager of the future (a future with a mature EV market), we propose a dynamic programming model that aids in determining policies for charging batteries at swap stations. In particular, our model considers the tradeoff between responding to dynamic electricity prices, and charging batteries just when they are needed (regardless of the price) in order to avoid the opportunity cost of committing capital to charging too early. To take an extreme, illustrative example, if the price of electricity is 5 cents per kWh for all of February and 10 cents per kWh for the entire month of March of the same year, it may yet be optimal to charge a battery to satisfy early April demand in late March rather than late February (in order to free up capital to seek greater return elsewhere during the month of March). The accounting of this opportunity cost requires the model to track when each battery in stock was charged; we use a "First-In First-Out" rule to insure that the opportunity cost properly accounts for the information known at the time of each charging decision. Additionally, using the model, we propose to determine the optimal number of batteries to hold at a station. We will demonstrate these models using data from the San Francisco Bay Area, where Better Place is planning to deploy charging stations in the near future. Note that we do not consider battery aging or other long term changes in the properties of the charging equipment, as our primary focus is on the short term decisions of swap station managers.

The primary contributions of our work are in the modeling of a new problem in an emerging and vital application field, and in the "FIFO" rule for accounting opportunity cost in an environment of dynamic prices.

II. Model

We present a dynamic programming formulation that minimizes the sum of electricity costs (incurred while charging batteries), opportunity cost (incurred on capital previously committed to charging batteries), and penalties for failing to satisfy customer demand. It is a backlogging model, i.e., customers who can not be served at the time of their arrival are assumed to wait until they can be served. Principally, the model sets up a tradeoff between minimizing opportunity cost (charging "just in time") and buying electricity when it is cheapest (and potentially stockpiling charged batteries far in advance of demand). An important wrinkle to the problem is that, in order to assess opportunity cost in each period, due to variable electricity prices, it must be known when each of the batteries in stock was charging. Thus, the state captures both the total number of charged batteries in stock (less backlogged demand) and the number of batteries that were set to charge in each previous period.

A. Formulation

We first assume that the initial number of batteries is known in advance.

We have the following state space:

S_t :Net Inventory Position in Period t
(Total number of charged batteries, or
-1$*$(Units of backlogged demand) if there
is unmet past demand)

$\hat{S}_t$:Vector Composed of Values S_{ti}, $\forall \;\; 0 \le i \le t-1$;
S_{ti} is the number of batteries charged in period i
that are still remaining and available for customers
in period t

There is a simple action variable X_t, which represents the number of batteries to start charging at time t. There is a simple exogenous stochastic variable W_t, corresponding to the demand for batteries in period t. (In a given period, demand occurs before decisions are made.)

The remaining parameters are:

L :Number of Periods Required to Charge a Battery
U :Total Number of Batteries in Initial Stock
p_t :Underage Cost per Unit Demand in Period t
γ :Discount Rate**
r :Opportunity Cost Rate Parameter
D :Purchase Price per Battery
h_t :The cost (of electricity) to charge one battery during period t.

**Discount rates are typically used in dynamic programs of this type to reflect the time value of money, which accounts for inflation, opportunity cost, uncertainty, and other factors.

The optimality equation reads:

$$
\begin{aligned}
&V_t(S_t, \hat{S}_t) = \\
&\min_{\substack{X_t \ge 0 \\ X_t + S_t + \sum_{i=t-L+1}^{t-1} S_{ti} \le U}} \{X_t \sum_{j=1}^{L} h_{t+j}\gamma^j + \tilde{C}_t(S_t) \\
&+ r[\sum_{i=0}^{t-L} S_{ti}(\sum_{j=i+1}^{i+L} h_j) + \sum_{i=t-L+1}^{t-1} S_{ti}(\sum_{j=i+1}^{t-1} h_j)] \\
&\qquad +\gamma\mathbb{E}[V_{t+1}(S_{t+1}, \hat{S}_{t+1})]\}
\end{aligned}
$$

where $\tilde{C}_t(a) = p_t(W_t - a)^+$.

The transition functions are governed by:

$$
\begin{aligned}
S_{t+1} =& S_t + S_{t,t-L+1} - W_{t+1} \\
S_{t+1,i} =& (S_{ti} - (W_{t+1} + (S_t)^- - \sum_{j=0}^{i-1} S_{tj})^+)^+, \\
& 0 \le i \le t-L \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
S_{t+1,i} =& S_{ti} \qquad t-L+1 \le i \le t-1 \\
S_{t+1,i} =& X_t \qquad i = t
\end{aligned}
$$

The S_{ti} state variables, tracking the number of batteries set to charge at time i that have yet to be distributed at time t, are updated in a First-In-First-Out fashion, so as to always assess opportunity cost on those batteries that were set to charge most recently. This is the appropriate method for calculating opportunity cost because, when the later batteries were set to charge, the decision maker knew about the earlier batteries - that is, the earlier batteries were already available to satisfy any demand that the later batteries would be able to satisfy. Thus, opportunity cost should continue to be charged on a battery until demand requires that the battery be used, which is when there are no older batteries available to satisfy that demand. This FIFO rule is represented by (1). The inner parentheses contain the sum of this period's demand and backlogged demand, less the total charged battery stock that was set to charge before the cohort of batteries currently under consideration. The outer parentheses contain the difference between the number of available charged batteries in the cohort currently under consideration and demand from this and earlier periods that could not be absorbed by batteries with charges of earlier vintage. For example, if $\hat{S}_t$ is $(0, 2, 6, 3, 5)$, W_t is 4, L is 2, and X_t is 1, then $\hat{S}_{t+1}$ is $(0, 0, 4, 3, 5, 1)$.

B. Solution Methodology

We approximate solutions to the dynamic program by fitting a separable piecewise linear approximation to the value function. In the optimality equation, the term $\mathbb{E}[V_{t+1}(S_{t+1}, \hat{S}_{t+1})]$ is replaced by the sum of a set of piecewise linear functions, each of which depends on only one of the state parameters $(S_{t+1}, \hat{S}_{t+1})$. This yields a compact estimate of the value function, over which we can easily optimize X_t, the number of batteries to charge in the period. Each component of the piecewise linear approximation is defined by a set of breakpoints and a corresponding set of slopes, such that the slope of the function to the right of each breakpoint is defined by the corresponding value in the slope vector.

To obtain the best function approximation, we simulate many periods (on the order of a few days, at 24 periods per day), using a random demand function and chosen initial values for the state variables at time 0 and incumbent value function approximation. At each iteration of the simulation, we find a derivative (using finite differences) of the function with respect to each state coordinate, and add the resulting breakpoints and slopes to the approximation. At the end of the time horizon, the state is reset to the initial value at time zero and the entire loop is repeated several times, continually adding breakpoints and slopes to the separable piecewise linear approximation function. At the end, after the final value function approximation is obtained, a separate simulation is executed that evaluates the objective value of the decisions produced by the value function. This simulation is very similar to the overall simulation optimization algorithm except that the value function is not being updated during these steps.

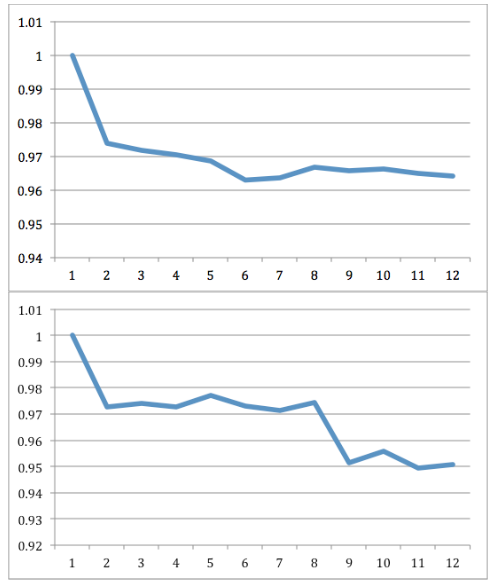

Fig. 1. Two representative runs of the approximate dynamic program, with the y-axis values scaled.

III. Data and Testing

We test the model on a hypothetical network of swap stations in the San Francisco Bay Area. One typical day of hourly electricity prices in June is extrapolated from historical price data. The discount rate and opportunity cost of capital rate are both set at 5%. The underage penalty (for being unable to satisfy a unit of demand) is set at \$20. Daily demand is roughly estimated to be 2,000 charges/day, and this quantity is broken down into hourly demand using hourly refueling data from refueling stations in Brazil. Additionally, a random component is added to the demand in each period (the random component is normal, with the demand for each period bounded below by zero). We set L, the number periods required to charge a battery, at 7, as the battery for the Renault Fluence Z.E., one of the models that Better Place is compatible with, charges in 6 to 8 hours at 220V. Faster charging is possible at higher voltages, but degrades the battery more rapidly, so we assume that a swap station will avoid such rapid charging. Because the Fluence Z.E.'s battery holds a total charge of 22 kWh, we multiply the electricity prices, estimated as described above, by 22/7 in order to generate the h_t parameters. [4]

IV. Conclusion

Further testing, including sensitivity testing on the underage penalty and interest rates, should be performed to better understand the properties and usefulness of the model.

The ultimate problem is to find how many batteries a swap station should hold in stock. Note that the value function V_t depends on U (the total stock of batteries), i.e., V_t^U. This number (of batteries) can be found by minimizing the sum of the value function presented above and the initial purchase cost of the batteries (as there is only one purchase, holding cost can be assumed to be contained in this purchase price):

$$\min_{U}\{V_0^U(S_0,\hat{S}_0)+D*U\},$$

where S_0 and $\hat{S}_0$ are set to reasonable estimates of their values at steady state.

We plan to investigate solution techniques that extend from standard ADP methods, and incorporate insights from optimization via simulation methods.

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